Self-Localization based on Ambient Signals

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Abstract. We present an approach for the localization of passive nodes in a communication network using ambient radio or sound signals. In our settings the communication nodes have unknown positions. They are synchronized but do not emit signals for localization and exchange only the time points when environmental signals are received, the time differences of arrival (TDOA). The signals occur at unknown positions and times, but can be distinguished. Since no anchors are available, the goal is to determine the relative positions of all communication nodes and the environmental signals.

The Ellipsoid TDOA method introduces a closed form solution assuming the signals originate from far distances. The TDOA characterize an ellipse from which the distances and angles between three network nodes can be inferred.

The approach is tested in numerous simulations and in indoor and outdoor settings where the relative positions of mobile devices are determined utilizing only the sound produced by assistants with noisemakers.

1 Introduction

The increasing mobility of computing devices like smart phones, PDAs, laptops, and tablet computers is a motivation to revisit the localization problem from a fresh perspective.

The usual approach is to include special hardware like GPS receivers, which adds extra monetary cost and power consumption. However, in shielded areas and for small distances such location hardware cannot solve the problem. This is in particular the case for sensor networks in houses or tunnels. Then, the standard approach is to use anchor points in the communication network and calculate the positions by the time of arrival (TOA), time difference of arrival (TDOA) or by the received signal strength indication (RSSI) of radio signals.

Our approach starts with the following idea. Suppose we have a number of devices with microphones in a room which are connected by a communication network, e.g. mobile phones or laptop computers. Now, somebody walks through the room snapping fingers. Solely based on the time when these sound signals are received, all distances and angles between network nodes are computed.

The practicability of our approach can easily be seen. Since most modern computing devices like laptops and smart phones are equipped with everything we need (microphone, wireless LAN) the software can be run without any cost or effort. Sound sources are widely available in crowded areas like market places or in an open air concert. The noises of the people might already be sufficient to be localized.

Or consider localization in a wireless sensor network which has been a time consuming task. Our scheme enables the experimenter to automatize positioning of sensor nodes equipped with microphones just by producing some sharp sound signals before or after a field test to determine the locations of the sensors.

Our software might be extended to use with radio signals instead of sound signals. This will require special hardware to detect time points of radio signals which have to be more precise due to the speed of light. Given such hardware it is possible to compute the relative positions of network nodes like notebook computers, mobile phones, tablet computers or PDAs by using ambient radio signals coming from WLAN base stations, radio or TV broadcast, TV satellites or lightnings. Of course such a localization method must be combined with anchors which give absolute locations.

The special quality of our approach is that we do not have to know the positions of the signal sources. We compute them as well. As a consequence, we can make use of any signal for localization. Even encrypted GPS signal from an unknown positioned satellite or just the signal of a mobile phone of a by-passer will function as an information source. This clearly separates our approach from prevalent approaches which use the information of time of flight, i.e. time of arrival (TOA) or direction of arrival (DOA).

1.1 Related Work

Localization with *known* receiver or sender positions has been a broad and intensive research topic with a variety of approaches. A popular application is GSM localization of mobile phones. Various techniques exist, including angle/direction of arrival (AOA/DOA), time of arrival (TOA, "time of flight"), and time difference of arrival (TDOA) [1]. U-TDOA is a provider-side GSM multilateration technique that needs at least three synchronized base stations. As a client-side implementation needs special hardware, it is hardly prevalent in common mobile phones. Instead, many approaches introduce a distance function based on the received signal strength indication (RSSI). Stable results in the range of meters can be achieved by fingerprinting using a map of base stations [2].

Similar is localization using the RSSI function of WiFi signals. Methods include Bayesian inference [3], semidefinite programming for convex constraint functions [4][5] a combination of WiFi and ultra sound for TOA measurements like the Cricket system [6] or combinations of methods [7].

