Performance Analysis of the Hierarchical Layer Graph for Wireless Networks

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Abstract

Recently, a promising network topology for wireless networks, called the Hierarchical Layer Graph (HL graph), has been introduced and analyzed by Meyer auf der Heide et al. 2004. This graph can be used as a topology for wireless networks with variable transmission ranges. In this paper we present a distributed, localized and resource-efficient algorithm for constructing this graph. The performance of the HL graph depends on the domination radius and the publication radius, which affect the amount of interference in the network. Worst case analysis shows a tradeoff between these parameters. We investigate the performance on randomly distributed vertex sets and show that the restrictions on these parameters are not so tight using realistic settings.

Here, we present the results of our extensive experimental evaluation, measuring congestion, dilation and energy. Congestion includes the load that is induced by interfering edges. We distinguish between congestion and realistic congestion where we take also the signal-to-interference ratio into account. Our extensive experiments show that the HL graph contains energy-efficient paths as well as paths with a few number of hops while preserving a low congestion.

1 Introduction and Overview

Topology control is an important issue in the field of wireless networks. Excellent surveys are presented by X.-Y. Li [Li03b, Li03a, Li03c] and R. Rajaraman [Raj02]. The general goal is to select certain connections to neighboring nodes that may be used for the network communication. On the one hand each node should have many connections to neighboring nodes to achieve fault-tolerance. On the other hand, if a node has many links, i.e. a high in-degree, then the probability of interference among these links is high and maintaining such a high number of links is not practicable. It is the task of the topology control algorithm – as

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part of the link layer – to select suitable links to neighboring nodes such that connectivity is guaranteed and the number of links per node as well as the amount of interference is low.

If the nodes can adjust their transmission power, then energy can be saved and interference can be reduced when using short links. Nevertheless, connectivity of the network must be guaranteed. The problem of assigning transmission ranges to the nodes of a wireless network so as to minimize the total power consumed under the constraint that adequate power is provided to the nodes to ensure that the network is strongly connected (i.e., each node can communicate along some path in the network to every other node) is called the *minimum range assignment problem*. This problem is known to be NP hard [KKKP00, CPS00]. Note, that we consider another model. We assume that every node is equipped with an power-variable omnidirectional antenna, i.e. every node is allowed to adjust its transmission range at any time to transmit a message. In [MSVG04] a promising network topology, called the Hierarchical Layer Graph (HL graph), for power-variable wireless networks has been introduced and analyzed. In this paper we present a distributed, localized and resource-efficient algorithm for constructing the HL graph. We have implemented this topology control algorithm in our simulation environment SAHNE [Vol02, RSVG03] and we present the results of our extensive experimental evaluations concerning the HL graph using realistic settings. We compare our results with the results using the *unit disk graph* as the network topology. An edge between two nodes is contained in the unit disk graph if and only if the Euclidean distance between the nodes is at most 1. The results of our extensive simulations show the impact of the topology on a hop-minimal and an energy-minimal routing.

The remainder of this paper is organized as follows. In Section 2 we review the formal definition of the HL graph introduced in [MSVG04]. In this paper we concentrate on the resources routing time, defined in terms of congestion and dilation, and energy. In Section 3 we present the measures that we use to compare wireless network topologies. In Section 4 we describe the main functionality of our topology control algorithm for the HL graph. In Section 5 we give the settings of our simulations before we present the results of our extensive simulations in Section 6. Finally, in Section 7 we conclude this work and discuss further directions.

