Congestion, Energy and Delay in Radio Networks (Technical Report)

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Abstract

We investigate the problem of path selection in radio networks for a given set of sites in two-dimensional space. For some given static point-to-point communication demand we define measures for congestion, energy consumption and dilation that take interferences between communication links into account.

We show that energy optimal path selection for radio networks can be computed in polynomial time. Then, we introduce the diversity $g(V)$ of a set $V \subseteq \mathbb{R}^2$. It can be used to upperbound the number of interfering edges. For real-world applications it can be regarded as $\Theta(\log n)$. A main result of the paper is that a t-spanner construction as a communication network allows to approximate the congestion optimal communication network by a factor of $O(g(V)^2)$.

Furthermore, we show that there are vertex sets where only one of the performance parameters congestion, energy and dilation can be optimized at a time. We show trade-offs lowerbounding congestion $\times$ delay and delay $\times$ energy. For congestion and energy the situation is even worse. It is only possible to find a reasonable approximation for either congestion or energy minimization, while the other parameter is at least a polynomial factor worse than in the optimal network.

Classification: Radio networks, wireless communication, routing

1 Introduction

In this paper we contribute to modeling radio networks, to modeling congestion, energy consumption and delay for routing in such networks, and to designing routing paths in order to minimize these cost measures. One major insight is the fact that trade-offs are unavoidable: minimizing one measure is only possible at the cost of enlarging another one.

Wireless ad hoc networks consist of nodes that can communicate via short-range wireless connections. Each node can be a source, a destination and a router for data packets, thus no explicit infrastructure is required to set up and maintain an ad hoc radio network. The area of

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application for radio networks is broad, especially in niches such as search and rescue missions or environmental monitoring. But ad hoc networks can also be used as a last-mile technology to provide access to the Internet in high-populated environments.

In wireless ad hoc networks, energy-expensive long-range connections should be avoided, and the overall distance between two communicating nodes respectively hop count should be minimized to achieve low latencies. To use the available network capacity efficiently and to achieve high bandwidths, congested connections should also be avoided by balancing the traffic over all reasonable connections.

These requirements can be expressed using three measurable quantities: congestion, energy and hop count. Traditional routing protocols such as AODV, DSDV and DSR [Per01] usually chooses the path with the lowest hop count. There also exist power-aware routing protocols that use different metrics (e.g., energy consumed per packet, variance in node power level) to choose the best route in order to extend the lifetime of individual nodes or the whole network [SR98, SW98, CT00]. The congestion of a route is usually not regarded directly, but some routing protocols choose routes with the shortest route discovery, assuming that the route with the quickest response is less congested (e.g., SSA [DRWT97]). However, to our knowledge, no practical work or theoretical studies exist that consider the interdependencies between these three quantities.

In radio networks it is not clear how to choose nodes as communication partners because links can interfere with each other. Our main goal in this paper is to determine the optimal choice of this network given a set of vertices $V \subseteq \mathbb{R}^2$ (Random choices of vertex sets have been investigated in [AS98, GK00]). Hence, we disregard the mobile and dynamic components of ad hoc-networking and determine the optimal static wireless network. We present a general model for congestion, energy and dilation for a given solution of the routing problem of the radio networks. (cf packet radio network model or more realistic wireless network models, for instance as in [AS98, UY98, GK00, ABBS01, KKKP00, ISP01]). Besides the load the congestion also measures the interferences between edges.

In Section 2 we start our considerations with the paths of all packets solving a routing problem in a radio network. The union of all these paths, called path systems, gives a natural definition of the communication network. These paths induce a load in the communication links, that can interfere with each other. Combining the load and the interferences we achieve an intuitive model for the congestion of an edge of the communication network. Our definition is very similar to those in [AS98], although they use a slightly different approach. Likewise in [AS98] we relate the congestion and the dilation, also known as hop-distance, to the routing time of the routing problem. Then, we define measures for energy consumption, which is important for autonomous nodes that have to "carry their energy".

The main contributions concern path selection in radio networks: Given a set of routing requests, find routing path so that the congestion, delay, and/or energy consumption is minimized. We introduce the notion of diversity to describe locations of vertex sets where high interferences are unavoidable. It turns out that if the diversity is small, i.e. all point to point distances differ only by polynomial factor, then the interferences of communication networks can be kept small. This is key factor for the congestion avoidance analysis in this paper.

In section 3 we present strategies for this path selection that provably optimize energy consumption and give a $O(g(V)^2)$-factor approximation of congestion. In section 4 as a main insight, we can conclude that not any two of these measures can be minimized simultaneously, but that trade-offs between measures are unavoidable. Finally, section 5 concludes the work.

2 Modeling Radio Networks

We consider a set $V \subseteq \mathbb{R}^2$ of n radio stations, featuring both transmitter and receivers, called sites or vertices, in 2-dimensional Euclidean space. Let $d = \max_{u,v \in V} |u, v|$ denote the geometric diameter of V.

As in the model of [MBH01] each node $u \in V$ can adjust its transmission radius to some $r \geq 0$ for sending a packet to a neighbor $v \in V$ in range r. Then, the communication network $N = (V, E)$

has the edge $\{u, v\}$, where $|u, v| = r$. To acknowledge this packet the receiving site adjusts its transmission radius to the same radius r. The transmission needs a unit time step and the area covered by sending and acknowledging a packet along $e = (u, v) \in E$ is $D(e) = D_r(u) \cup D_r(v)$, where $D_r(u)$ denotes a disk with center u and radius r. The edge e' interferes with e if $D(e')$ contains u or v (cp. Figure 1).