RSSI evaluation usually comes with difficulties for indoor localization due to the unpredictability of signal propagation [8]. We focus on TDOA analysis in our approach. For TDOA localization of sound and RF signals there is a basic scheme of four or more known sensors locating one signal source. This is solved in closed form [9][10] or with iterative methods [11]. TDOA determination can be done by cross correlation of pairs of signals. An optimal shift between signals is calculated, corresponding to the angle of the signal [12][13]. However, we use signals with a characteristic peak.

Moses et al. [14] use DOA and TDOA information to solve the problem of *unknown* sender and receiver positions. Though sounding similar to our problem, both problem settings differ fundamentally. The additional DOA information enables the authors to apply some sort of "bootstrapping": Initial starting points can be found to solve the problem incrementally.

Raykar et al. locate unknown receivers with onboard audio emitters by time of flight information [15]. Lim et al. locate mobile devices using the RSSI information of unknown WiFi access points [16] given some anchor points in space.

To our knowledge our problem setting of unknown sender and receiver positions with no further information but TDOA has never been addressed so far.

1.2 Problem setting

Given a communication network of n synchronized nodes $\mathbf{M}_1, \ldots, \mathbf{M}_n$, where $\mathbf{M}_i \in \mathbb{R}^2$ denotes the unknown position in two-dimensional Euclidean space. Now m sound (or radio) signals are produced at unknown time points $t_{\mathbf{S}_1}, \ldots, t_{\mathbf{S}_m}$ and at unknown locations $\mathbf{S}_1, \ldots, \mathbf{S}_m \in \mathbb{R}^2$. Each signal $\mathbf{S}_j$ arrives at receiver $\mathbf{M}_i$ at time $t_{\mathbf{M}_i,\mathbf{S}_j}$ which is the only input given in this problem setting. We can measure this time up to an error margin which we assume to be Gaussian distributed. We assume that the signals propagate in a straight line from the sources to the receivers with the constant signal speed c and that they are distinguishable.

The problem is to compute all the distances and angles between receivers, solely from the times when environmental signals are received. Of course then, the signal directions can be computed from this information. The mathematical constraints can be described using the signal velocity c, the time $t_{\mathbf{S}_j}$ of signal creation and the time $t_{\mathbf{M}_i,\mathbf{S}_j}$ when the signal is received at $\mathbf{M}_i$:

$$c\,(t_{\mathbf{S}_j} - t_{\mathbf{M}_i,\mathbf{S}_j}) = |\mathbf{S}_j - \mathbf{M}_i|_2 \tag{1}$$

where $|\mathbf{S} - \mathbf{M}|_2$ denotes the Euclidean distance in two-dimensional space.

By squaring the equations of form (1) we yield a quadratic equation system which can be written in quadratic form. Depending on the number of signals and receivers this system is under-defined, well-defined or even over-defined. It can be rewritten as an optimization problem where a polynomial function of degree four needs to be minimized. There is only small hope for an efficient solution for such problems in general.

Our solution considers the case where the signal sources are so far from the receivers that the time difference at two receivers depends only on the angle between the signal beam and the line between the two receivers. The Ellipsoid TDOA method is an elegant closed form solution for three receivers in the two-dimensional space. The solution is tested in numerical simulations of sound sources with realistic distributions of gaussian error.

Finally, we show how our algorithm performs in real-world indoor and outdoor experiments. Here, we generate series of signals at random positions on circles around the computers by clanking a bottle or two wooden planks. This is the sole information we need to compute the relative distances of the computers.

2 Ellipsoid TDOA method for distant sources

We consider the case where the signal origins are very far from the receivers. Under this assumption we develop an approximative approach to reveal distances and angles between a fixed number of three receivers in two-dimensional space. In this special case a smaller number of sound signals is sufficient to compute the relative locations than in the general case. Furthermore, the solution of the problem can be expressed in a closed form.

Once the receiver triangle ABC has been reconstructed we determine the direction of the signal origins.

