Storage Networks
Optimizing Heterogeneous Data Distribution

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Literature

- André Brinkmann, Kay Salzwedel, Christian Scheideler, Compact, Adaptive Placement Schemes for Non-Uniform Capacities, 14th ACM Symposium on Parallelism in Algorithms and Architectures 2002 (SPAA 2002)
- Christian Schindelhauer, Gunnar Schomaker, Weighted Distributed Hash Tables, 17th ACM Symposium on Parallelism in Algorithms and Architectures 2005 (SPAA 2005)
- Christian Schindelhauer, Gunnar Schomaker, SAN Optimal Multi Parameter Access Scheme, ICN 2006, International Conference on Networking, Mauritius, April 23-26, 2006

The Problem in Storage Networks

- $A_{d,s}$: Number of bytes of document d assigned to storage s
- Distributed Algorithm:
  - Use DHHT to split each document into |S| parts
  - Store corresponding blocks on the server
- Can be also achieved by a centralized algorithm
- Straight forward generalization of fair balance
  - Distribute data according to a (m x n) distribution matrix A where
    $\forall s : \sum_d A_{d,s} \leq |s|$ and $\forall d : \sum_s A_{d,s} = |d|$
- DHHT
  - assigns $A_{d,s}(1 \pm \varepsilon)$ elements of d ∈ D to s ∈ S
  - Information needed: File-IDs, Server-IDs, and matrix A
  - If matrix A changes to A´ $(1+\varepsilon)\sum_{d,s} \left| A_{d,s} - A'_{d,s} \right|$
    data reassignments are needed

How to Balance

- A fair balance like $A_{d,s} = |d| \cdot \frac{|s|}{\sum_{s' \in S} |s'|}$ is not always the best to do
- Servers are different in capacity and bandwidth
- Documents are different in size and popularity

- Goal: Optimize Time

- Assumption
  - All sizes can be modeled as real numbers

Which Time ?

- b(s) = bandwidth of server s
  - b(s) = number of bytes per second
- p(d) = popularity of document d
  - p(d) = number of read/write accesses
- Sequential time for a document d and an assignment A

$$\mathrm{SeqTime}_A(d) := \sum_{s \in S} \frac{A_{d,s}}{b(s)}$$

- Parallel time for a document d and an assignment A

$$\mathrm{ParTime}_A(d) := \max_{s \in S} \left\{ \frac{A_{d,s}}{b(s)} \right\}$$

- Observation
  - Popular bytes cause more traffic than less popular once
  - Costs are defined by the traffic per byte

Sequential Time

- Sequential time
  - load all parts of a document from all servers sequentially

$$\text{SeqTime}_A(d) := \sum_{s \in \mathcal{S}} \frac{A_{d,s}}{b(s)}$$

- Worst case sequential time

$$\text{WSeqTime} := \max_d \{\text{SeqTime}_A(d)\}$$

- Average sequential time

$$\text{AvSeqTime} := \sum_{d \in \mathcal{D}} p(d) \ \text{SeqTime}_A(d)$$

- where
  - S: set of servers with bandwidth b(s) and capacity |s| for each server s
  - D: set of documents with size |d| and popularity p(d) for each document

Parallel Time

- Parallel time
  - load all parts of a document from all servers simultaneously

$$\text{ParTime}_A(d) := \max_{s \in \mathcal{S}} \left\{ \frac{A_{d,s}}{b(s)} \right\}$$

- Worst case parallel time

$$\text{WParTime} := \max_d \{\text{ParTime}_A(d)\}$$

- Average parallel time

$$\text{AvParTime} := \sum_{d \in \mathcal{D}} p(d) \ \text{ParTime}_A(d)$$

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  - S: set of servers with bandwidth b(s) and capacity |s| for each server s
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Sequential Bandwidth

- Sequential time
  - load all parts of a document from all servers sequentially

$$\mathrm{SeqTime}_A(d) := \sum_{s \in \mathcal{S}} \frac{A_{d,s}}{b(s)}$$

- Sequential bandwidth
  - download speed of a document d

$$\mathrm{SeqBandwidth}_A(d) := \frac{|d|}{\mathrm{SeqTime}_A(d)}$$

- Worst case sequential bandwidth
  WBandwidth := mind {SeqBandwidthA(d)}
- Average sequential bandwidth
  AvBandwidth := $\sum_{d \in \mathcal{D}} p(d)$ SeqBandwidth(d)
- where
  - S: set of servers with bandwidth b(s) and capacity |s| for each server s
  - D: set of documents with size |d| and popularity p(d) for each document

Parallel Bandwidth

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- Parallel time
  - load all parts of a document from all servers in parallel

$$\mathrm{ParTime}_A(d) := \max_{s \in \mathcal{S}} \left\{ \frac{A_{d,s}}{b(s)} \right\}$$

- Parallel bandwidth
  - download speed of a datum d

$$\mathrm{ParBandwidth}_A(d) := \frac{|d|}{\mathrm{ParTime}_A(d)}$$

- Worst case parallel bandwidth

$$\mathrm{WParBandwidth} := \min_d \{\mathrm{ParBandwidth}_A(d)\}$$

- Average parallel bandwidth time

$$\mathrm{AvParBandwidth} := \sum_{d \in \mathcal{D}} p(d) \; \mathrm{ParBandwidth}_A(d)$$

- where
  - S: set of servers with bandwidth b(s) and capacity |s| for each server s
  - D: set of documents with size |d| and popularity p(d) for each document

Most Reasonable Time Measures

- Minimize the expected sequential time based on popularity of the document:

$$\text{AvSeqTime}(p, A) = \sum_{d \in \mathcal{D}} \sum_{s \in \mathcal{S}} p(d) \frac{A_{d,s}}{b(s)}$$

- Minimize the expected parallel time based on the popularity of the document

$$\text{AvParTime}(p, A) = \sum_{d \in D} \max_{s \in S} \frac{A_{d,s}}{b(s)} p(d)$$

Solution by Linear Program

$$\forall s : \sum_d A_{d,s} \le |s| \qquad \forall d : \sum_s A_{d,s} = |d|$$

Measure	Linear programm	Add. variables	Additional restraint	Optimize
AvSeqTime	yes	—	—	$\min \sum_{s\in\mathcal{S}} \sum_{d\in\mathcal{D}} p(d) \frac{A_{d,s}}{b(s)}$
WSeqTime	yes	m	$\forall d \in \mathcal{D} : \sum_{s\in\mathcal{S}} \frac{A_{d,s}}{b(s)} \le m$	$\min m$
AvParTime	yes	$(m_d)_{d\in\mathcal{D}}$	$\forall s \in \mathcal{S}, \forall d \in \mathcal{D} : \frac{A_{d,s}}{b(s)} \le m_d$	$\min \sum_{d\in\mathcal{D}} p(d) m_d$
WParTime	yes	m	$\forall s \in \mathcal{S}, \forall d \in \mathcal{D} : \frac{A_{d,s}}{b(s)} \le m$	$\min M$
AvSeqBandwidth	no	—	—	$\max \sum_{d\in\mathcal{D}} \frac{p(d)\lvert d\rvert}{\sum_{s\in\mathcal{S}} \frac{A_{d,s}}{b(s)}}$
WSeqBandwidth	yes	m	$\forall d \in \mathcal{D} : \sum_{s\in\mathcal{S}} \frac{A_{d,s}}{\lvert d\rvert b(s)} \le m$	$\min m$
AvParBandwidth	no	$(m_d)_{d\in\mathcal{D}}$	$\forall d \in \mathcal{D} : \sum_{s\in\mathcal{S}} \frac{A_{d,s}}{b(s)\lvert d\rvert} \le m_d$	$\max \sum_{d\in\mathcal{D}} \frac{p(d)}{m_d}$
WParBandwidth	yes	m	$\forall s \in \mathcal{S}, \forall d \in \mathcal{D} : \frac{A_{d,s}}{\lvert d\rvert b(s)} \le m$	$\min m$

How to Describe AvParTime as a LP

AvParTime

$$= \sum_{d \in D} p(d) \cdot \underbrace{\max_{s \in S} \frac{A_{d,s}}{b(s)}}_{m_d}$$

$$= \sum_{d \in D} p(d) \cdot m_d$$

Variables: $A_{d,s}, m_d$

Restraints: $\sum_s A_{d,s} = |d|$

$$\sum_d A_{d,s} \le |s|$$

$$m_d = \max_{s \in S} \frac{A_{d,s}}{b(s)}$$

Additional Restraints

$$\left\{ \begin{array}{l} m_d \ge \frac{1}{b(s_1)} \cdot A_{d,s_1} \\ m_d \ge \frac{1}{b(s_2)} \cdot A_{d,s_2} \\ \vdots \end{array} \right.$$

Example

- **Storage device**
  - s_1: 500 GB, 100 MB/s
  - s_2: 100 GB, 50 MB/s
  - s_3: 1 GB 1000 MB/s
- **Documents**
  - d_1: 100 GB, popularity 1/111
  - d_2: 5 GB, popularity 100/111
  - d_3: 100 GB, popularity 10/111

$A_{d,s}$	s_1	s_2	s_3	Σ
d_1	100	0	0	100
d_2	2	2	1	5
d_3	2	98	0	100
Σ	≤ 500	≤ 100	≤ 1	

	SeqTime	SeqBand width	ParTime	ParBand width
d_1	1000	100	1000	100
d_2	61	82	40	125
d_3	1980	51	1960	51
Av	1864	121	1827	160
Worst case	1980	51	1960	51

Excursion: Linear Programming

- Linear Program (Linear Optimization)
- Given: m × n matrix A

  m-dimensional vector b

  n-dimensional vector c
- Find: n-dimensional vector $x=(x_1, ..., x_n)$
- such that

  - $x \geq 0$, i.e. for all j: $x_j \geq 0$
  - A x = b, i. e. $$\sum_{j=1}^{n}\sum_{i=1}^{m} A_{ij}x_j = b_j$$
  - $z = c^T x$ is minimized, i.e. $z = \sum_{j=1}^{n} c_j x_j$ is minimal

Linear Programming 2

- Linear Programming (LP2)
- Given: m × n matrix A

  m-dimensional vector b

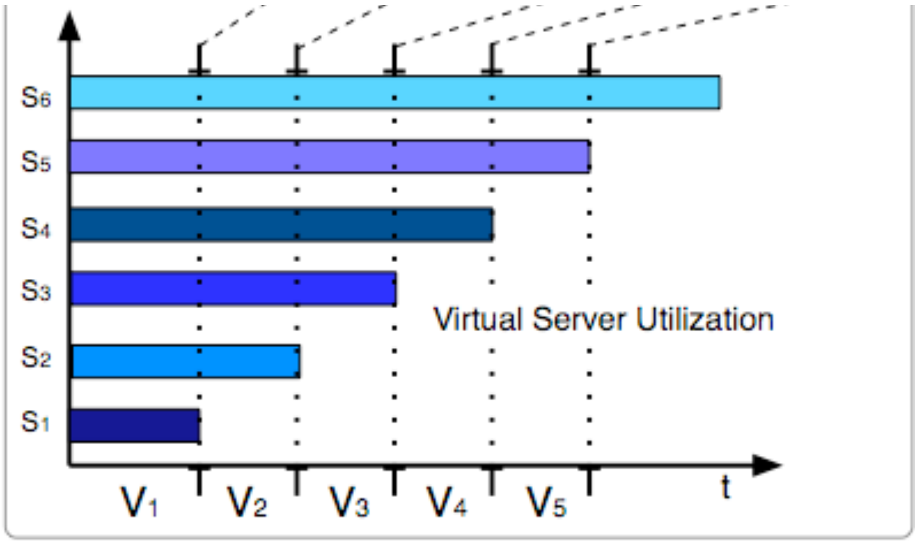
  n-dimensional vector c
- Find: n-dimensional vector $x=(x_1, ..., x_n)$
- such that
  - $x \geq 0$
  - $A x \leq b$
  - $z = c^T x$ is maximal