We define the set of interfering edges by $\mathrm{Int}(e) := \{e' \in E(N) \mid e' \text{ interferes with } e\}$. Note that sending a packet along e is successful only if no edge from $\mathrm{Int}(e)$ sends concurrently. These interferences of network N describe the directed *interference graph* $G_{Int}(N)$. Its vertex set are all edges of N and its edges describe all interferences, i.e. $(c, e) \in E(G_{\mathrm{Int}}(N))$ iff $c \in \mathrm{Int}(e)$. The in-degree of an edge in the interference graph is called the *interference number* of a communication link. The maximum interference number of a site u is the maximum interference number of all edges with receiving site u. The interference number of the network is the maximum interference number of all edges.

Now consider a routing problem $w : V \times V \to \mathbb{N}$, where $w(u, v)$ packets have to be sent from u to v. We subdivide the design of a routing strategy for w into the following steps:

- *Path selection*: Select a system $\mathcal{P}$ of paths R_p from source to destination for the packets p in the graph on V. The union of all edges $E_{\mathcal{P}}$ of the path system gives the links of communication network $N = (V, E_{\mathcal{P}})$.
- *Collision avoidance*: As noted above sending a packet along edge e is only successful if no $e' \in \mathrm{Int}(e)$ sends at the same time.

Therefore the following holds: Consider any routing strategy that routes w in T steps using the path system $\mathcal{P}$. Let $\Gamma(e) \subseteq \{1, \ldots, T\}$ denote the time steps in which e sends successfully. Then clearly

$$|\Gamma(e)| + \sum_{e' \in \mathrm{Int}(e)} |\Gamma(e')| \leq T \ .$$

As $|\Gamma(e)|$ is just the load $\ell(e)$ of e, i.e. the number of packets whose path goes through e, the above quantity is $\ell(e) + \sum_{e' \in \mathrm{Int}(e)} \ell(e')$. We denote this quantity (which is uniquely defined by the path system $\mathcal{P}$) as congestion of the edge $C_{\mathcal{P}}(e)$. The **congestion of the path system** $\mathcal{P}$ is defined by

$$C_{\mathcal{P}}(V) := \max_{e \in E_{\mathcal{P}}} \{C_{\mathcal{P}}(e)\} \ .$$

We will denote by the dilation $D_{\mathcal{P}}(V)$ the length of a longest path in $\mathcal{P}$, also known as the hop-distance. By definition the optimal routing time T using $\mathcal{P}$ fulfills

$$T \ \geq \ \max\{C_{\mathcal{P}}(V), D_{\mathcal{P}}(V)\} \ .$$

Our first result will show, that the routing time can be pretty well upper and lower bounded in terms of congestion and dilation.

Theorem 1 *Consider a radio network M with path system $\mathcal{P}$, and a routing problem w with dilation D and congestion C. Let T be its optimal routing time, when the path system $\mathcal{P}$ is used. The following holds.*

1. $T \ \geq \ \max\{C, D\}$.
2. *There is an offline routing protocol with routing time $3D \cdot C \cdot (2 \log n + \log C)$.*
3. *If C and the path system is known, then there is a randomized distributed online protocol that needs expected time $O(C \cdot D)$ for each packet.*

A proof of 2. and 3. using similar techniques as in [AS98] is given in the appendix. Note that the online protocol only guarantees expected time $O(C \cdot D)$ for each packet, but not for the slowest packet.

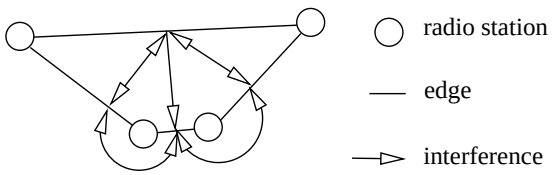

Figure 1: Radio stations, edges and induced interferences

The variable choice of the transmitter power allows to reduce the energy consumption, saving on the tight resources of batteries in portable radio stations and reducing interferences. Theoretically, the energy needed to send over a distance of r is given by $O(r^2)$. It turns out that in practice one can model the energy by $O(r^4)$ or even $O(r^5)$. Throughout this paper we model energy costs by $O(r^2)$. However, all results in this paper can be easily transferred for higher exponents.

We distinguish two energy models. In the first model, called **unit energy model**, we assume that maintaining a communication link e is proportional to $O(|e|^2)$, where $|e|$ denotes its Euclidean length. Therefore the unit energy *U-Energy* used by radio network N is given by

$$\text{U-Energy}_{\mathcal{P}}(V) := \sum_{e \in E_{\mathcal{P}}(N)} |e|^2 \ .$$

The **flow energy model** reflects the energy actually consumed by transmitting all packets. Here, the power consumption of a communication link is weighted by the actual load $\ell(e)$ on an edge e:

$$\text{F-Energy}_{\mathcal{P}}(V) := \sum_{e \in E_{\mathcal{P}}(N)} \ell(e)|e|^2 \ .$$

In this paper we focus on the question: Given some sites which path selection is best possible to obtain small congestion, low energy consumption and small dilation. Clearly, the optimal network for hop-distance is the complete graph. Hence, we investigate only energy and congestion.

3 Minimizing Energy and Congestion

3.1 Energy

The unit energy of a path system for a radio network is defined as the energy consumption necessary to deliver one packet on each communication link. It turns out that the minimal spanning tree optimizes unit energy. Note that the hardness results shown in [KKKP00, CPS00] do not apply because in our model the transmission radii are adjusted for each packet.