2.1 TDOA ellipse

For three receivers A, B, C in the plane and a distant source S the discrete signal is received by the receivers at time points t_A, t_B and t_C, see Fig. 1. Define

$$\Delta t_1 = t_B - t_A \tag{2}$$

$$\Delta t_2 = t_C - t_A \tag{3}$$

where Δt_1 and Δt_2 are the time differences of arrival (TDOA) between A and B, resp. A and C. For $\alpha = \angle_{CAB}$ and using the assumption of infinite distant signal origins we state:

$$x := \Delta t_1 = d_1 \cos(\gamma - \alpha/2) \tag{4}$$

$$y := \Delta t_2 = d_2 \cos(\gamma + \alpha/2) \tag{5}$$

where γ denotes the direction of $\mathbf{s}$ with respect to the bisection of α. Combining the equations we derive the following ellipse equation:

$$x^2 \frac{1}{d_1^2} + y^2 \frac{1}{d_2^2} + xy \frac{-2\cos\alpha}{d_1 d_2} = \underbrace{\frac{1}{2} - \frac{1}{2}\cos 2\alpha}_{\sin^2\alpha} \tag{6}$$

Normalization by division by $\sin^2\alpha$ (under the assumption $\alpha \notin \{0, \pi\}$, i.e. A, B, C are collinear) leads to the ellipse parameters

$$a = \frac{1}{d_1^2 \sin^2\alpha}\,, \qquad b = \frac{1}{d_2^2 \sin^2\alpha}\,, \qquad c = \frac{-2\cos\alpha}{d_1 d_2 \sin^2\alpha}$$

for $ax^2 + by^2 + cxy = 1$.

Ellipsoid TDOA localization requires at least three pairs of time differences (Δt_1, Δt_2) from different distant signal origins. From these points we compute the ellipse equation with parameters a, b, c, see Fig. 2. Then, we use the above equations to compute d_1, d_2, α which can be done by the following equations:

$$d_1 = 2\sqrt{\frac{b}{4ab - c^2}}\,, \qquad d_2 = 2\sqrt{\frac{a}{4ab - c^2}}\,, \qquad \alpha = \arccos\frac{-c}{2\sqrt{ab}}$$

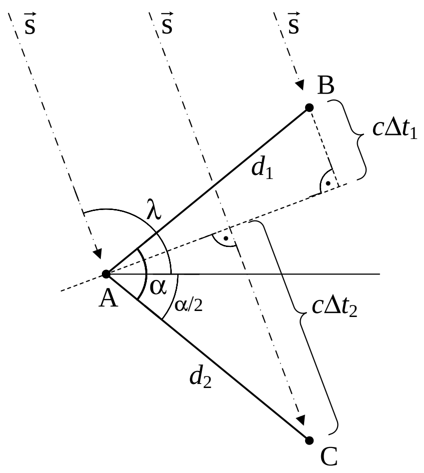

Fig. 1. Three receivers A, B, C and a signal on the horizon with direction $\mathbf{s}$.

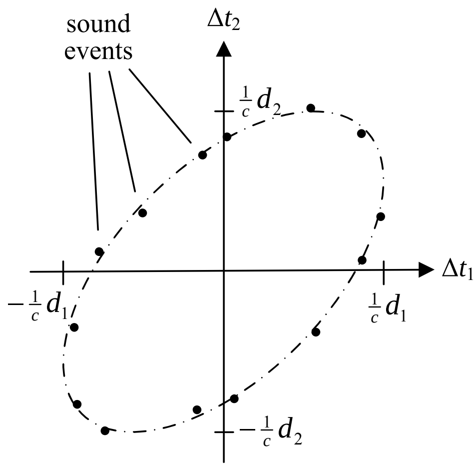

Fig. 2. Multiple distant signal sources with time difference pairs $(\Delta t_1, \Delta t_2)$ in two dimensions form an ellipse.