Performance of Linear Programming

- Worst case time behavior of the Simplex algorithm is exponential
  - A simplex can have an exponential number of edges
- For randomized inputs, the running time of Simplex is polynomial on the expectation
- The Ellipsoid algorithm is a different method with polynomial worst case behavior
  - In practice it is usually outperformed by the Simplex algorithm

ParTime = SeqTime with virtual servers

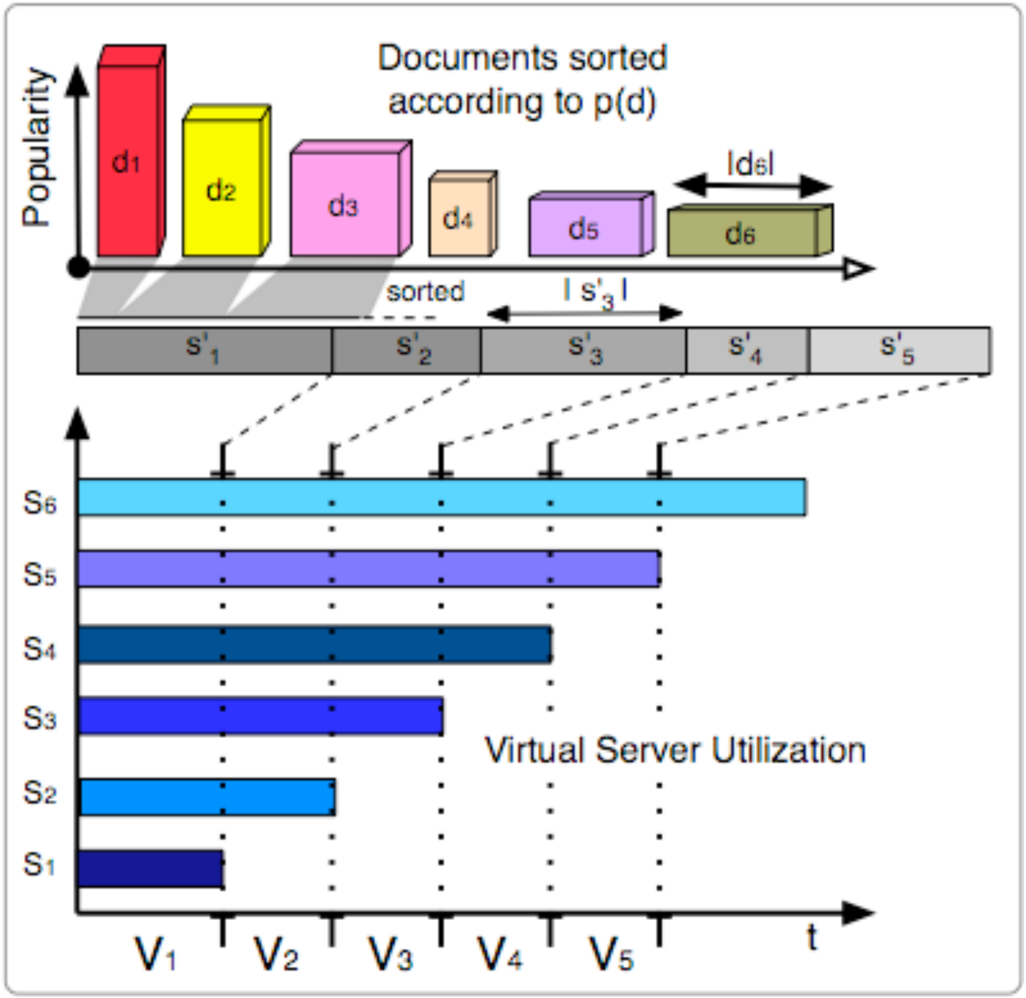
- **Reduce optimal solution for LP of ParTime to the optimal solution of LP of SeqTime**
  - Combining capacity of many disks in parallel
- **Define new sequential virtual servers $s'_1, \ldots, s'_m$**
  - Sort s_i such that $\frac{|s_j|}{b(s_j)} \leq \frac{|s_{j+1}|}{b(s_{j+1})}$
  - Server s'_j parallelizes servers $s_j, \ldots, s_{|S|}$
  - Virtual servers s'_i are then sorted such that $b(s'_i) > b(s'_{i+1})$
  - Size of s'_i:



$$t_j = \frac{|s_j|}{b(s_j)} - \sum_{i=1}^{j-1} t_i \qquad s'_j = b(s'_j) \cdot t_j$$

Solve the LP of AvSeqTime

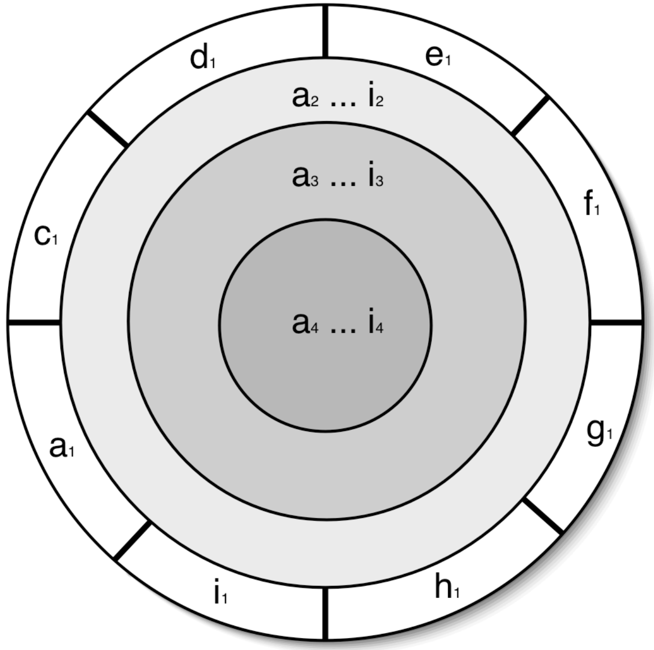
- **Simple optimal greedy solution**
- **Repeat until all documents are assinged:**
  - Assign most popular document on fastest sequential (virtual) server
  - Reduce the storage of the server by the document size and remove the document



Applications in Storage Networks

- **Object storage with different popularity zones**
  - e.g. movies with varying popularities over time
  - Fragmentation is done automatically
  - Includes dynamics for adding and removing documents
  - The same for servers

- **Use different bandwidth**
  - Each disk has different bandwidths
  - Exporting different zone classes as sequential servers



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Optimal File-Distribution in Heterogeneous and Asymmetric Storage Networks

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Department for Information Technology and Electrical Engineering
ETH Zurich, Switzerland

SOFSEM '11
Nový Smokovec

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Distributed Storage is Ubiquitous

Motivation

- Storing files on multiple machines is a common habit
  - P2P overlays basically represent storage networks
  - File-hosting services like Amazon S3, Rapidshare, etc.

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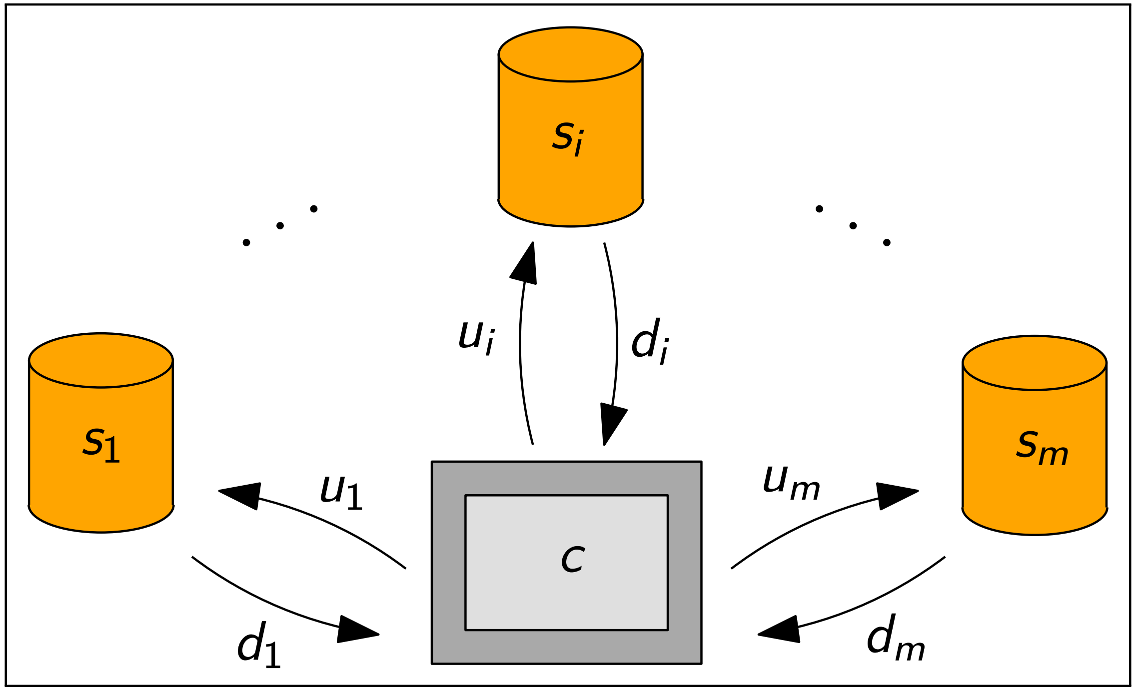
  - File-hosting services like Amazon S3, Rapidshare, etc.
- Gigantic storage requirements by special applications (Google, Amazon, CERN)
- Most existing approaches specifically tailored for data centers with homogeneous and symmetric network connections.
- Little attention to asymmetric bandwidths typical for end-user connections