Theorem 2 *The minimal spanning tree is an optimal path system for a radio network with respect to the unit energy.*

Proof: Consider the graph defined by all edges $E \subseteq V \times V$ with edge weight $|e|^2$. The minimum energy network can be constructed using Prim's or Kruskal's algorithm for minimum spanning tree. Note that the decisions in this algorithm are based on comparison of the length of some edges e and e', i.e. $|e| \leq |e'|$. Thus, the minimal network for energy is also the minimum spanning tree for Euclidean distances. ■

For the flow energy model, the minimal network is not necessarily a tree. However, one can compute the minimal flow energy network in polynomial time.

Theorem 3 *For a given vertex set V a sub-graph of the Gabriel Graph [GS69, JT92] is an optimal path system for a radio network with respect to the flow energy.*

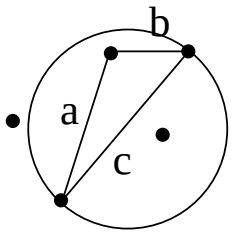

Figure 2: For an edge c of a Gabriel-graph no node may be in inside the disk (otherwise energy is not optimal because $a^2 + b^2 < c^2$)

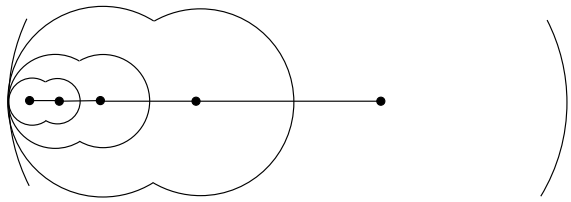

Figure 3: The high diversity of the vertex set increases the interference number, resulting in high congestion

Proof: If in the interior of the circle defined by the diameter (u, v) there exists a vertex w, then the edges $(u, w), (w, v)$ need less energy than the original edge. This follows by the Theorem of Thales. Therefore, one can add an edge into the communication network iff in there are no sites in the interior its circle, see Fig. 2. This matches the definition of a Gabriel graph of V.

For two vertices u and v the sub-graph providing the lowest energy for routing information from u to v is given by the shortest path in the Gabriel graph if the length of an edge is redefined by $|e|^2$. The flow energy of the optimal network consists of a linear combination of these lowest-energy-paths between pairs. Using an all-pair-shortest-path algorithm gives the optimal network. ■

Note, that there are situations where edges of the Gabriel-graph can be replaced by less energy-consuming paths, even if no site lies inside the disk described by the edge. Then, the edge of the Gabriel graph is not part of any energy optimal route.

3.2 Diversity of a Vertex Set

Sometimes the location of the radio stations does not allow small congested routes for the radio networks at all. Consider a vertex set $V = \{v_1, \ldots, v_n\}$ on a line, with distances $|v_i, v_{i+1}| = 2^i$. The edge (v_i, v_{i+1}) interferes with all edges (v_j, v_{j+1}) for $j \leq i$, see Fig. 3. Therefore the interference number of the network is $n - 1$. Suppose only v_1 and v_n want to communicate, then the better solution for congestion is to disconnect all interior points and to realize only the edge (v_1, v_n). Of course this is not an option when interior nodes need to communicate.

It turns out that a determining parameter for the realization of optimal communication networks for radio networks, is the number of magnitudes of distances. Distances have different magnitude if they differ more than a factor of 2.

Definition 1 *The diversity $g(V)$ of a point set V in Euclidean space is defined by*

$$g(V) := |\{m \mid \exists u, v \in V \ : \ \lfloor \log |u, v| \rfloor = m\}| \ .$$

Note that in the above scenario we observe the maximum diversity of n (and a high interference number). For point sets V on the line with small diversity the interference number is small, too. It is easy to see, that the interference number for a vertex set V on the line is at most $O(g(V))$.

Fact 1 *The diversity of n points is at least $\frac{\log n}{2}$ and at most $2n$. For a point set randomly distributed in a square the diversity is $O(\log n)$ with high probability (i.e. $1 - n^{-c}$ for any fixed constant $c > 0$). Furthermore,*

$$g(V) \leq 1 + \log \frac{\max_{u,v \in V} |u, v|}{\min_{u,v \in V \wedge u \neq v} |u, v|} \ .$$

The last inequality follows directly from the definition. This inequality implies logarithmic diversity for random point sets since the probability to choose a vertex within a n^{-c-2}-neighborhood

```
Algorithm AS-spanner(V, η, w)
                (* 0 < η < π/4, 0 ≤ w < ½(cos η − sin η) *)
begin
    L := list of all pairs of V sorted according to their distances;
    E := ∅
    for all ordered pairs (p, q) ∈ L do
        add := true
        for each edge (r, s) ∈ E do
            if angle(q − p, s − r) ≤ η then
                add := add ∧ (|p, r| > w|r, s|)
            fi
            if angle(p − q, r − s) ≤ η then
                add := add ∧ (|q, s| > w|r, s|)
            fi
        od
    if add = true then E := E ∪ {(p, q)} fi
    od
    return(E)
end
```

Figure 4: The algorithm of Arya and Smid [AS94] for the construction of a t-spanner.

of another can be bound by n^{-c-1}. Hence, for all vertices the probability that $g(V) \geq (c+2)\log n$ can be bounded by at most n^{-c}.

There are many reasons why in the real world the diversity can always be estimated by $O(\log n)$, e.g. the accuracy of determining locations; and the ratio between the physical size of a radio station and its transmitting range.

3.3 Congestion

To approximate congestion-optimal communication networks for radio networks we will use the t-spanner with bounded degree introduced in [AS94]. A t-spanner is a graph such that for each pair (u, v) there exists a path of at most length $t|u, v|$. Note that t-spanners are a common choice for the communication links in radio networks, e.g. see [GGH+01].