2.2 Linear regression

Three ambient signals are sufficient to find the ellipse for two dimensions. Since ambient radio or sound signals are no scarce resource the additional signals can be used to overcome the inaccuracies caused by imprecise time measurements and other error sources. Given a sufficient number of $m \geq 3$ signal sources that form a set of (x, y)-tuples we obtain a system of linear equations

$$ax_i^2 + by_i^2 + cx_iy_i = 1 \tag{7}$$

where $1 \leq i \leq m$. We use linear regression to reconstruct the parameters of this ellipse. In matrix notation this is:

$$\underbrace{\begin{pmatrix} x_1^2 & y_1^2 & x_1y_1 \\ \vdots & \vdots & \vdots \\ x_m^2 & y_m^2 & x_my_m \end{pmatrix}}_{\mathbf{Q}} \underbrace{\begin{pmatrix} a \\ b \\ c \end{pmatrix}}_{\mathbf{x}} = \mathbf{1} \tag{8}$$

If $m > 3$ we use the least squares method and solve for the ellipse parameters:

$$\left(\mathbf{Q}^T\mathbf{Q}\right)\mathbf{x} = \mathbf{Q}^T\mathbf{1} \quad \Rightarrow \quad \mathbf{x} = \left(\mathbf{Q}^T\mathbf{Q}\right)^{-1}\left(\mathbf{Q}^T\mathbf{1}\right) \tag{9}$$

If $m = 3$ we solve $\mathbf{x} = \mathbf{Q}^{-1}\mathbf{1}$. For $m < 3$ the system is under-determined and cannot be solved uniquely.

Then, we use the equations of the previous subsection to compute the geometry of the triangle ABC. Since the assumption of infinitely far senders is not realistic this approach results in an approximative solution of the problem.

However, this is the best one can offer if only three signal sources are available, since the problem for three general signal positions is under-defined. Later on, we present simulations which indicate that the approximation behaves well if the signals are a small constant factor farer than the longest edge of the receiver triangle.

2.3 Simulation

We have tested the accuracy of this approximation algorithm with a computer algebra system. A simulation cycle consists of a number of sound sources arbitrarily arranged on a circle with a fixed radius around the origin. Three microphones A, B and C are positioned on a circle with a fixed radius of about 2.3 m forming a triangle with an edge length of 4 m. The sound sources are received by the microphones at time points t_A, t_B and t_C depending on the distance and the speed of sound. A probabilistic Gaussian error model has been added to each timestamp to simulate measurement errors.

For a set of different radii up to 20 m a series of 1,000 tests with 8 sound sources is run. The distance results d_1 and d_2 and the angle α between A and B are subtracted from the real values, which are read from the triangle properties. Failed runs occur if the approximated quadratic equation does not describe an ellipse. For successful runs we calculated the average and the standard deviation of the distance and angle differences.

The results show a systematic under-estimation of the distances between microphones for short ranges which improves after the perimeter of the microphones has been left at about 5 m (Fig. 3). The angle errors show high variance within the perimeter of the microphones which stabilizes quickly upon leaving it, at a range of 5 m (Fig. 4). Failing localizations occur especially if the sound source radius is equal to the microphone radius with up to 4%, but the rate drops quickly to below 1% (Fig. 5).

A stress test was run to observe the behavior of the approximation in case of runtime variances. Distant sound sources were assumed (radius of 1,000 m) and the gaussian runtime error was increased up to a standard deviation of 2.0 ms. For comparison: In 1.0 ms a sound wave travels about 34 cm. Results show a slight over-estimation of the microphone distances and a moderate increase in angular variance. Failures increase to about 5 %. However, a Gaussian distributed error of 2 ms ranges inside the limits of nearly 3 m, which is a lot for a scenario with an edge length of 4 m. The time differences of this magnitude, drawn as x/y-plot, are hardly recognizable as an ellipse any more (Fig. 6). In our real-world experiments we observed runtime errors with a standard deviation of about 0.2 ms, which is way below the errors we induced here.