Synopsis

Distribution Network

Problem Setting

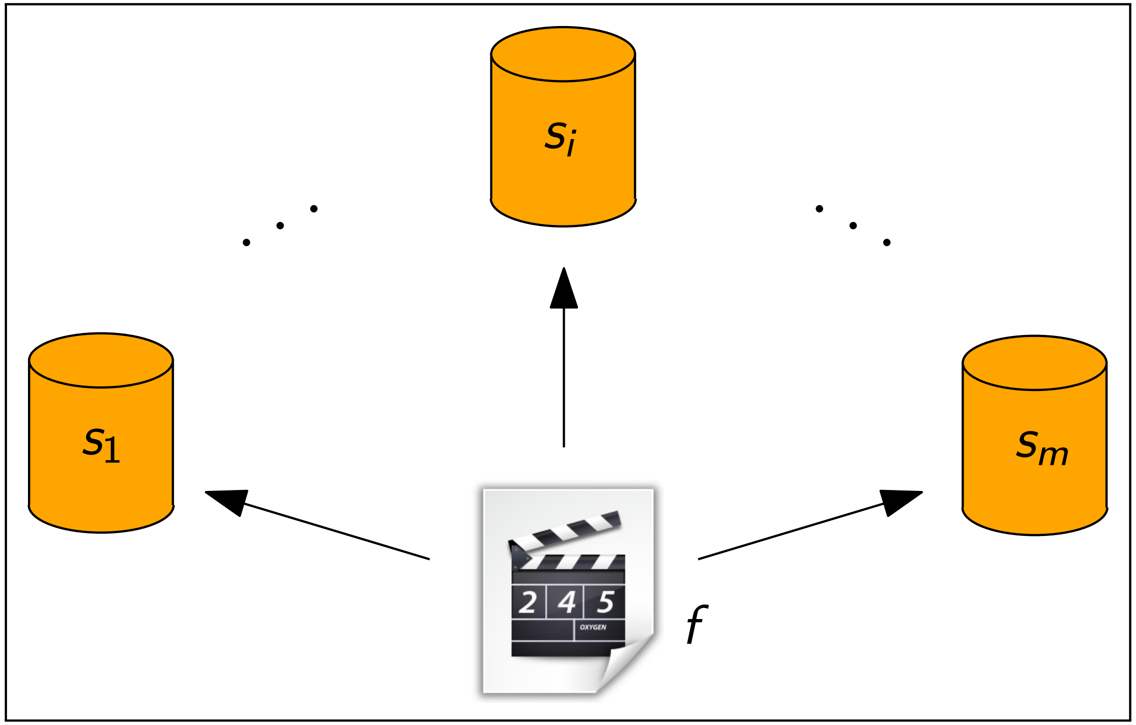


- Servers s_i with $i \in S = \{1, \ldots, m\}$, and one client c
- Each server s_i is characterized by its upload bandwidth u_i and download bandwidth d_i

Distribution Problem

Problem Setting

- How do we split a file f of size $|f|$ into fragments for the servers to minimize the time for one upload and n downloads?



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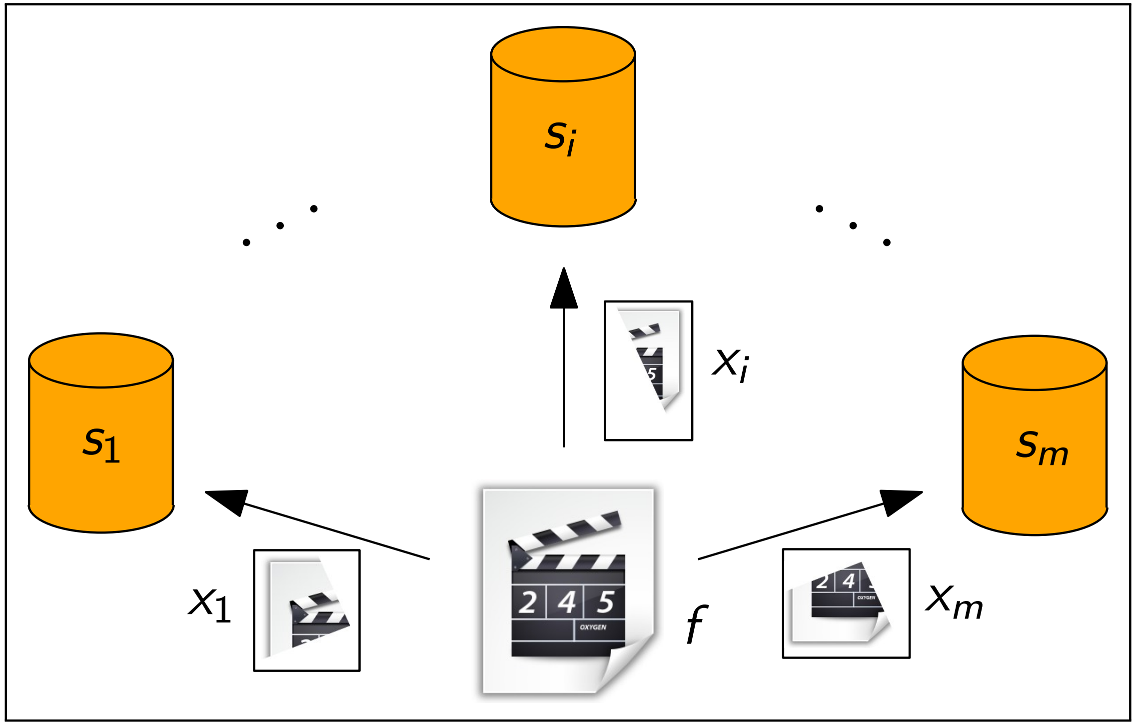

- Distribution vector $\mathbf{x} = (x_1, \ldots, x_m)$ with $\sum x_i = |f|$. x_i is the size of the fragment of server s_i.

Distribution Problem

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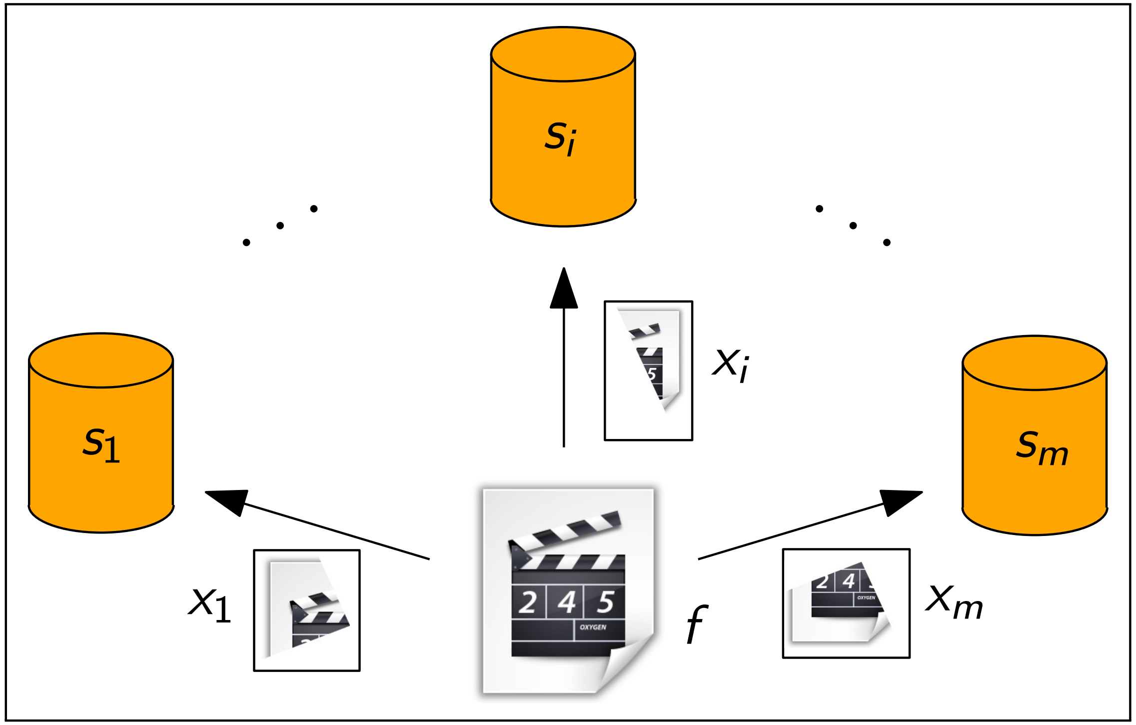

- Distribution vector $\mathbf{x} = (x_1, \ldots, x_m)$ with $\sum x_i = |f|$. x_i is the size of the fragment of server s_i.
- Equivalent simplified problem with $|f| = n = 1$.

Transfer Times

Problem Setting

The quality of a distribution $\mathbf{x}$ is given by the transfer times:

- Upload time:

$$t_u(\mathbf{x}) = \max_{i \in S} \left\{ \frac{x_i}{u_i} \right\}$$

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- Download time:

$$t_d(\mathbf{x}) = \max_{i \in S} \left\{ \frac{x_i}{d_i} \right\}$$

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- Download time:

$$t_d(\mathbf{x}) = \max_{i \in S} \left\{ \frac{x_i}{d_i} \right\}$$

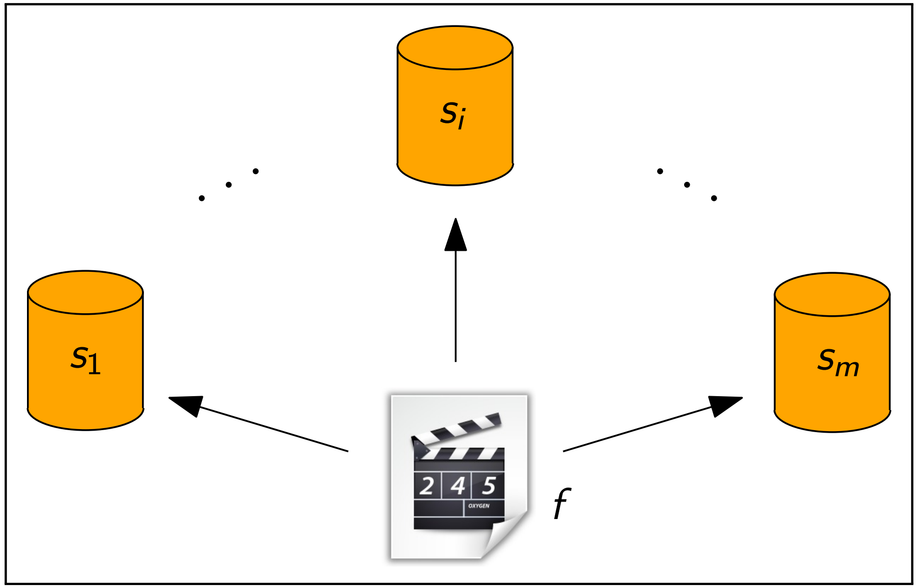
- Total time:

$$\mathrm{val}(\mathbf{x}) = t_u(\mathbf{x}) + (n \cdot)\, t_d(\mathbf{x})$$

Objective

Problem Setting

We are given a distribution network and a file f with size 1.