2 The Hierarchical Layer Graph

The Hierarchical Layer Graph (HL graph) was first introduced in [GLSV02]. The set of nodes is divided into several layers. The idea is to establish many short edges on the lower layers, that constitute an energy-optimal path system, and create only a small number of long edges on the upper layers, that ensure connectivity and allow short paths, i.e. paths with a small number of hops. The HL graph consists of the layers $L_0, L_1, \ldots, L_w$. The lowest layer L_0 contains all the nodes. The next layers contain fewer and fewer nodes. Finally, in the uppermost layer only one node remains. If a node v belongs to the set of layer-i nodes, i.e. $v \in V(L_i)$, then it belongs also to each layer $V(L_j)$ with $0 \leq j < i$. In each layer a minimal distance between the nodes is required: $\forall u, v \in V(L_i) : |u, v| \geq r_i$. All

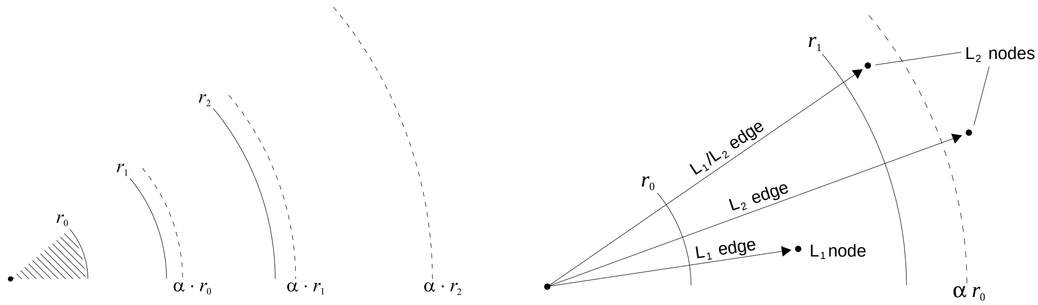

Figure 1: The radii of the HL graph and the edge sets of the HL graph ($\beta < \alpha < \beta^2$)

the nodes in the layers below must be located within the radius r_i around a node in $V(L_i)$: $\forall w \in V(L_{i-1}) \exists v \in V(L_i) : |w,v| \geq r_i$. The distance constraints are defined by the parameters $\alpha \geq \beta > 1$. The smallest radius r_0 is chosen such that $r_0 < \min_{u,v}\{|u,v| : u,v \in V\}$. The other radii are defined as follows: $r_i := \beta^i \cdot r_0$. These radii also define the edge set of each layer (see Figure 1). An edge (u,v) with $u,v \in V(L_i)$ belongs to the edge set of the i-th layer $E(L_i)$, if its length does not exceed the minimal distance r_i by the factor α: $E(L_i) := \{(u,v)|u,v \in V \wedge |u,v| \leq \alpha \cdot r_i\}$. The HL graph contains only symmetric edges. In this paper we assume that all nodes have random and unique IDs which serve for breaking the symmetry in leader election. A node belongs to the i-th layer, if there is no other node within distance r_i with higher priority, i.e. higher ID: $v \in V(L_i) \Leftrightarrow \neg\exists\, w : |v,w| \leq r_i \wedge ID(v) < ID(w)$. A node in $V(L_i)$ is a leader in $V(L_{i-1})$ and all the layers below. The **rank** of a node v denotes the number of the highest layer it belongs to: $r = \text{Rank}(v) \Leftrightarrow v \in \bigcup_{i=0}^{r} V(L_i) \wedge v \notin V(L_{i+1})$. In the following we refer to r_i as **domination radius** (β-radius) and to $\alpha \cdot r_i$ as **publication radius** (α-radius). According to the definition of the HL graph, the domination radii determine which nodes belong to a layer and the publication radii determine which edges are established (see Figure 1).

3 Measures for Network Topologies

The quality of a routing scheme depends on the quality of the topology of the network. Instead of focussing on one specific routing algorithm we consider path systems that use the topology and investigate the quality of the paths systems using the measures congestion, dilation, and energy which have been proposed in [MSVG02, GLSV02]: Given a path system $\mathcal{P}$. The **dilation** is given by the maximum of the lengths of all paths in $\mathcal{P}$. Regarding energy we distinguish between unit energy, which reflects the power consumption of maintaining the edges, and flow energy, which reflects the power consumption of using the paths for communication. Maintaining a communication link e is proportional to $|e|^2$, where $|e|$ denotes the Euclidean length of e. Then, the **unit energy** is defined by $\sum_{e \in E(\mathcal{P})} |e|^2$. If we take the **load**