3 Real-world experiments

We have tested this theoretical approach in several real-world experiments. For this we use a network of mobile devices as network nodes. Our software establishes UDP communication via local area network (LAN) between several

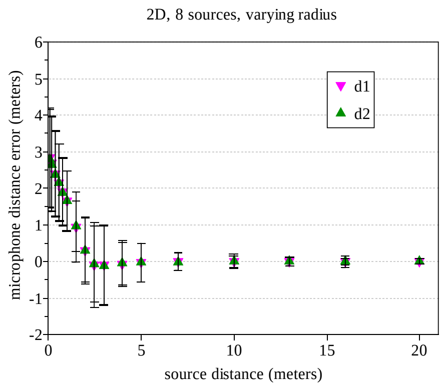

Fig. 3. Increasing sound source distances above 4 m result in distance errors below 0.1 m.

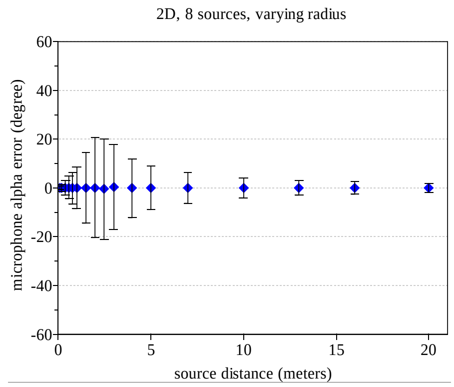

Fig. 4. Increasing sound source distances above 4 m result in angle approximation errors below 2°.

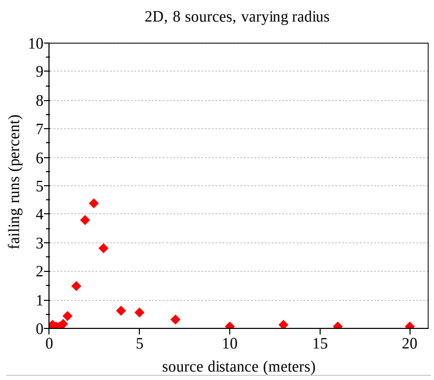

Fig. 5. After reaching a maximum at a sound source distance of 2.5 m the failure rate drops below 1 % for greater distances.

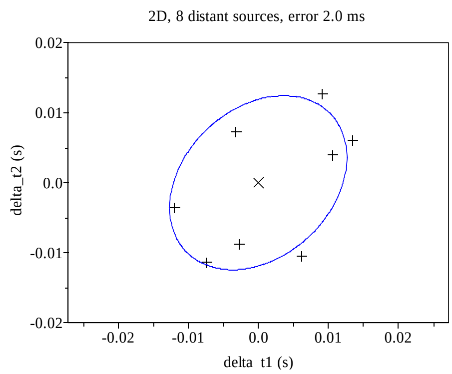

Fig. 6. Eight signals with Gaussian error of $\sigma = 2\,\text{ms}$ were received. The elliptical form is hardly recognizable due to this high error.

devices and assures precise time synchronization. With the built-in microphones we record sound signals. The audio track is searched for sharp sound events, like clapping or finger snapping and their points in time are determined. As a peculiar mark for a sound event we use the moment when the signal rises above a environment noise dependent threshold for the first time.

Threshold comparisons showed to be the robustest approach with only little drawbacks in precision. Maximum searches, either directly or derivative (edge detection) showed to be slightly more precise but prove to be ambiguous with fatal results in cases when hosts chose different maxima.

The detected signals are exchanged between the nodes. With this information given each node can compute the relative locations using the algorithm described before.

3.1 Time synchronisation

Common TDOA localization requires precise synchronization among receivers. While unsynchronized localization is generally possible, time synchronization reduces the number of required sound events. To get a global time reference the nodes elect a master based on priority IDs and synchronize to the master clock. The synchronization is achieved with a series of pings between master and all other nodes to get a good estimation of the round trip time (RTT) to the master. The exchanged reference timestamps are filtered for high RTT (outliers), which results from network jitter, and corrected by 1/2 RTT, assuming the network packet took the same runtime in both directions.