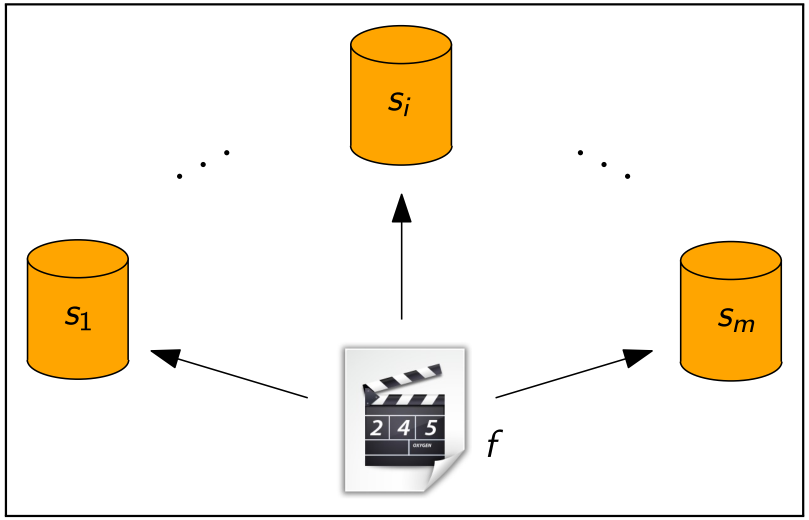
Objective

Find the optimal distribution $\mathbf{x}^*$ that minimizes the total time

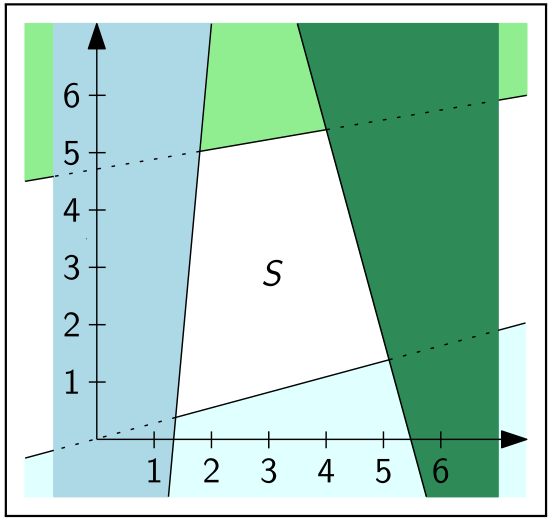
$$\mathrm{val}(\mathbf{x}) = t_u(\mathbf{x}) + t_d(\mathbf{x}) \quad .$$

Important Properties

Problem Setting



$\Rightarrow$



- The distribution problem can be formulated as linear program.
  $\Rightarrow$ Its solution space is a convex region.
  $\Rightarrow$ Every local minimum is global.

Non-Linear Optimization Problem

Linear Program

Upload/download bandwidths u_1 to u_m and d_1 to d_m

File distribution $x = (x_1, \ldots, x_m)$

Non-Linear Program

$$\begin{aligned} \text{minimize} \quad & \max\left\{\frac{x_1}{u_1}, \frac{x_2}{u_2}, \ldots, \frac{x_m}{u_m}\right\} + \max\left\{\frac{x_1}{d_1}, \frac{x_2}{d_2}, \ldots, \frac{x_m}{d_m}\right\} \\ \text{subject to} \quad & \sum_{i=1}^{m} x_i = 1 \\ & x_i \geq 0 \quad \text{for all } i \in \{1, \ldots, m\} \end{aligned}$$

Linear Optimization Problem

Linear Program

Linear Program

$$
\begin{aligned}
\text{minimize} \quad & t_u + t_d \\
\text{subject to} \quad & \sum_{i=1}^{m} x_i = 1 \\
& \frac{x_i}{u_i} \leq t_u \qquad \text{for all } i \in \{1, \ldots, m\} \\
& \frac{x_i}{d_i} \leq t_d \qquad \text{for all } i \in \{1, \ldots, m\} \\
& x_i \geq 0 \qquad \text{for all } i \in \{1, \ldots, m\}
\end{aligned}
$$

$\boxed{\textbf{Dimension: } m + 2}$

Synopsis

The Distribution Problem as Flow Problem

Maximum Flow in Distribution Problems

- Assume we are given upload time t_u and download time t_d

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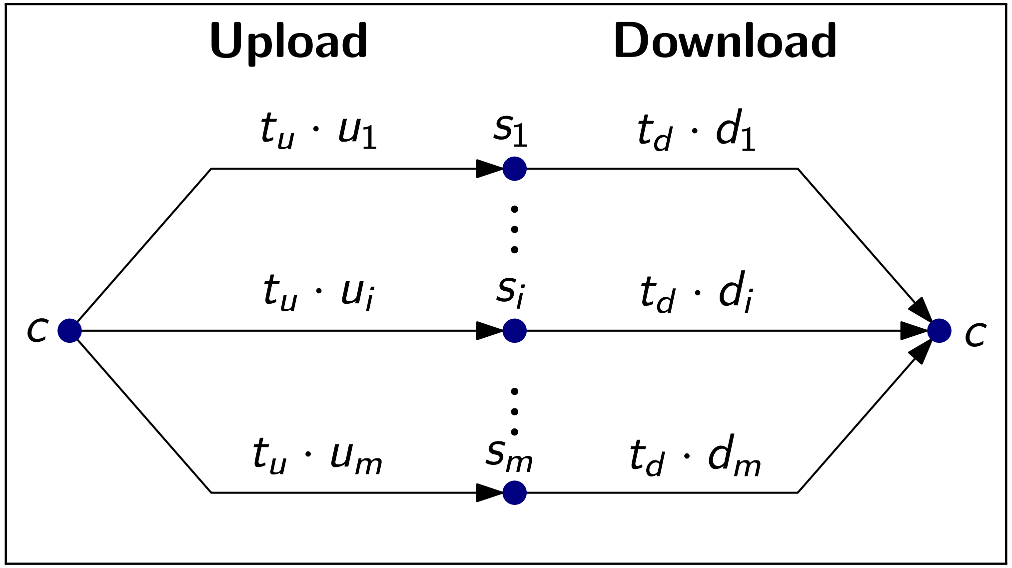

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Maximum Flow in Distribution Problems

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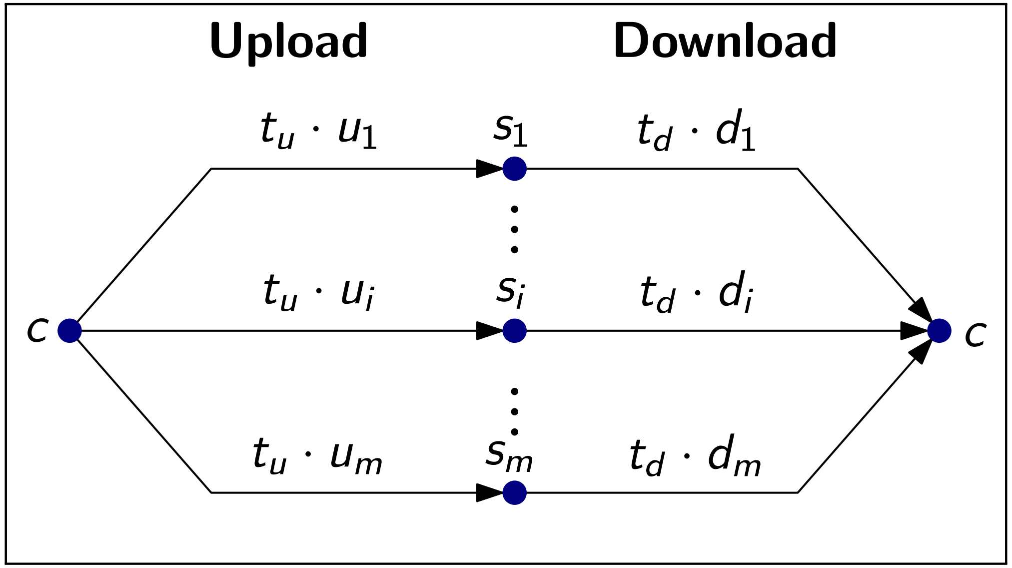

- The maximal amount of data that can be uploaded within t_u and downloaded within t_d corresponds to a maximum flow.

The Distribution Problem as Flow Problem

Maximum Flow in Distribution Problems

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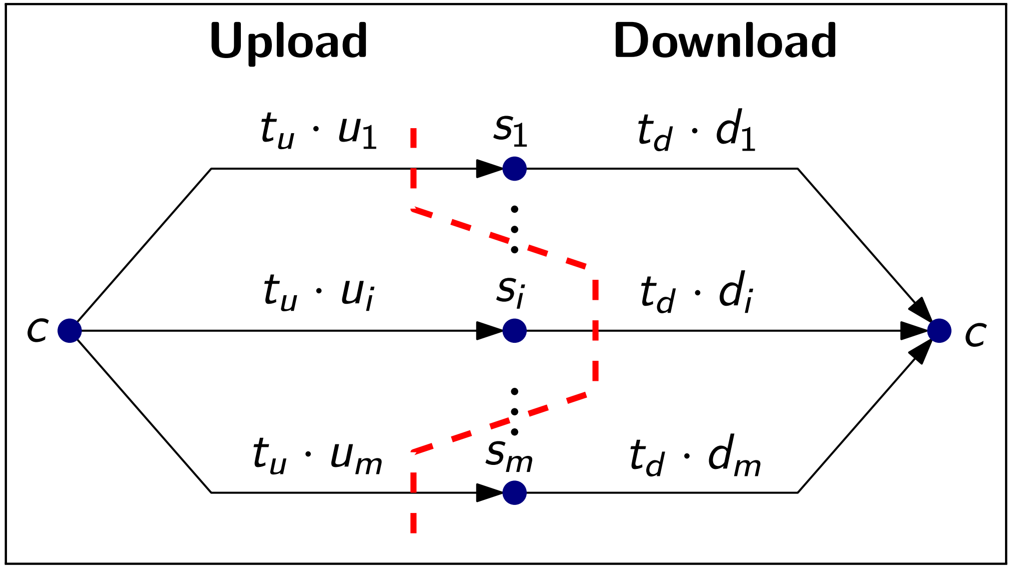

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- Value of maximum flow f^* equals capacity of minimal cut

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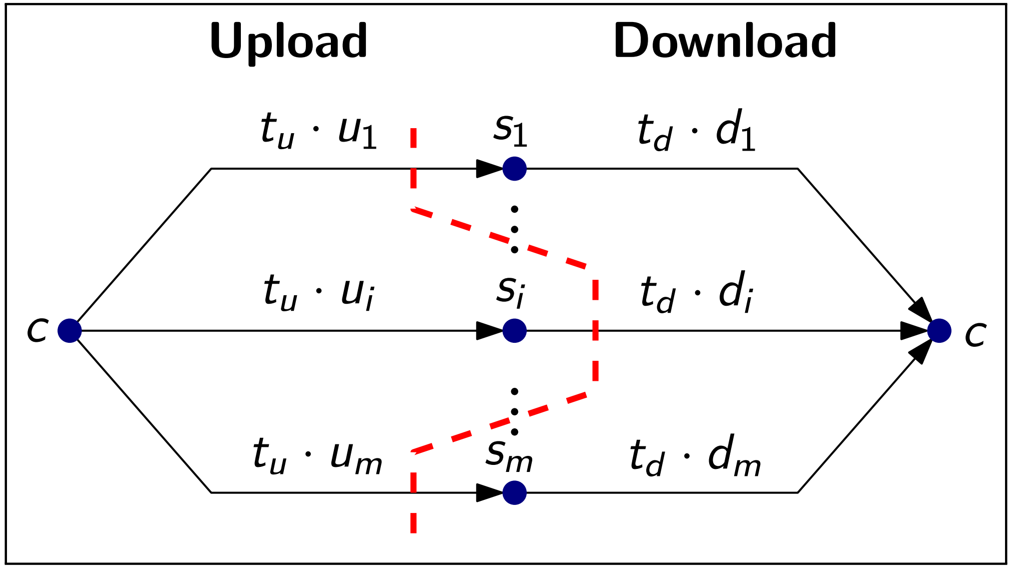

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- Value of maximum flow f^* equals capacity of minimal cut

$$\mathrm{val}(f^*) = \sum_{i=1}^{m} \min\{t_u\, u_i,\ t_d\, d_i\}$$

Decision Predicate

Maximum Flow in Distribution Problems

- There exists a distribution such that f with $|f| = 1$ can be uploaded and downloaded in time t_u and t_d, respectively, if

$$\boxed{\sum_{i=1}^{m} \min\{t_u\, u_i,\ t_d\, d_i\} \geq 1}$$

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- We want to find the minimum values for t_u and t_d for which the above predicate holds.