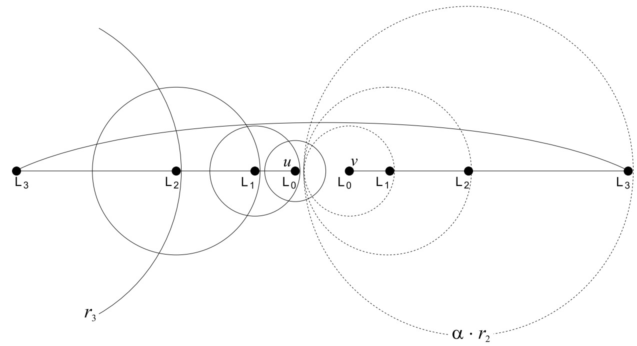

Figure 2: Worst case construction for the HL graph

of an edge $\ell(e)$ (i.e. the number of paths in $\mathcal{P}$ using this edge) into account we obtain the **flow energy**, which is defined by $\sum_{e \in E(\mathcal{P})} \ell(e)|e|^2$.

The **load** of a path system is the maximum load of an edge: $L(\mathcal{P}) := \max_{e \in E(\mathcal{P})} \ell(e)$ (for wired networks this is often called congestion, see [Lei92]). The definition of congestion for wireless networks contains also the load which is induced by interfering edges. The **congestion of an edge** e is given by $C(e) := \ell(e) + \sum_{e' \in \mathrm{Int}(e)} \ell(e')$ where $\mathrm{Int}(e)$ is the set of edges e' that interfere with e. The **congestion** of a path system is defined by $C(\mathcal{P}) := \max_{e \in E(\mathcal{P})} C(e)$.

The set of interfering edges $\mathrm{Int}(e)$ can also be described according to the model used in [MSVG02]: An edge $e' = (v_1, v_2)$ interferes with $e = (u_1, u_2)$ if $\exists\, v \in e', \exists\, u \in e \;:\; ||v - u|| \leq r$, where r is the transmission range used by node v. The motivation for modeling interference this way is that communication in networks usually includes the exchange of acknowledgements, so that interference is also a problem for the sender who expects to receive an acknowledgement. Beside this model, we use a more realistic model in the simulation, including the power attenuation according to the free space propagation model and the signal-to-interference ratio. In particular, we consider the **realistic congestion** $C_r(\mathcal{P})$ that we introduced in [RSVG03]. The realistic congestion includes load, interference, and properties of the propagation model. The definition is the same as for congestion, but for counting the interfering edges, we take the power attenuation and the signal-to-interference ratio (SIR) into account. Let us assume, that transmissions take place on all edges. An edge $e' = (v_1, v_2)$ interfers with an edge $e = (u_1, u_2)$ if the transmission on e' causes a received power p' at u_1 or u_2 that is higher than the received power p caused by a transmission on e divided by the SIR (i.e. $p/p' < \mathrm{SIR}$).

For the computation of congestion, we consider two path systems which are constructed by solving an all-pairs-shortest-path-problem w.r.t. hop-optimal and energy-optimal paths. The hop-optimal path system $\mathcal{P}_d$ optimizes dilation, whereas the energy-optimal path system $\mathcal{P}_e$ optimizes flow energy.

In [MSVG04] it is shown that a c-spanner allows us to approximate an energy-optimal