The algorithm *AS-spanner*(V, η, w) shown in Fig. 4 constructs such a spanner for a point set V in $\mathbb{R}^2$. The algorithm considers all ordered pairs (p, q) of sites in increasing order of their distances. The edge (p, q) is added to the graph iff there is no edge (r, s) in the current graph such that (p, q) and (r, s) have roughly the same direction and the points p and r are close to each other, or the points q and s are close to each other.

Theorem 4 *[AS94] The graph constructed by algorithm* AS-spanner *is a t-spanner with bounded degree $\frac{2\pi}{\eta}$ for $t = 1/(\cos\eta - \sin\eta - 2w)$.*

Such a t-spanner causes only a small number of interferences, if the vertex set V has a small diversity.

Lemma 1 *For a vertex set V with diversity $g(V)$ the interference number of the t-spanner constructed by algorithm* AS-spanner *is bounded by $O(g(V))$.*

Proof: We choose $w = \frac{1}{4}(\cos\eta - \sin\eta)$ for $\eta = \pi/8$. Hence the stretch-factor is given by $t = 2/(\cos\eta - \sin\eta)$.

We define $Q := \{q_1, q_2, \ldots, q_g\} \subset \mathbb{Z}$ such that $Q := \{m \mid \exists u, v \in V \ : \ \lfloor \log|u, v| \rfloor = m\}$. We consider an edge $e = (u, v)$ of length $l \in [2^{q_i}, 2 \cdot 2^{q_i}]$. We try to insert as many edges interfering

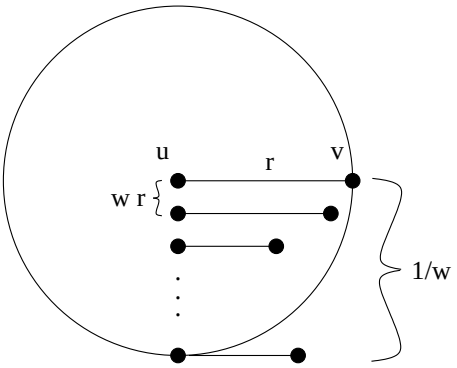

Figure 5: For an edge e to a neighbored node exist at most $c \cdot \frac{1}{w}$ parallel edges that interfere with e

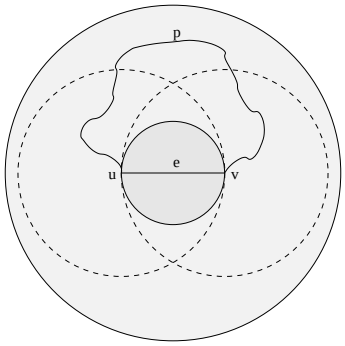

Figure 6: The edge e interferes with other edges (at least) within the central disk. Its information is rerouted on p, lying completely within the outer-disk with radius $\frac{1}{2}t|e|$

e as possible. Their number is bounded by $O(\frac{2\pi}{\eta w})$, since the constructed spanner has bounded degree 16 and for every edge to a neighbored node we can construct at most $O(\frac{1}{w}) = O(1)$ parallel edges that interfere with e because of the restriction that the distance between the edges must be at least $w \cdot l$ (see figure 5). We have $g(V) = |Q|$ and therefore the number of interferences is bounded by $O(\frac{g(V)}{\eta \cdot w})$. Using that η and w are constant, this completes the proof. ■

A typical feature of radio communication is that transmitting information blocks a region for other transmission. We formalize this observation and define the capacity of a region following a similar approach presented in [GK00]. Let $A(R)$ denote the area of a geometric region R.

Definition 2 *The capacity $\kappa(R)$ of a geometric region R is defined as follows:*

1. *If in every point of R the same set of edges E interfere then $\kappa(R) := \sum_{e \in E} \ell(e) \cdot A(R)$, where $A(R)$ denotes the area of R. Such a region is called elementary.*
2. *Otherwise partition R into elementary regions $R_1, \ldots, R_m$ and define $\kappa(R) := \sum_{i=1}^{m} \kappa(R_i)$.*

This definition implies the following relationship between capacity, area and congestion.

Lemma 2 *Let R be a region and C the congestions of a path system $\mathcal{P}$. Then, the capacity of R is bounded by $\kappa(R) \leq A(R) \cdot C$.*

Every edge e with load $\ell(e)$ has a certain impact on the capacity of the area covered by the radio-network.

Lemma 3 *An edge e with load $\ell(e)$ occupies the capacity $c\ell(e)|e|^2$ for a constants $c > 0$.*

The proof follows from the definition of the interference area.

Lemma 4 *Let C^* be the congestion of the congestion-optimal path system $\mathcal{P}^*$ for a vertex set V. Then, every t-spanner N can host a path system $\mathcal{P}'$ such that the induced load $\ell(e)$ in N is bounded by $\ell(e) \leq c' g(V)\ C^*$ for a positive constant c'.*

Proof: Given a path P of the path system $\mathcal{P}^*$, we replace every edge $e = (u, v)$ that does not exist in the t-spanner N with the shortest path p from u to v in N (which by definition has length of at most $|p| \leq t|u, v|$). Therefore, the new route lies completely inside a disk $C_t(e)$ of radius $\frac{1}{2}t|u, v|$ and center $\frac{1}{2}(u + v)$.

For the path system $\mathcal{P}^*$ there may have been interferences between e and other edges. For simplicity we underestimate the area where e can interfere other communication by the disk $C_1(e)$ with center $\frac{1}{2}(u + v)$ and radius $\frac{1}{2}|u, v|$ (see Fig. 6).