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- We want to find the minimum values for t_u and t_d for which the above predicate holds.
- Substitute t_d by $T - t_u$ to obtain the total data function.

$$\delta_T(t_u) = \sum_{i=1}^{m} \min\{t_u\, u_i,\ (T - t_u) \cdot d_i\}$$

Total Data Function

Total Data Function

- The total data function for a fixed value of T:

$$\delta_T(t_u) = \sum_{i=1}^{m} \min\{t_u\, u_i,\ (T - t_u) \cdot d_i\}$$

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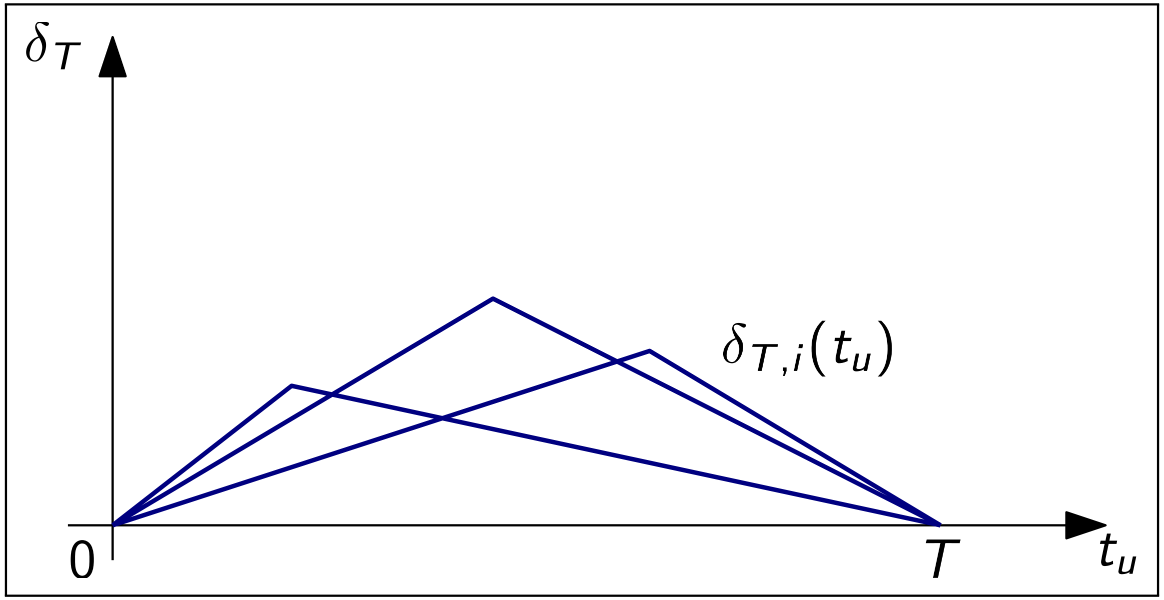

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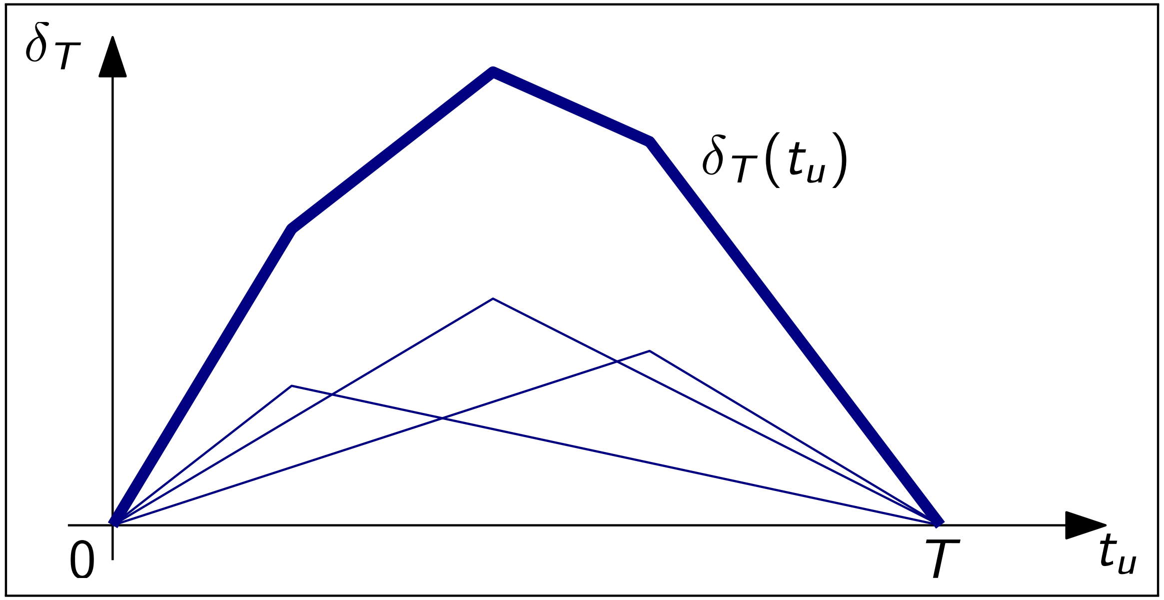

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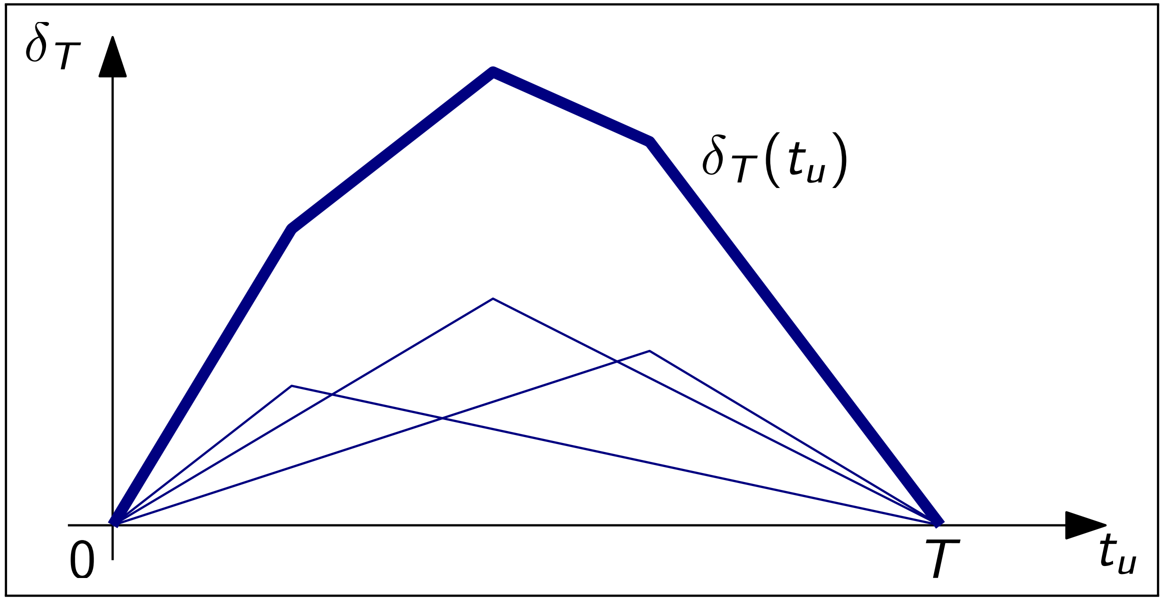
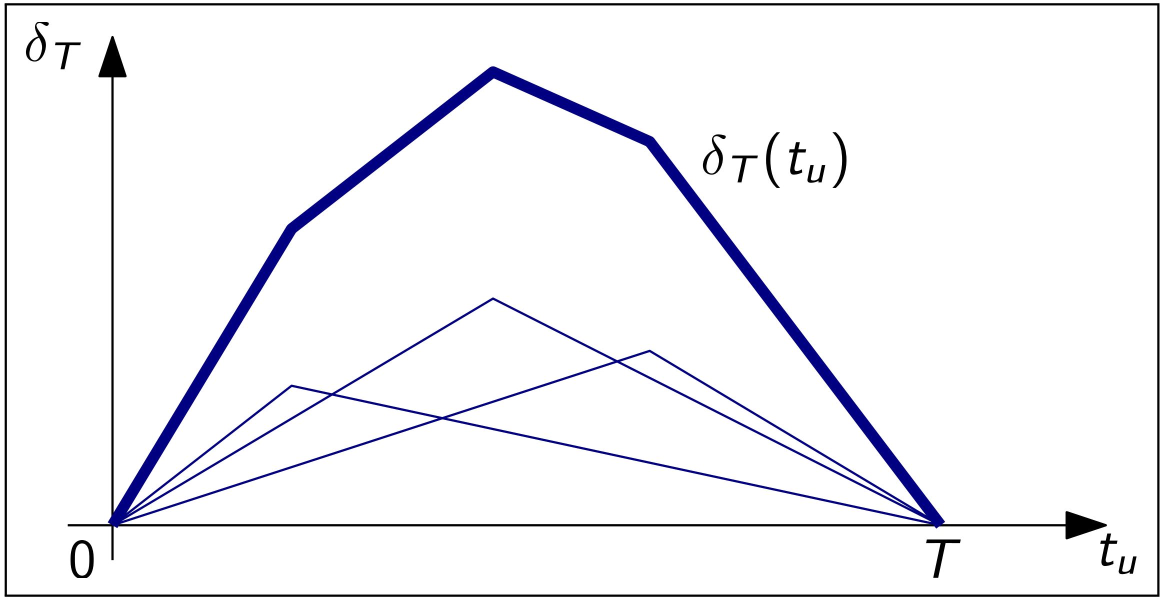