Preliminaries
RANDOM($x_0, \dots, x_i$) generates a random number uniformly chosen from $\{x_0, \dots, x_i\} \subset \mathbb{N}$.
SEND(`type`, `target`, p, i) sends a message to the node with the address `target` in layer i with transmission power p.
r is the rank of the node.
$1/\rho$ is the expected length of an interval between updating a neighbor.
M is a list of messages a node receives.
$p_\alpha[i]$ is the power for the publication radius of layer i
$p_\beta[i]$ is the power necessary for the domination radius of layer i

```
HLTC()
r ← 0
for t ← 1 to ∞
do with Probability ρ
    if RANDOM(0, 1) = 0
      then ℓ ← RANDOM(0, ..., r)
            SEND(NNP, undef, p_α[ℓ], ℓ)
      else  a ← 1
            repeat
                 M ← ∅
                 interference ← false
                 SEND(CFL, undef, p_β[r + 1], r + 1)
                 wait 2^a time steps
                 and add received DIS-Messages to M
                 update interference
                 a ← a + 1
              until interference = false or M ≠ ∅
            if M = ∅
              then r = r + 1
```

Figure 3: The hierarchical layer topology control algorithm (HLTC)

path system with a constant factor and a congestion-optimal path system with a factor of $O(\log n)$ for n nodes in general positions. Note, that in a c-spanner there is at least one path between two arbitrary nodes u and v of length at most $c \cdot |u, v|$ where $|u, v|$ denotes the Euclidean distance between u and v (for more details we refer to [SVZ04]). If $\alpha = \beta$ then $\alpha \cdot r_i = r_{i+1}$, i.e. the publication radius of layer i and the domination radius of layer $i + 1$ are congruent. In this case the HL graph is not a c-spanner for any constant c (see Figure 2). However, due to the definition, the graph is still connected, unless the maximum transmission radius is limited. For $\alpha > 2\frac{\beta}{\beta-1}$ the HL graph is a c-spanner with $c = \beta \, \frac{\alpha(\beta-1)+2\beta}{\alpha(\beta-1)-2\beta}$ [MSVG04]. It is known that the amount of interference in the HL graph can be upper bounded by $O(\log n)$. Hence, the HL graph allows an approximation of a congestion-optimal path system by a factor of $\mathcal{O}(\log^2 n)$ for n nodes in general positions.

4 The HL Topology Control Algorithm

The hierarchical layer topology control algorithm basically consists of two parts (see Figure 3): Leader election and link establishment. As the topology control is meant to be a small part of the protocol stack we use a simple leader election mechanism: We assume that the nodes have unique IDs which are regarded as priorities. Initially, each node is in layer 0. Then each node tries to become leader in layer 0 which means that it becomes member of layer 1. Therefor it sends a *claim for leadership* (CFL) message with the transmission range that is covering the domination-radius and increases its rank by one. If other nodes with a higher priority receive this message, then they respond with a *disagreement* (DIS). A node that receives a disagreement has to decrease its rank. If it receives no disagreement it can try to become leader on the next upper layer.

The second part of the protocol, the link establishment is done the following way: A node sends neighbor notification packets (NNP) for a certain layer ℓ. The layer is chosen randomly between 0 and the rank of the node. If a message for layer ℓ is received by a node with a rank less than ℓ, then it is ignored. Each node belonging to layer ℓ that receives the NNP message includes the sender in the list of neighbors[1].

5 Simulation Settings

In this section we present our experimental settings. We want to study the quality of the resulting topology in large-scale networks with high node density. One would assume, that such networks can be characterized by high interference and high congestion. We use our simulator for mobile ad hoc networks, SAHNE [Vol02, RSVG03], that enables us to perform simulations of wireless networks with a large number of nodes. SAHNE has been designed for this purpose and is characterized by a lean model of the physical layer. It's data structures enable efficient range queries in order to determine the nodes that possibly receive a packet.

We consider the following scenario (cf. Table 1): From 50 up to 300 nodes are deployed (randomly and identically distributed) over an area of size 50m $\times$ 50m. The nodes communicate with a low power radio transceiver which supports variable transmission ranges, e.g., the CC1000 from ChipCon[2]. We assume 256 transmission power levels within this range and a maximum transmission range of 50m. We use the signal propagation proposed in [HM04] for flat rural environments, which is similar to the free space propagation model but with a path loss exponent of 3. All the transceivers use one frequency. Instead of a statistical error model, we use a stronger condition for the successful reception of a packet: A packet can be successfully received only if the signal-to-interference ratio is at least 10.