Our experiments pointed out that clock drift correction is essential even with the utilized high precision event timer (HPET). Although running with accurately constant speed, drift rates between different clocks of 0.03 % were observed, which is too high for our purposes, if untreated. Both time offset and clock drift between client and master are obtained by linear regression of the timestamp set. The precision we achieve is within 0.1 ms in a wireless LAN with an RTT of about 10 ms and within 0.01 ms in a wired LAN with an RTT of about 1 ms.

3.2 Experiments

The first real-world test was situated in a large lecture hall with a size of 17 m × 13 m at the University of Freiburg. We arranged 3 laptops A, B, and C in a small triangle residing on a circle with radius 2.3 m and connected them with an ethernet based LAN switch for communication. The triangle was placed in a corner of the hall to test far distant sound sources up to 16 m. To examine the measurement results we noted down the positions of the laptop microphones with a precision of 2 cm and the sound sources with a precision of 10 cm. The distances between the laptops were $d_{AB} = 4.30\,\text{m}$, $d_{AC} = 4.14\,\text{m}$ and $d_{BC} = 3.47\,\text{m}$, which results in $\angle_{CAB} = 48.6°$.

In the experiment, we generated several sound events with an empty glass bottle and a spoon on concentric circles with varying radii around the laptop

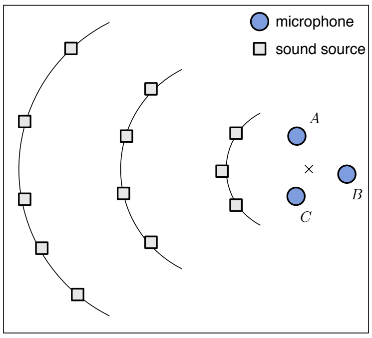

Fig. 7. Series of random signals on concentric circles with varying radii around the computers.

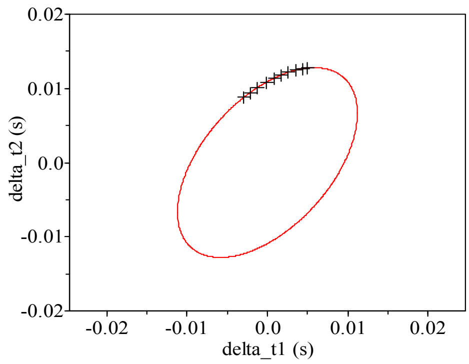

Fig. 8. Time differences from the approximation experiment as x/y-plot. Sound signals from a distance of 13 m arrive from only one direction.

triangle. The audio signals were recorded with the built-in microphones to detect timestamps for the sound events. The Ellipsoid TDOA method was executed with the timestamps of a single radius as the only input to compute the distances between the microphones. Implausible sound signals with a time difference of more than 20 ms (corresponding to 6 m) were filtered. This is to be done automatically in the future.

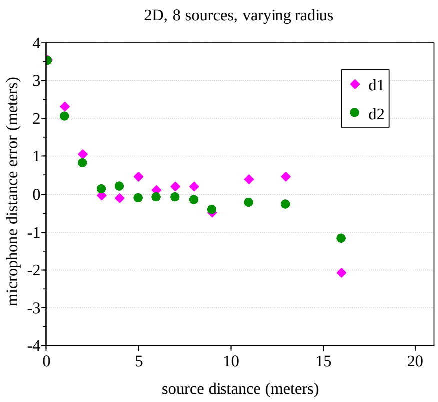

Fig. 9. Distance errors of d_1 and d_2 for the indoor experiment. Errors decrease quickly except for an outlier at 16 m.