- Can be evaluated in $\mathcal{O}(m \log m)$ steps

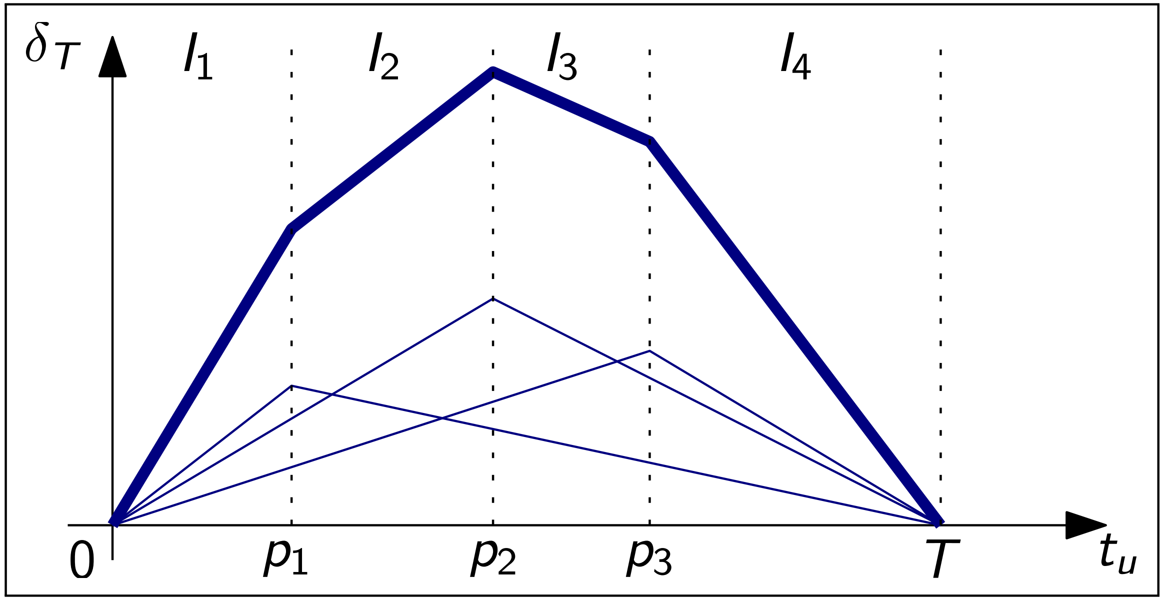
Evaluation

The Total Data Function



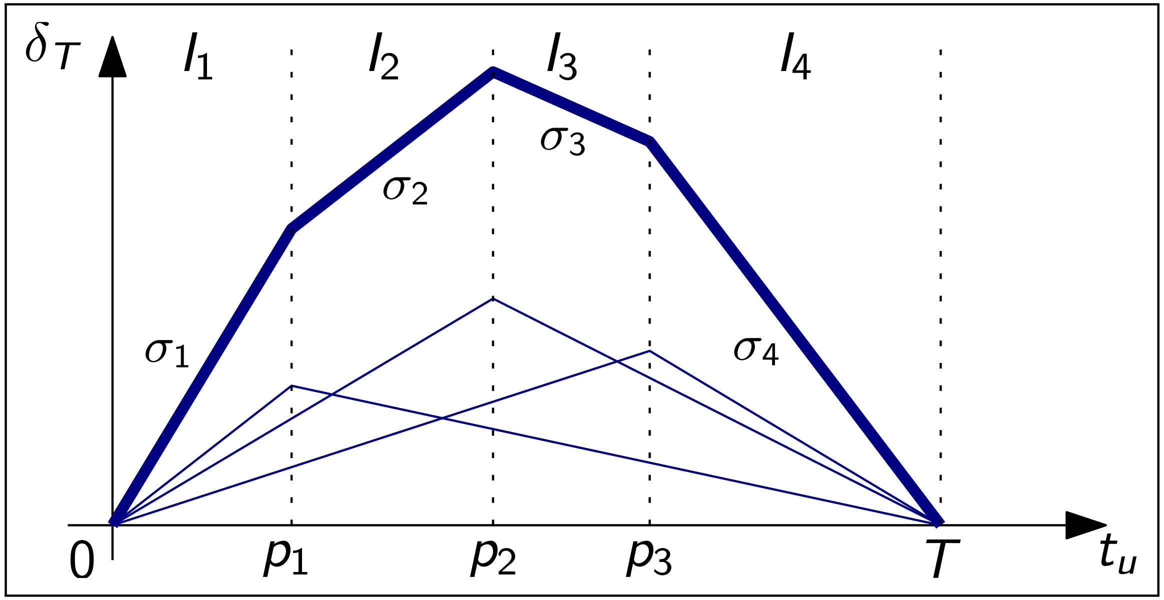
Evaluation

The Total Data Function



Evaluation

The Total Data Function



σ_i is slope in interval I_i.

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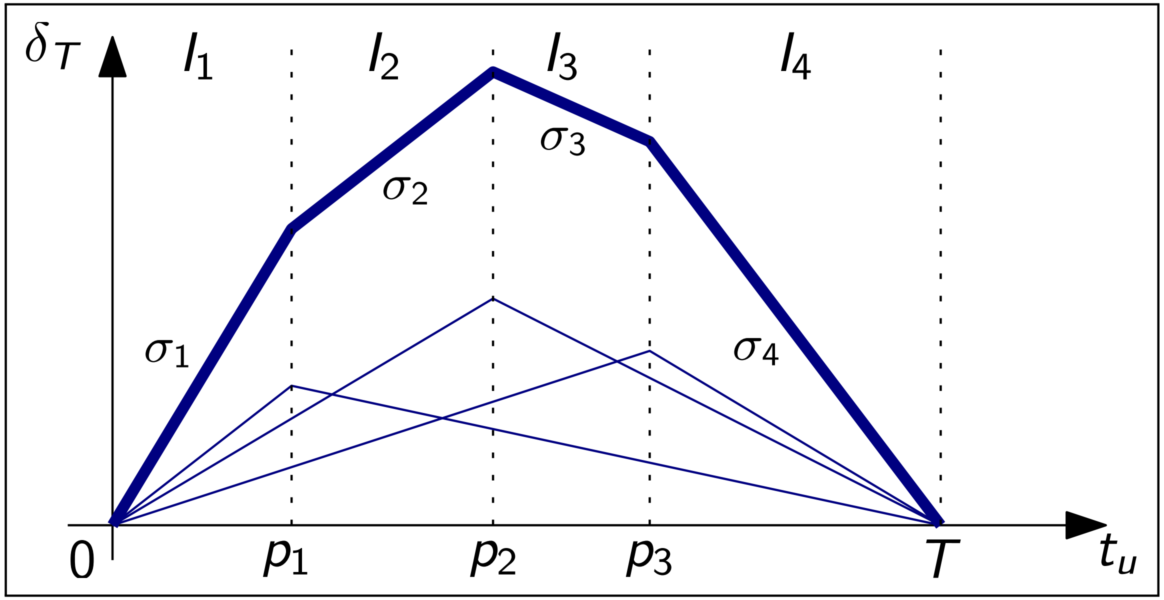

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Recursion formula

$$\delta_T(p_i) = \delta_T(p_{i-1}) + \sigma_i \cdot (p_i - p_{i-1})$$

Scaling the Total Data Function

Total Data Function in 3D

- The total data function $\delta_T(t_u)$ determines how much data can be uploaded to and downloaded again from the servers within time T for different values of t_u.

Scaling the Total Data Function

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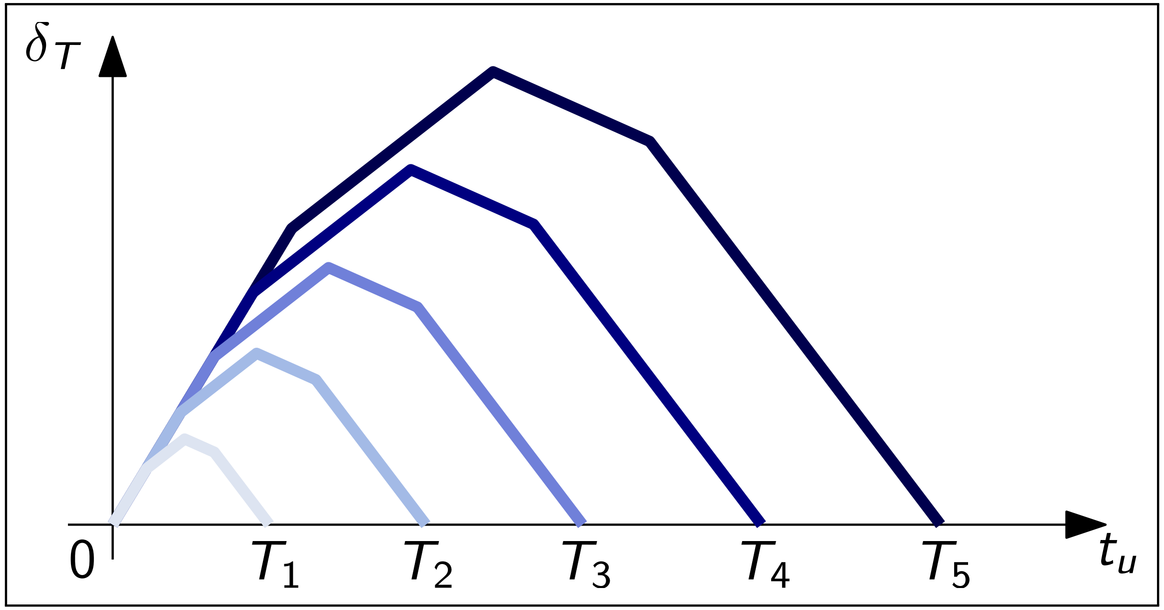
- Varying values of T only scale the total data function



Two-Variable Total Data Function

Total Data Function in 3D

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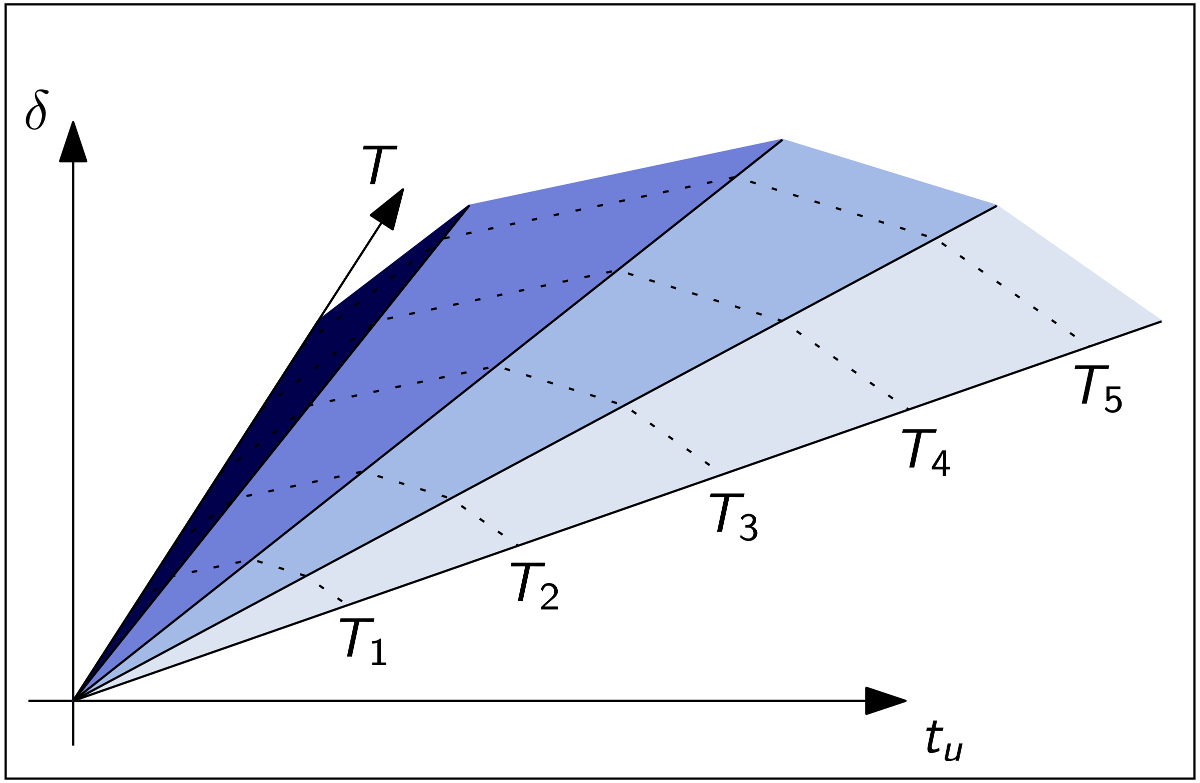