We assume synchronized transmission and simulate the exchange of packets by the topology control algorithm. We do not simulate multi-hop data transmission because we do not want to focus on a specific routing algorithm. Instead, we assess the quality of the constructed topology as a whole using the measures defined in Section 3. Therefor the network simulator calculates shortest paths for all pairs of nodes based on the current network topology in

[1] In practical environments the antennae often have deformed radiation patterns. In this case the receiver of a message cannot assume to reach the sender with the same transmission power. For the link establishment one would use additional acknowledgements for the NNP packets

[2] ChipCon SA, www.chipcon.com. The CC1000 has an programmable output ranging from -20 dBm to 5 dBm

simulation area	50 m $\times$ 50 m
number of nodes	up to 300
max. transmission power	5 dBm
min. transmission power	-20 dBm
min. reception power	-107 dBm
path loss exponent	3
signal-to-interference ratio	10
max. transmission range	50 m

Table 1: Simulation parameters

order to determine the load on the edges of the network graph. According to the simplified theoretical interference model and the more realistic interference model, the interference of the edges is calculated, which is needed to determine the congestion. As distance metric we use either the hop-distance or the energy-consumption of the links. So we can see whether the topology is suitable for shortest path routing in general.

At the beginning of a simulation the nodes are placed uniformly at random in the fixed simulation area, i.e. with an increasing number of nodes the density of the nodes also increases, and with a higher density the probability of interference grows. The maximum transmission is also fixed such that it is unlikely that nodes are not connected. This is important for the all-pairs-shortest-path routing: Missing connections would reduce the load and thus affect the calculation of congestion. Values for unit energy and flow energy are normalized, such that the transmission at maximum power requires the amount of energy of one. We call this unit of measurement **standard energy**. All the simulations are done for 20 node sets. In the tables and diagrams the average values (together with the standard error) are given.

6 Experimental Results

We performed numerous experiments with different combinations of the parameters α and β that determine the publication radius and the domination radius. In general, the HL graph is a c-spanner for a constant c if $\alpha > 2\frac{\beta}{\beta-1}$. In our simulations sometimes we deliberately chose these parameters such that this inequality is not fulfilled to see the behavior using random node placements. A comparision between different settings of α and β is difficult, so we decided to choose a fixed number of layers w and a factor $\delta = \alpha/\beta$, which is the ratio of publication radius and domination radius (α and β are derived from w and δ). We performed experiments with a fixed number of layers ranging from four to ten. With more than ten layers, the radii cannot be assigned to different power levels.

For comparison we constructed a unit disk graph (UDG), see Section 1, with a fixed transmission range of 25m. With a lower transmission range most of the resulting networks are not connected (if 100 nodes are equally distributed over an area of 50m $\times$ 50m). Note, that for the HL graph the maximum transmission range is 50m which can cause more interference. The results of these experiments are shown in Table 2 and Table 3.

The HL graph yields a small load for a high number of layers and $\delta \geq 2$. In this case the