We want to describe the impact of rerouting of all edges in $E(N^*)$ to a specific edge $e_0 \in E(N)$ in the t-spanner N. If this edge $e_0 = (u_0, v_0) \in E(N)$ transmits the traffic of a detour of an edge $e = (u, v) \in E(N^*)$, then the distance between the central points $z_0 := \frac{1}{2}(u_0 + v_0)$ of e_0 and $C = \frac{1}{2}(u + v)$ is bounded by $|z_0, z| \leq \frac{1}{2}t|e|$.

Now consider the edge set $E_{i,e_0} \subseteq E(N^*)$ of edges e with length $|e| \in [2^i, 2^{i+1}]$ for $i \in \mathbb{Z}$ which reroute their traffic to e_0. Their center points are located inside a disk with radius $2^i t$ and center z_0. The interference area of every edge e is described by $C_1(e)$. It occupies an area of at least $\pi 2^{2i}$, which lies completely inside a disk D with radius $2^i(t+1)$ and center z_0. The area of D is $\pi 2^{2i}(t+1)^2$.

Lemma 3 shows that every edge e reduces the capacity in D by at least $c\ell(e)2^{2i}$. Because of Lemma 2, the over-all capacity of C is at most $A(D) = \pi 2^{2i}(t+1)^2 C^*$. Therefore we have for the sum of the loads $\ell(e)$ for $e \in E_{i,e_0}$ that $\sum_{e \in E_{i,e_0}} \ell(e) \leq \pi(t+1)^2 C^*/c$. By definition there are at most $g(V)$ non-empty sets E_{i,e_0}. This implies for the sum of loads $\ell(e)$ of the set $E_{e_0} \subseteq E(N^*)$: $\sum_{e \in E_{e_0}} \ell(e) \leq g(V)(t+1)^2 \pi C^*/c = c' C^* g(V)$, where $c' := (t+1)^2\pi/c$. ■

Combining the last two lemmas we can show that for the t-spanner approximates the optimal network by $O(g(V)^2)$. Since in practice the diversity can be seen as a logarithmic term, such a t-spanner provides a $O((\log n)^2)$-approximation for the congestion.

Theorem 5 *The t-spanner of [AS94] contains a path system $\mathcal{P}$ with congestion $O(g(V)^2 C_{\mathcal{P}^*}(V))$, where $\mathcal{P}^*$ denotes the congestion optimal path system for the vertex set V.*

Proof: From Lemma 4 it follows that there exist a routing on a t-spanner such that the load of an edge e is bounded by $\ell(e) \leq c' g(V) C_{\mathcal{P}^*}(V)$. Lemma 1 shows that the interference number of the network is bounded by $O(g(V))$. This implies $C_{\mathcal{P}}(V) = O(g(V)^2 C_{\mathcal{P}^*}(V))$. ■

4 Trade-Offs

As we have seen there are efficient ways for selecting paths to optimize energy and approximate congestion. One might wonder whether an algorithm can compute a path system for a radio network such that energy, congestion and dilation can be optimized at the same time. It turns out that this is not the case.

4.1 Congestion versus Dilation

For a vertex set G_n placed on the crossings of a $\sqrt{n} \times \sqrt{n}$-grid the best choice to minimize congestion is to connect grid points only to their neighbors given the demand $w(u, v) = W/n^2$ for all vertices (Fig. 7). Then the congestion is $O(W/\sqrt{n})$ and the dilation is given by $O(\sqrt{n})$. In [GK00] it is shown that such a congestion is best possible in a radio network. A fast realization is given by a tree featuring a hop-distance of $O(\log n)$ and congestion $O(W \log n)$ (Such a tree-construction for the Cost-distance problem is presented in [SW01]). In both cases we observe $C_{\mathcal{P}}(G_n) D_{\mathcal{P}}(G_n) \geq \Omega(W)$. This also is true for any other path selection:

Theorem 6 *Given the grid vertex set G_n then for every path system $\mathcal{P}$ the following trade-off between delay $D_{\mathcal{P}}(G_n)$ and congestion $C_{\mathcal{P}}(G_n)$ exists: $C_{\mathcal{P}}(G_n) \cdot D_{\mathcal{P}}(G_n) \geq \Omega(W)$.*

Proof: For $n = 9p^2$ partition the grid into three $p \times 3p$ rectangle shaped vertex sets V_1, V_2, V_3, such that V_1 contains all left vertices, V_3 all right vertices and V_2 the vertices in the middle.

We consider only an $\frac{1}{9}$th of the demand starting at V_1 heading for vertices in V_3. Let $D \leq 3p$ be the delay of the network and $p_{i,j}$ denote the route from vertex v_i to vertex v_j. Let $r(p_{i,j}) = w(u_i, u_j)$ denote the information flow on path $p_{i,j}$.

Consider two vertices $v_i \in V_1$ and $v_j \in V_3$. Then the path $p_{i,j}$ has at most $D_{\mathcal{P}}(G_n)$ edges. The induced capacity $\kappa(p_{i,j})$ of the path $p_{i,j}$ is at least $\kappa(p_{i,j}) \geq c_1 \ell(p_{i,j}) \sum_{e \in p_{i,j}} |e|^2$. This term is minimized if the path uses the maximum possible number $D_{\mathcal{P}}(G_n)$ of edges with equal length of at least $\frac{d}{3D_{\mathcal{P}}(G_n)}$. Then, we have $\kappa(p_{i,j}) \geq \frac{c_1 d^2 W}{9n^2 D_{\mathcal{P}}(G_n)}$.