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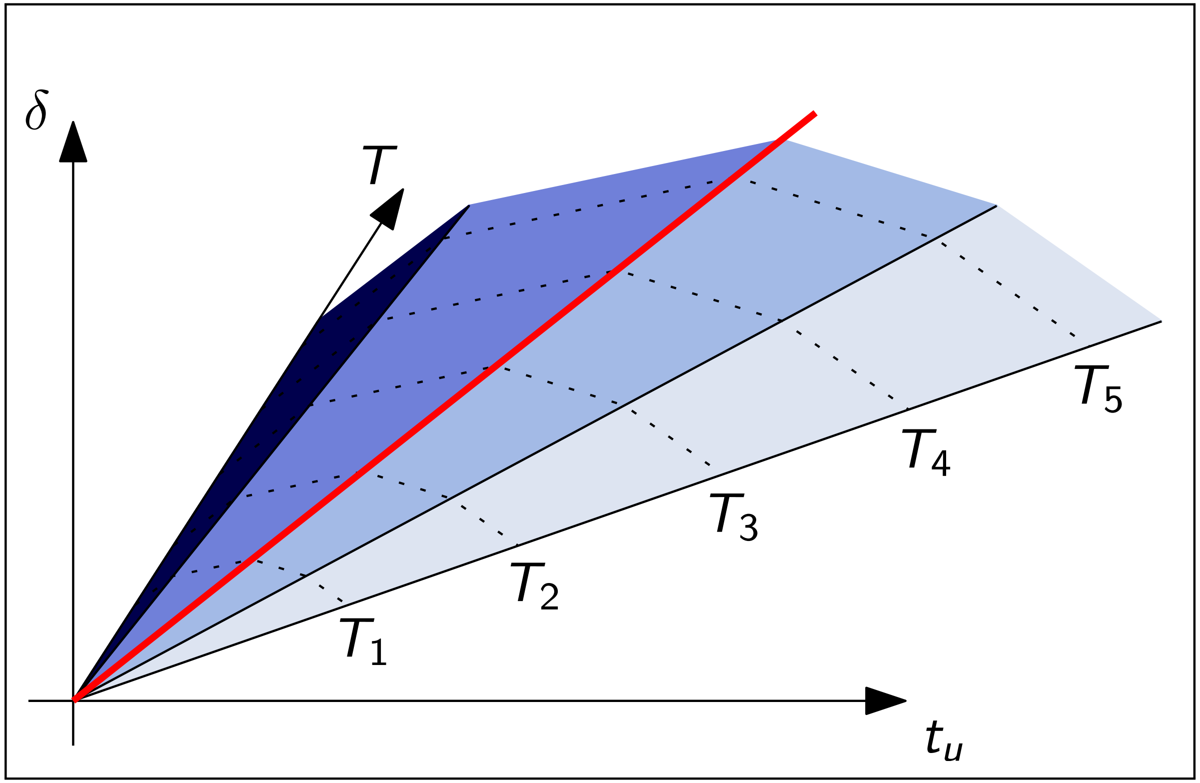

- To find the minimal value of T with $\delta(T, t_u) \geq 1$ for some t_u, establish straight line through the highest point of $\delta(T, t_u)$

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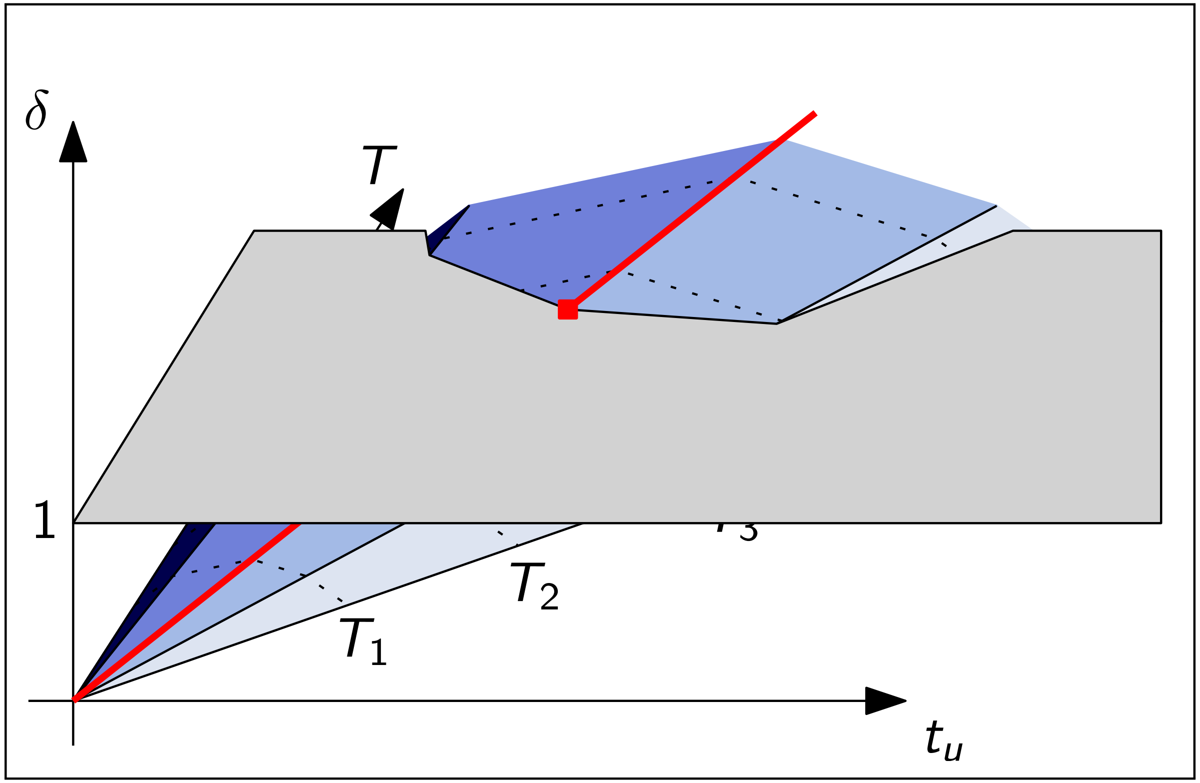
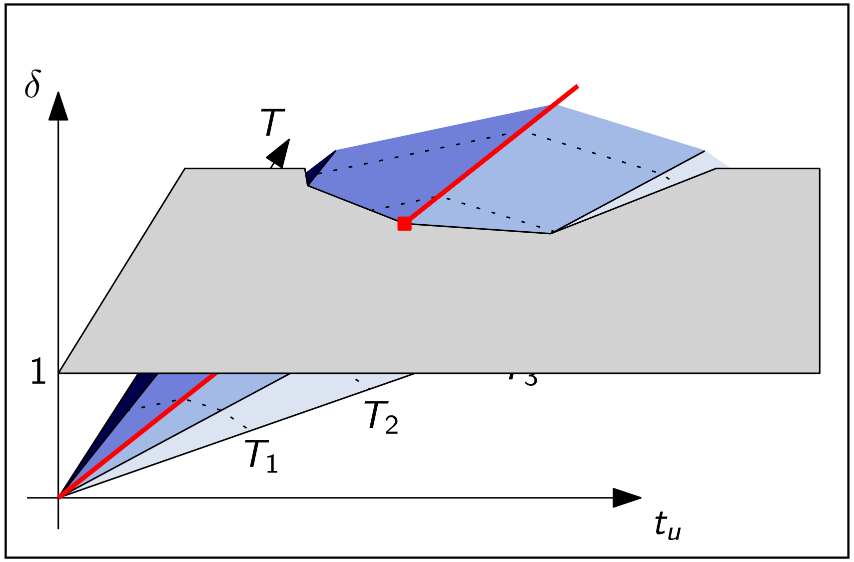


- To find the minimal value of T with $\delta(T, t_u) \geq 1$ for some t_u, establish straight line through the highest point of $\delta(T, t_u)$
- Determine where it intersects the plane $\delta = 1$

Determining the Actual Distribution

The Algorithm

- We determine the minimal value of T as depicted.