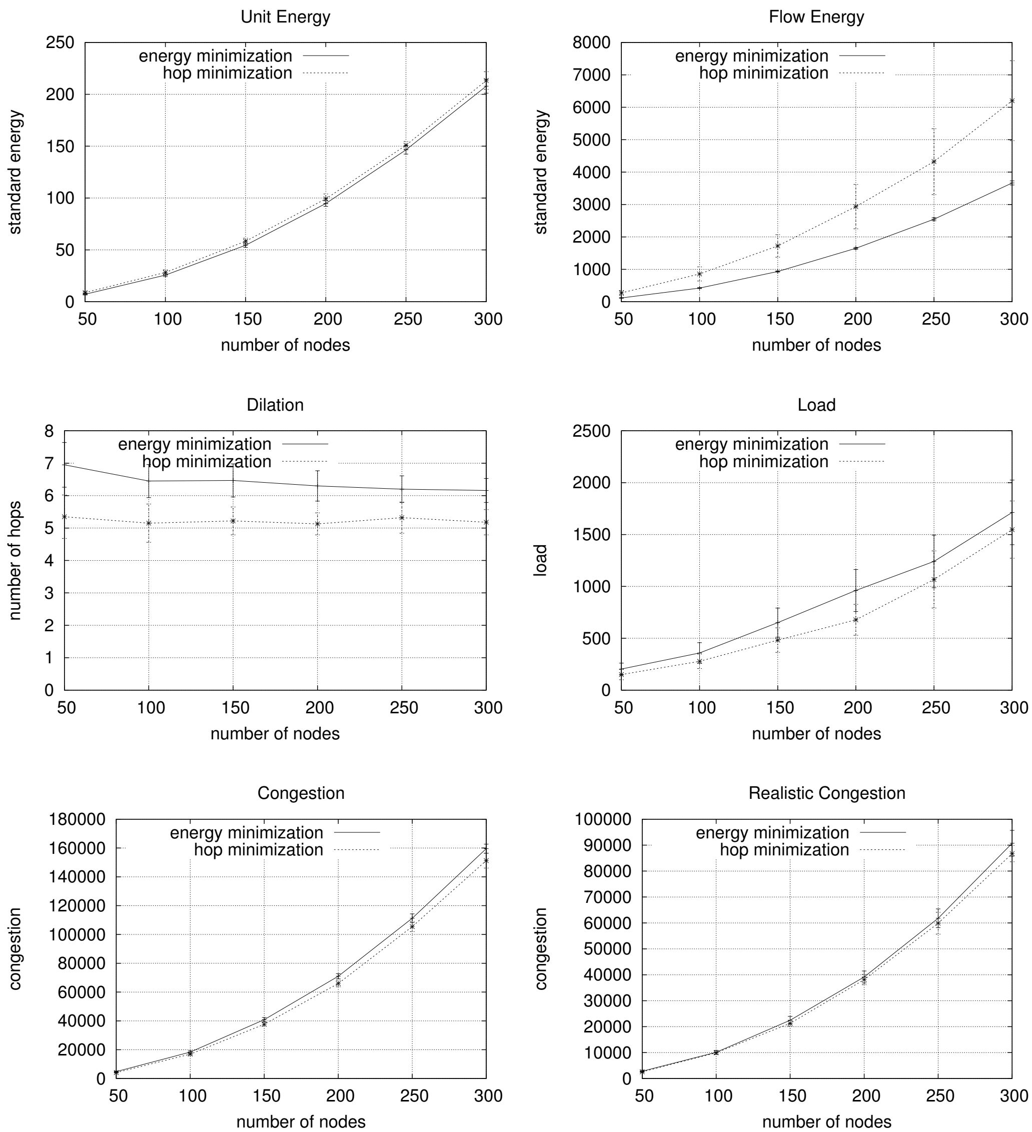

Figure 4: Energy, dilation, load and congestion of the HL graph with 10 layers and $\delta = 2$ for up to 300 nodes (average values of 20 node sets)

hop minimization of the path system results in a lower congestion than the energy minimization. One would expect that in an energy-optimal path system the short edges on the lower layers would be preferred. These edges usually cause fewer interferences. But paths using these edges are longer and (larger hop-count). This causes the relative high load values of the energy-optimal path system. A comparison between the load and the congestion shows the great impact of interferences. Congestion includes the load of interfering edges, which is significantly higher than the original load.

Note, that for constructing the HL graph no range adaption or assignment is performed. Choosing different power levels for the different layers is only a coarse-grained approximation of the optimal transmission range. Thus, the transmission power is not optimal for reaching a certain neighboring node. This makes the comparison of scenarios with a different number of layers difficult. But it can explain why a choice of ten layers yields better results than fewer layers, because with more layers a better approximation of the optimal transmission ranges can be achieved.