The sum of the capacity over all paths cannot extend the capacity of the $(3d) \times (3d)$-square containing all possible interference areas. This gives: $\sum_{v_i \in V_1} \sum_{v_j \in V_3} \kappa(p_{i,j}) \leq 9d^2$. Combining the inequalities states the claim: $C_{\mathcal{P}}(G_n) D_{\mathcal{P}}(G_n) \geq \frac{c_1}{81} W$. ■

4.2 Dilation versus Energy

The simplest location of sites is the line vertex set L_n as investigated in [KKKP00], see Fig. 8. Here all vertices $L_n = \{v_1, \ldots, v_n\}$ are placed on a line with equal distances $|v_i, v_{i+1}| = \frac{d}{n}$. Only the first and the last node want to exchange messages, i.e. $w(v_1, v_n) = W$ and $w(v, w) = 0$ for all other pairs (v, w). The energy-optimal network for unit and flow energy is the path $(v_1, v_2, \ldots, v_n)$, given the unit energy U-Energy$_{\mathcal{P}}(L_n) = \frac{d^2}{n}$, the flow energy F-Energy$_{\mathcal{P}}(L_n) = \frac{d^2 W}{n}$ and the delay n.

The fasted network realizes only the edge (v_1, v_n) with hop-distance 1 and unit energy d^2 (and flow energy Wd^2). There are path systems that can give a compromise between these extremes. However, it turns out that the product of delay and energy cannot be decreased:

Theorem 7 *Given the vertex set L_n with diameter d then for every path system $\mathcal{P}$ the following trade-offs between delay D and unit energy* U-Energy *(resp. flow energy* F-Energy*) exist:*

$$\begin{aligned} D_{\mathcal{P}}(L_n) \cdot \textit{U-Energy}_{\mathcal{P}}(L_n) &\geq \Omega(d^2) \ , \\ D_{\mathcal{P}}(L_n) \cdot \textit{F-Energy}_{\mathcal{P}}(L_n) &\geq \Omega(d^2 W) \ . \end{aligned}$$

Proof: Let m be the number of edges of the longest path of the radio network (wlog we assume that there are only edges with non-zero information flow $\ell(e) > 0$). For the unit energy model we can assume that there is only a path p from v_1 to v_n (because introducing more edges needs additional energy without decreasing the delay). We have to minimize U-Energy$_{\mathcal{P}}(p) := \sum_{i=1}^{m} (\ell_i)^2$ defined by the edge lengths $\ell_1, \ldots, \ell_m$, where $\sum_{i=1}^{m} \ell_i = d$. Clearly, the sum is minimal for $\ell_1 = \ell_2 = \cdots = \ell_m = \frac{d}{m}$ giving U-Energy$_{\mathcal{P}}(p) \cdot D_{\mathcal{P}}(p) \geq d^2$.

The bound for the flow energy follows analogously. ■

4.3 The Incompatibility of Congestion and Energy

We will show that for some vertex sets congestion and energy are incompatible. This is worse than a trade-off-situation since there is no compromise possible for energy and congestion.

The vertex set $U_{\alpha,n}$ for $\alpha \in [0, \frac{1}{2}]$ consists of two vertical parallel line graphs L_{n^α}. Neighbored (and opposing) vertices have distance $\frac{d}{n^\alpha}$. There is only demand W/n^α between the horizontal pairs of opposing vertices of the line graphs. The rest of the $n - n^{-\alpha}$ vertices are equidistantly placed between the vertices of each line graph and the lowest horizontal pair of vertices, see Fig. 9.

The minimum spanning tree consists of n vertices where all edges have length $\Theta(dn^{-1})$. This results in a total unit energy of U-Energy$_{\mathrm{MST}}(U_{\alpha,n}) = O(d^2 n^{-1})$ and congestion $C_{\mathrm{MST}}(U_{\alpha,n}) = O(W)$. The flow energy of the (same) minimal network is given by F-Energy$_{\mathrm{MST}}(U_{\alpha,n}) = O(Wd^2 n^{-1})$.

The congestion optimal path system $\mathcal{P}'$ connects only vertices with non-zero demand. Its congestion is $C_{\mathcal{P}'}(U_{\alpha,n}) = O(Wn^{-\alpha})$ and its unit energy is U-Energy$_{\mathcal{P}'}(U_{\alpha,n}) = O(d^2 n^{-\alpha})$. The flow energy is given by F-Energy$_{\mathcal{P}'}(U_{\alpha,n}) = O(Wn^{-2\alpha} d^2)$.

Lemma 5 *For $\alpha \in [0, \frac{1}{2})$ and the vertex set $U_{\alpha,n}$ with diameter d let $x \in \{0, \ldots, n^\alpha\}$ be the number of edges of length $dn^{-\alpha}$ of a path system for the radio network and let $f \in [0, W]$ be the*

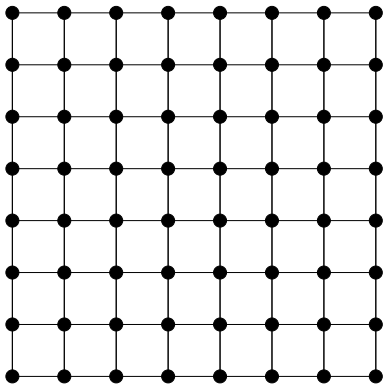

Figure 7: The grid G_n

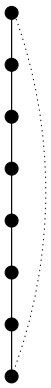

Figure 8: The line L_n

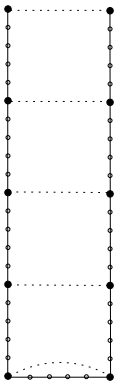

Figure 9: Vertex set U_α

information flow on these edges. Then, we have for the congestion C, unit energy and flow energy:

$$\begin{aligned}
\textit{U-Energy}_{\mathcal{P}}(U_{\alpha,n}) &\geq \frac{d^2}{n} + x\left(\frac{d^2}{n^{2\alpha}} - \frac{d^2}{n^{1+\alpha}}\right) , && (1)\\
C_{\mathcal{P}}(U_{\alpha,n}) &\geq \frac{W}{x+1} , && (2)\\
\textit{F-Energy}_{\mathcal{P}}(U_{\alpha,n}) &\geq f\frac{d^2}{n^{2\alpha}} + (W-f)\left(\frac{d^2}{n} - \left\lfloor \frac{f}{n^\alpha} \right\rfloor \frac{d^2}{n}\right) , && (3)\\
C_{\mathcal{P}}(U_{\alpha,n}) &\geq \frac{f}{n^\alpha} + (W-f) . && (4)
\end{aligned}$$