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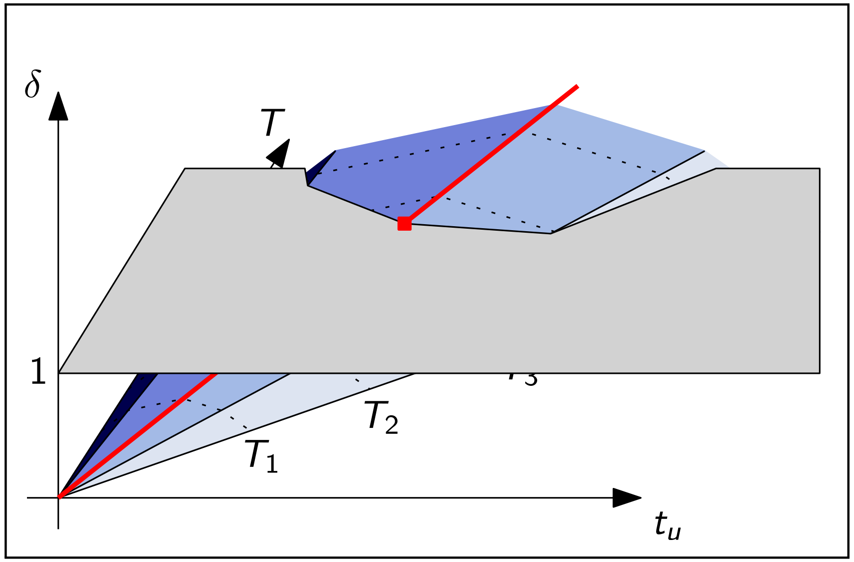

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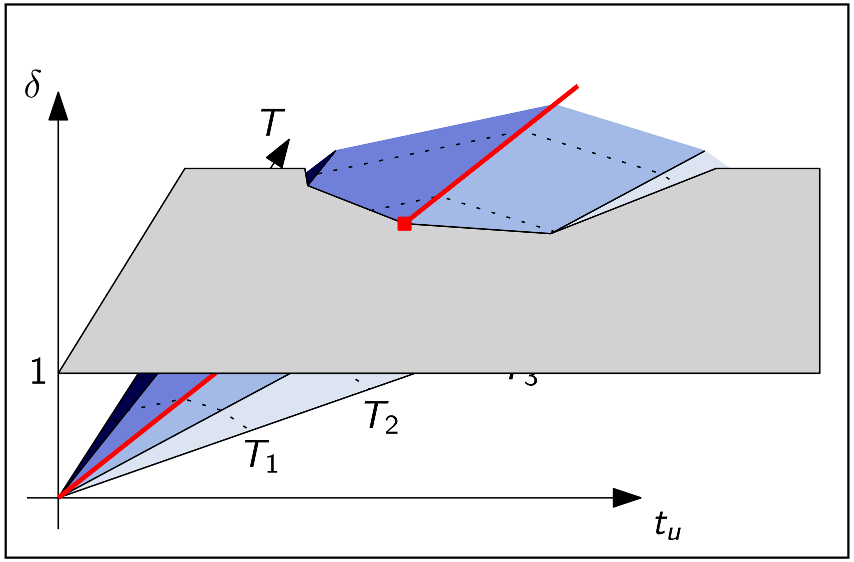

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- Through the feasibility predicate

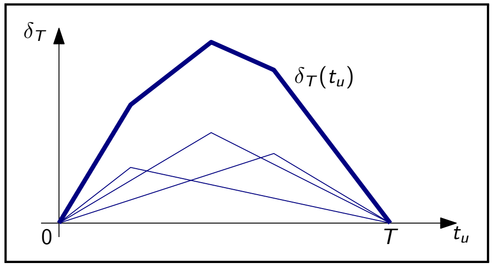
$$\sum_{i=1}^{m} \min\{t_u\, u_i,\ t_d\, d_i\} = 1$$

we know that $\sum x_i = 1$ and thus $\mathbf{x}$ is a valid distribution.

Runtime Complexity

The Algorithm

- Evaluating the total data function for a fixed value of T runs in $\mathcal{O}(m \log m)$ steps.



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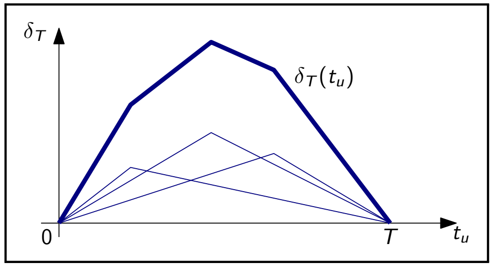

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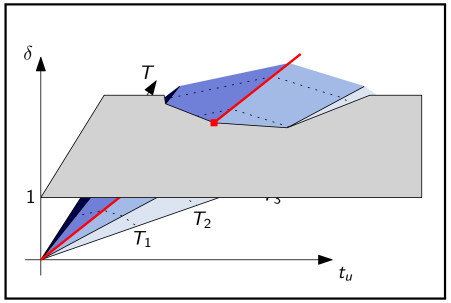

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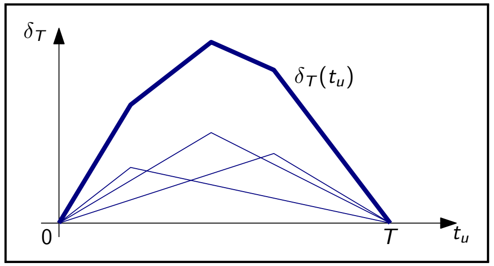

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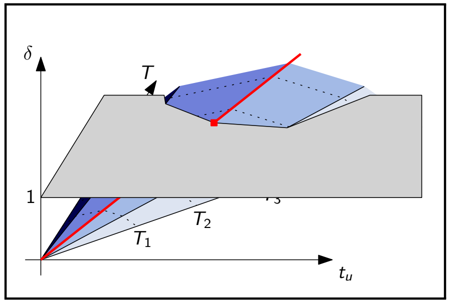

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Synopsis

Summary

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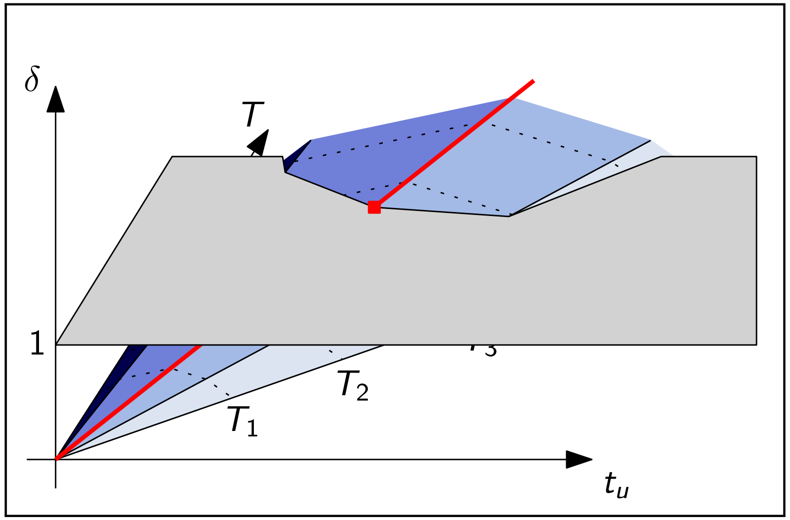

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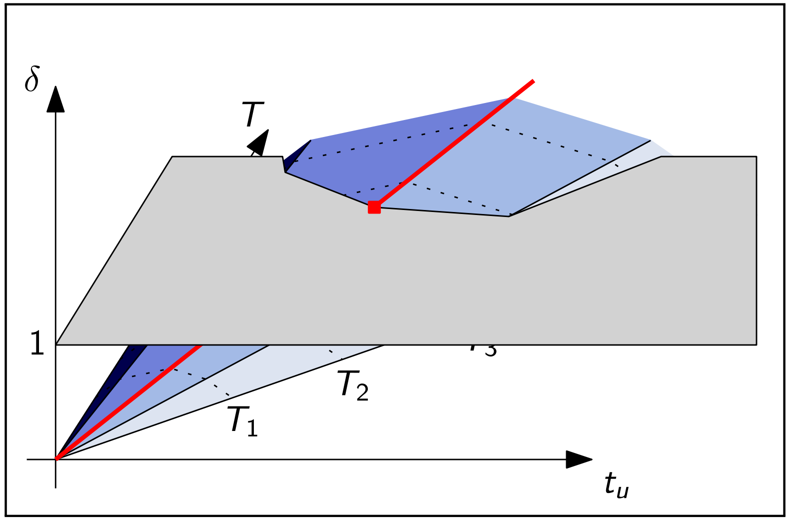
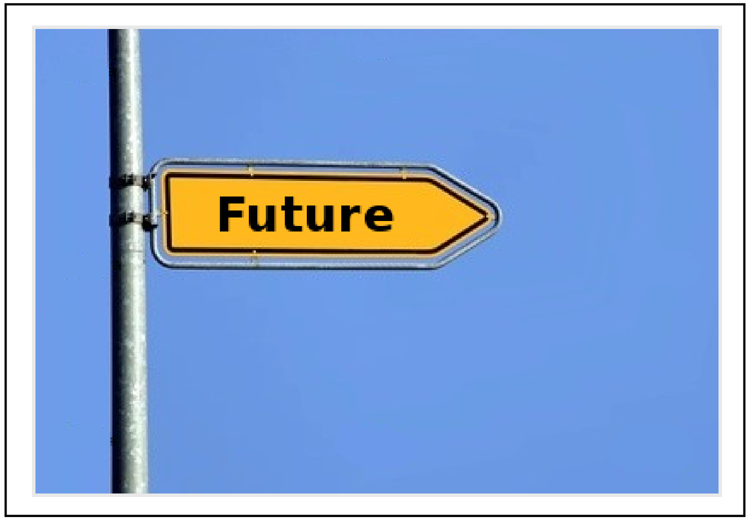


Future Directions



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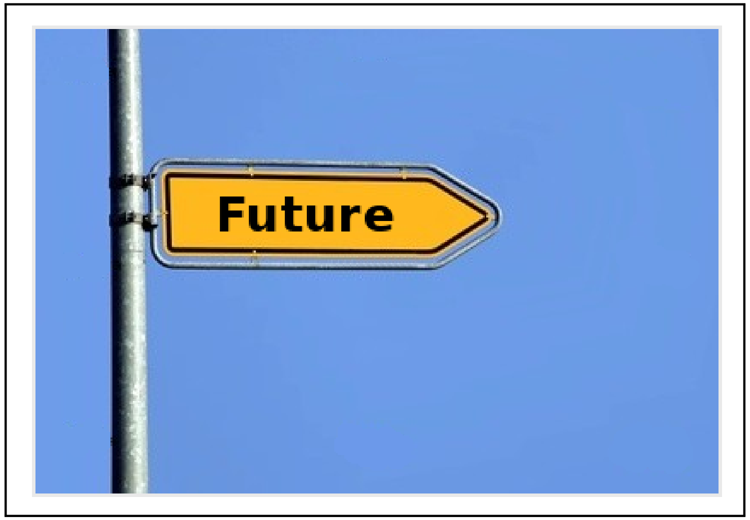

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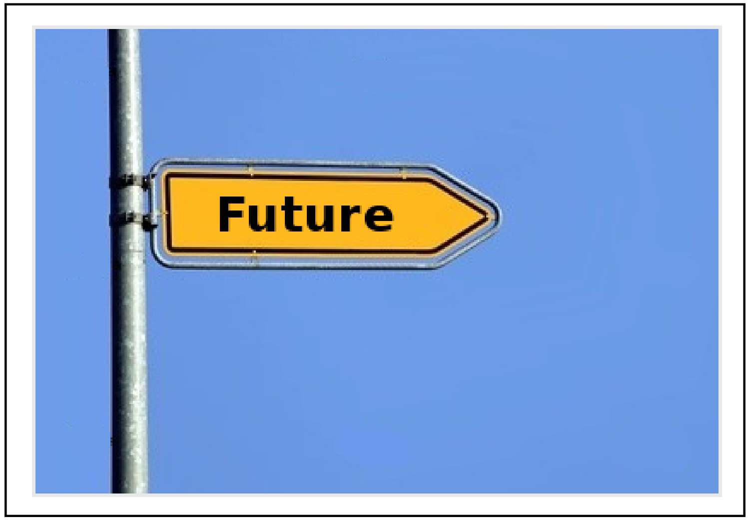

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- Cope with non-static, fluctuating bandwidths
- Respect limited storage capacity of servers
- Incorporation of redundancy to allow for failure tolerance

The End

Thank you for your attention!

Questions?