The unit disk graph contains path systems with a higher load, but with a smaller congestion. This is due to the fact that in the HL graph longer edges are allowed. If an edge on the top layer of the HL graph has a high load, then its load is assigned to most of the other edges. This effect is not so strong in the UDG, because the transmission range is limited. Note, that the path systems are not constructed with respect to congestion optimization.

For ten layers and $\delta = 2$ we performed further experiments for hop-optimal and energy-optimal path systems. The results are shown in Figure 4. We observe a super-linear increase of congestion and energy, due to the fact that the area is fixed and thus the node density increases.

A high number of layers yields also a small dilation in combination with $\delta \geq 2$. Especially with hop minimization a very small dilation can be achieved. Here, energy minimization can reduce unit energy and flow energy. Also a small δ, that reduces the publication radius and causes shorter edges on the lower layers, causes a great reduction of unit energy and flow energy. But even with $\delta = 2$ the HL graph yields a flow energy that is about six times smaller than that of an energy minimized path system in the UDG. Yet, the unit energy of the HL graph is about four times smaller.

This demonstrates the advantages of the HL graph: It contains energy-saving paths as well as paths with a small number of hops. In comparison to a unit disk graph with a smaller restricted transmission range the HL graph has a smaller load but a higher congestion. So the routing strategy can decide, if a packet has to be delivered quickly or with low power consumption – the HL graph contains suitable paths for both cases.

7 Conclusion

In this paper we presented the results of our extensive experimental evaluations concerning the hierarchical layer graph (HL graph) for power-variable wireless networks. For this purpose we have developed and implemented the hierarchical topology control algorithm that allows us to construct the HL graph in a local and distributed way. We could show by simulations that the restrictions on the domination radius and the publication radius are not so tight using realistic settings, i.e. the HL graph gives a well-suited topology concerning congestion, dilation and energy also for values for α and β that do not fulfill the inequality $\alpha > 2\frac{\beta}{\beta-1}$ for nodes in general position. For practical considerations the simulation results show how the power levels of a radio transceiver should be set to achieve a good network topology. With a high number of layers the benefit of range adaption becomes visible: With ten layers and $\delta = 2$ we could achieve moderate congestion and small flow energy. Finally, the decision which paths to choose is left to the routing algorithm – the HL graph contains short paths (small hop-count) as well as energy efficient paths.

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energy-optimal path system						
w	δ	α	β	load	congestion C	realistic congestion C_r
4	1	2.52	2.52	1376.29 +/- 411.12	20023.40 +/- 1997.95	11240.70 +/- 976.70
4	2	4.00	2.00	461.15 +/- 139.10	18797.30 +/- 1078.45	10329.70 +/- 885.49
4	3	5.25	1.75	299.40 +/- 64.19	18849.80 +/- 537.63	11628.20 +/- 694.02
6	1	1.74	1.74	1573.00 +/- 517.08	22202.50 +/- 4310.91	12258.50 +/- 2387.25
6	2	3.03	1.52	429.05 +/- 82.04	18656.00 +/- 830.34	10104.50 +/- 1095.90
6	3	4.20	1.40	269.95 +/- 74.30	18748.00 +/- 517.35	11373.80 +/- 821.95
8	1	1.49	1.49	1612.38 +/- 433.30	20119.50 +/- 3588.20	11183.20 +/- 1696.22
8	2	2.69	1.35	409.15 +/- 146.06	17913.20 +/- 979.74	9913.30 +/- 731.78
8	3	3.81	1.27	244.105 +/- 59.27	18244.10 +/- 406.21	11294.10 +/- 556.85
10	1	1.36	1.36	1165.80 +/- 532.90	17270.00 +/- 3298.22	9001.60 +/- 1612.68
10	2	2.52	1.26	358.50 +/- 99.13	18357.40 +/- 835.89	10120.30 +/- 701.61
10	3	3.61	1.21	256.63 +/- 42.27	18148.40 +/- 469.53	11554.60 +/- 907.49
UDG r=25m				731.83 +/- 177.87	9545.28 +/- 1458.02	5904.89 +/- 989.62
hop-optimal path system						
w	δ	α	β	load	congestion C	realistic congestion C_r
4	1	2.52	2.52	686.00 +/- 202.31	16859.90 +/- 687.142	11176.30 +/- 926.78
4	2	4.00	2.00	242.00 +/- 51.81	16187.40 +/- 688.529	10433.90 +/- 333.77
4	3	5.25	1.75	141.95 +/- 46.02	16524.80 +/- 476.963	11140.60 +/- 584.62
6	1	1.74	1.74	1148.46 +/- 314.28	18343.10 +/- 3316.36	11786.90 +/- 2302.61
6	2	3.03	1.52	234.20 +/- 58.96	16364.90 +/- 627.88	9943.85 +/- 755.67
6	3	4.20	1.40	198.32 +/- 99.78	17174.30 +/- 355.95	11053.60 +/- 629.66
8	1	1.49	1.49	1099.75 +/- 196.14	17992.80 +/- 978.47	10103.60 +/- 1261.27
8	2	2.69	1.35	281.05 +/- 72.37	16635.20 +/- 633.17	9986.75 +/- 470.35
8	3	3.81	1.27	194.95 +/- 77.85	17242.50 +/- 588.09	10922.10 +/- 864.51
10	1	1.36	1.36	1137.27 +/- 252.81	17346.50 +/- 1735.72	9618.55 +/- 1556.63
10	2	2.52	1.26	278.05 +/- 69.68	16907.20 +/- 653.95	9925.10 +/- 713.20
10	3	3.61	1.21	195.00 +/- 49.55	17493.70 +/- 348.91	11340.00 +/- 585.41
UDG r=25m				808.79 +/- 369.47	10104.10 +/- 1530.98	6177.32 +/- 1088.30