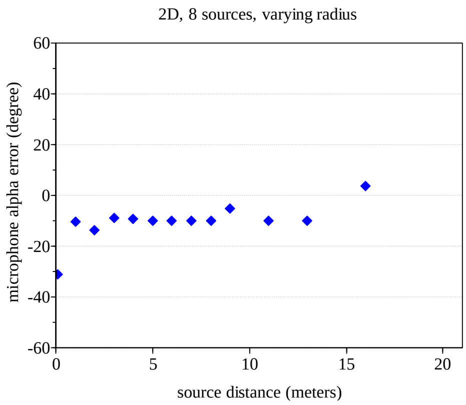

Fig. 10. Angle error of α for the indoor experiment. Angle errors decrease except for a slight over-estimation of about 10°.

The evaluation showed a good convergence of the microphone distance approximations d_1 and d_2 at circle radii of 4 m and above (Fig. 9). Errors fall below 0.5 m. Angle α resides at about 58°, which is an over-estimation of 10° (Fig. 8). With increasing sound signal distance results degrade, which we attribute to the narrowing sector of sound origins. Due to limited room size they come from only one direction, thus making it harder to describe an ellipse, see Fig. 8. This seems to be a drawback of the technique. Obviously we need signals from different angles to reconstruct the ellipse properly.

A second experiment was performed outdoors. We expected to find more realistic conditions like wind noise, birdsong, and the nearby rapid transit system. On the other hand there would be space to generate sound events from all directions, facilitating the ellipse regression. Eight nodes, consisting of four laptops and four Apple iPhones with our software running were placed randomly on a green area of the campus in an area of 30 m × 30 m. Their positions were measured precisely to within 20 cm. A WLAN access point established communication between nodes for synchronization and timestamp exchange.

A series of sound events was produced by an assistant circling the experiment perimeter in varying distances. He generated clearly audible sound signals by clapping two wooden planks. We obtained a series of 50 sounds of which none were filtered.

The Ellipsoid TDOA method was applied to all combinations of three nodes with a total of $n(n-1)(n-2) = 336$ combinations. From every Ellipsoid method run only the two distance measures d_1 and d_2 were used while angle α was discarded. Symmetric duplicates were removed, which resulted in 12 measures for each of the 28 node pairs. The measures belonging to the same node pair were averaged. They form a complete graph of known node distances.

By optimization we calculated the relative positions (x_i, y_i) of the microphones from the node distances d_{ij}:

$$\min_{x,y} \left(\sum_{i=1}^{n} \sum_{j=i+1}^{n} (x_i - x_j)^2 + (y_i - y_j)^2 - d_{ij}^2 \right)$$

The resulting point set was mapped onto the real-world positions by a congruent rotation and translation. This was done by calculating the SVD (Singular Value Decomposition) of the point set correlation which provides an optimal transformation to minimize distances of associated points in the least squares sense.

The average distance from ground truth after mapping was 38 cm with a standard deviation of 14 cm. Fig. 11 shows the mapped point set and the real-world positions. Fig. 12 depicts the ellipse for node (1), (3) and (8) as an example. For the distant sound signals the marks reside on the ellipse. Only when the assistant came closer to the microphones the infinite distance assumption was violated and the marks lie inside the ellipse. However, this did not affect the robust ellipse regression. Neither did the environmental noise affect our results, as they have no influence on the sound velocity and our signals were loud enough to predominate the noise.

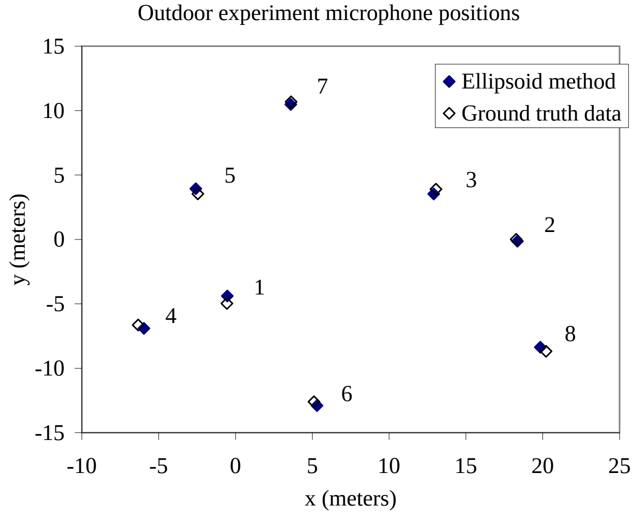

Fig. 11. Relative positions of the microphones mapped onto the ground truth data. The average distance from ground truth is 38 cm (standard deviation: 14 cm).