Proof: The minimum unit energy network is given by the MST which is a U-shaped path. Note that no shortcut within the left and right vertical bars of this path can reduce energy or congestion. Therefore the only reasonable choice for an edge is to connect some (x) of the horizontal vertices (and possibly to disconnect a route to a vertical neighbors). Adding the horizontal channel implies additional energy consumption of $\frac{d^2}{n^{2\alpha}}$. For $x+1$ horizontal routes (including the original low energy route) the best choice is to fairly distributed the traffic.

For the flow energy the argument is analogous. ■

Theorem 8 *There exists a vertex set V with a path system minimizing congestion to C^*, and another path system optimizing unit energy by U-Energy* and minimal flow energy by F-Energy* such we have for any path system $\mathcal{P}$ on this vertex set V we have*

$$\begin{aligned}
C_{\mathcal{P}}(V) \geq \Omega(n^{1/3}C^*) \quad &\textit{or} \quad \textit{U-Energy}_{\mathcal{P}}(V) \geq \Omega(n^{1/3}\,\textit{U-Energy}^*) ,\\
C_{\mathcal{P}}(V) \geq \Omega(n^{1/3}C^*) \quad &\textit{or} \quad \textit{F-Energy}_{\mathcal{P}}(V) \geq \Omega(n^{1/3}\,\textit{F-Energy}^*) .
\end{aligned}$$

Proof: follows by Lemma 5 using the graph $V = U_{1/3,n}$. ■

Hence, there is no hope that communication networks can optimize more than one parameter at a time. The network designer has to decide whether to go for small congestion or low energy consumption.

5 Conclusions and Further Work

The main difference between wired networks and radio networks is that the choice of communication links in wireless networks influences the quality of the edges. We model the type of influences by the interference graph, which gives a very general description how links can interfere. If the sending and receiving characteristics of the radio stations are known, this interference graph can be described by the geometric properties like the location of sites and transmitter power.

	Congestion	Dilation	Unit Energy	Flow Energy
Structure	AS-spanner	Complete Network	MST	Gabriel Sub-Graph
Approx.-factor	$O(\log^2 n)$	optimal	optimal	optimal

Table 1: Approximation results for logarithmic diversity

However, the main difference is still that choosing a certain communication link for some time decreases the ability of transmitting information in some other parts of the radio network. Since the analysis of point-to-point communication (or permutation networks) in wireless networks is relatively young (see [AS98]), we start our investigation with a static simplified model: The point-to-point communication and the location of the sites is fixed (unlike in mobile ad hoc networks). You can also see this model as a snapshot of a more dynamic model (where research has just begun [ABBS01]).

We investigate the question, what is the optimal choice of communication links to achieve the best possible network. We measure the quality by congestion, energy and delay. Given a path system for the packets we present a sound definition of congestion, which takes into account the actual information flow, i.e. load, over a link and the interferences of other links.

There already exists a probabilistic solution for solving interferences if the network parameters are known [AS98]. We show how this algorithm can be applied to our setting. Further, we relate the routing to our notion of congestion and dilation, which is the maximum length of a path.

We prove that for our notion of energy (depending on the packet flow) the optimal path system can be computed in polynomial time. Furthermore, we prove that a t-spanner construction for the communication networks allow path systems with small congestion. Concretely, we show an approximation of a factor of $O(g(V)^2)$ of the minimal congestion, where $g(V)$ denotes the diversity of the vertex set. We introduce this measure to characterize malformed vertex locations. For practical applications we have $g(V) = \Theta(\log)$, e.g. if the vertex set is random, or if the ratio of maximum and minimum distance of nodes is at most polynomial. An overview of these results is shown in Table 1.

However there are situations where it is not possible to optimize two of this measures at the same time, see Table 2. We prove trade-off results for congestion versus dilation and energy versus dilation. For congestion and energy we show that every path system trying to approximate one the congestion within a smaller factor than $o(n^{1/3})$ of the optimal congestion, suffers under an increased energy consumption of at least a factor of $\Omega(n^{1/3})$, and vice versa. Hence, energy and congestion minimization in radio networks are incompatible tasks.

Besides the standard model of omni-directional communication we are currently investigating a sector model where sender and receiver can focus signals (e.g. infrared). Such sector communication is a special case of so-called space multiplexing techniques to increase the network capacity (e.g. by using directional antennas [KSV00]). All results shown in this paper can be easily transferred to such a model.

Another possibility to decrease interferences is to use multiple frequencies, (as done in Bluetooth [Miy00] or IEEE 802.11 [IEE97]). As long as number f of frequencies is small (which is the case in practice, because of governments' regulation all frequency spectra) this may improve the congestion by this factor f. However, using frequency hopping cannot completely resolve the shown the trade-off and incompatibility problems, shown here.