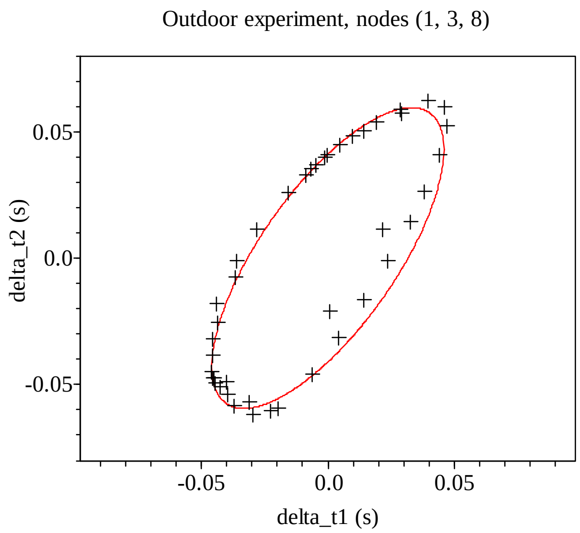

Fig. 12. Ellipse of time differences for the nodes (1), (3) and (8) with the distances $d_{(1)(3)}$ and $d_{(1)(8)}$. Nearby sound events deform the ellipse, instead of residing on the ellipse border (*lower right sector*).

4 Conclusion

To our knowledge, we are the first to consider the problem of relative localization of nodes using nothing but TDOA information of ambient signals. The Ellipsoid TDOA method does not need any anchor points in space. In our method, we only need distinguishable sound events which we assume to travel with a constant speed on a direct line.

Our considerations about the degrees of freedom point out that position reconstruction without any given anchors cannot be done with less than four receivers. However, the approximation scheme enables us to state some propositions about the receiver positions and the direction of the signal sources even with three receivers.

The technique requires a minimum number of three signals in two-dimensional space. However, it directly benefits from an increased number of signal events. These are cheaply available in many environments. Then, our technique becomes very robust, even for noisy data.

Simulation and real-world tests suggest that our assumption of infinitely remote signal sources is not far-fetched. The parallax decreases quickly, as soon as we are outside the receiver perimeter. This allows us to use the approximation even in close-ranged scenarios.

The approximation scheme fails if receiver positions collapse on a line or a plane. In this case, the time differences form a line of which no ellipse can be extracted. However, this singular case can be detected and treated particularly.

In some cases of noisy data we found a slight, systematic over-estimation of receiver distances and angles. For the stress test runs this resulted in higher variance. Visual analysis of the time differences showed that the resulting ellipse does not fit the corpus of the noisy data properly. This seems to be a result of deficient ellipse regression.

4.1 Future work

It is very obvious that time synchronization is hard to achieve for radio signals due to the much higher speed of light. The Ellipsoid TDOA method can be extended to work without time synchronization between computers. Then, the minimum number of sound signals increases from three to five. However, sound signals are not a scarce resource.

We have also seen some room for improvement in the approximation of the TDOA ellipse. While our regression minimizes the error "in some least squares sense" [17], there are more sophisticated techniques available like *geometric fit* proposed by Gander et al. [17].

Further research will involve the use of non-discrete continuous signals, e.g. voices, traffic noise or analogous radio signals. By testing for best overlaps of such signals it should be possible to compute a time difference analogously to sharp signals. This would dramatically increase the information basis of the algorithm.

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