This work is part of a project where a prototype communication system is developed based on infrared directed communication. The prototype will be able to communicate in eight sectors independently with adjustable transmission powers. Furthermore, it can be used as an extension module for the mobile mini robot Khepera ([MFG99, KTe00]). Thus, realistic scenarios for ad hoc networks can be reproduced by performing experiments with these mini robots. Thus, beside computer simulations, our communication strategies will also be validated under practical conditions. Such a network is technically more complicated, but our goal is to show that it is possible to set up a geometric spanner graph as a communication network. Notably, this paper shows that such

	Delay	Congestion
Congestion	$C_{\mathcal{P}}(V) \cdot D_{\mathcal{P}}(V) \geq \Omega(W)$	—
Unit Energy	$D_{\mathcal{P}}(V) \cdot \mathrm{UE}_{\mathcal{P}}(V) \geq \Omega(d^2)$	$C_{\mathcal{P}}(V) \geq \Omega(n^{1/3} C^*_{\mathcal{P}}(V))$ or $\mathrm{UE}_{\mathcal{P}}(V) \geq \Omega(n^{1/3} \mathrm{UE}^*_{\mathcal{P}}(V))$
Flow Energy	$D_{\mathcal{P}}(V) \cdot \mathrm{FE}_{\mathcal{P}}(V) \geq \Omega(d^2 W)$	$C_{\mathcal{P}}(V) \geq \Omega(n^{1/3} C^*_{\mathcal{P}}(V))$ or $\mathrm{FE}_{\mathcal{P}}(V) \geq \Omega(n^{1/3} \mathrm{FE}^*_{\mathcal{P}}(V))$

Table 2: Trade-Offs and Incompatibilities on network parameters

geometric spanners always provide good solutions for congestion minimization in radio networks.

Appendix

Theorem 9 *Consider a radio network M with path system $\mathcal{P}$, and a routing problem w with dilation D and congestion C. Let T be its optimal routing time, when the path system $\mathcal{P}$ is used. The following holds.*

1. $T \geq \max\{C, D\}$.
2. *There is an offline routing protocol with routing time $3D \cdot C \cdot (2 \log n + \log C)$.*
3. *If C and the path system is known, then there is a randomized distributed online protocol that needs expected time $O(C \cdot D)$ for each packet.*

Proof:

2. We consider super-rounds consisting of $3\, C \log W$ consecutive synchronous rounds, where $W := \sum_{u,v \in V} w(u,v)$ is the number of packets. During a super-round every packet will advance one step towards its destination.

   The number of packets at link e within a super-round is at most $\ell(e)$. Now consider the graph $L = (P, E')$ where the nodes are all packets P and the edges E' are defined as follows. $e(p)$ denotes the link a packet p passes during a super-round. We define the directed edges of L by:

   $$E' := \{(p, p') \mid e(p) \in \mathrm{Int}(e(p'))\} \ .$$

   Clearly, the graph L has in-degree C and $|W| \leq |E|C \leq Cn^2$ vertices. From Lemma 6 it follows that the undirected version of this graph can be colored with $3C \log |W|$ colors. Every color corresponds to a time point within the super-round when the packet advances over its edge e.

   Hence, after D super-rounds, every packet arrives at its origin.

3. We define a probability $p(e)$ for activating a link of the communication network. If no other edge interferes, then a packet advances one step over this edge.

   We choose $p(e) := \min\{\frac{1}{2}, \frac{\ell(e)}{C}\}$. Then it holds:

   $$p(e) + \sum_{e' \in \mathrm{Int}(e)} p(e) \ \leq \ 1 \ .$$

   By Lemma 7 it follows that the probability that the transmission over this link is successful is at least $\frac{1}{4} p(e)$. The expected time the first packet is transferred over e is therefore at most $\frac{4}{p(e)}$. Hence, the expected time that all at most $\ell(e)$ packets, waiting at a link, are delivered is at most $\frac{4\ell(e)}{p(e)} \leq 4C$. Hence, the over-all expected time a packet needs to be routed is bounded by $4\ C \cdot D$.

■

Lemma 6 *The undirected version of a directed Graph $G = (V, E)$ with in-degree d can be colored with at most $3d \log |V|$ colors.*

Proof: Let $P(v)$ be the set of direct predecessors of a node v. Define $L_0 := V$ and $L_{i+1} := \{v \in L_i \mid |P(v) \cap L_i| \geq 2d\}$. The degree of the undirected version of the subgraph G_i of G induced by the vertex set $L_i \setminus L_{i+1}$ is bounded by $3d - 1$. Therefore, G_i can be colored using $3d$ colors.

Note that $|L_{i+1}| \leq \frac{1}{2}|L_i|$. Thus, there are at most $\log |V|$ non-empty vertex sets L_i. We color every vertex set L_i with a disjoint set of $3d$ colors which implies that no conflicts between G_i and G_j can appear. Therefore the over-all number of colors is at most $3d \log |V|$. ■

Lemma 7 *The probability of a successful transmission on a link e is at least $\frac{1}{4}p(e)$.*

Proof: Note, that $1 - p \geq \frac{1}{4^p}$ for $p \in [0, \frac{1}{2}]$. Let e_i, $i \in \{1..m\}$, $m \in \mathbb{N}$, be defined as the edges $e_1, ..., e_m$ that interfere with e. Then it holds:

$$\begin{aligned} \text{Prob[Transmission on link } e \text{ is successful]} &= p(e) \prod_{i=1}^{m} (1 - p(e_i)) \\ &\geq p(e) \prod_{i=1}^{m} \frac{1}{4^{p(e_i)}} = p(e) \frac{1}{4^{\sum_{i=1}^{m} p(e_i)}} \geq \frac{1}{4} p(e) \ . \end{aligned}$$

■

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