Table 2: Congestion of the HL graph with 100 nodes (average values of 20 node sets with standard error)

energy-optimal path system						
w	δ	α	β	dilation	unit energy	flow energy
4	1	2.52	2.52	14.93 +/- 2.53	3.50 +/- 0.59	209.58 +/- 40.48
4	2	4.00	2.00	6.90 +/- 0.45	33.02 +/- 2.73	432.44 +/- 16.78
4	3	5.25	1.75	5.00 +/- 0.00	118.86 +/- 8.88	756.46 +/- 23.64
6	1	1.74	1.74	16.82 +/- 4.17	2.64 +/- 0.36	221.77 +/- 56.69
6	2	3.03	1.52	6.85 +/- 0.49	28.42 +/- 1.38	431.52 +/- 12.46
6	3	4.20	1.40	5.00 +/- 0.00	108.18 +/- 8.55	749.98 +/- 23.77
8	1	1.49	1.49	17.25 +/- 1.83	2.35 +/- 0.10	227.50 +/- 38.76
8	2	2.69	1.35	6.60 +/- 0.50	25.76 +/- 1.54	437.28 +/- 21.97
8	3	3.81	1.27	4.79 +/- 0.42	97.12 +/- 5.62	751.62 +/- 29.17
10	1	1.36	1.36	13.20 +/- 0.84	2.26 +/- 0.22	181.25 +/- 19.43
10	2	2.52	1.26	6.45 +/- 0.51	25.67 +/- 1.37	422.47 +/- 15.48
10	3	3.61	1.21	4.74 +/- 0.45	95.68 +/- 7.08	743.09 +/- 24.96
UDG r=25m				9.50 +/- 0.79	116.86 +/- 8.33	5118.71 +/- 161.63

hop-optimal path system						
w	δ	α	β	dilation	unit energy	flow energy
4	1	2.52	2.52	7.75 +/- 0.93	8.84 +/- 2.46	1139.20 +/- 238.72
4	2	4.00	2.00	4.45 +/- 0.51	59.26 +/- 10.99	1126.91 +/- 154.73
4	3	5.25	1.75	3.95 +/- 0.23	161.93 +/- 23.37	1249.81 +/- 129.22
6	1	1.74	1.74	10.62 +/- 2.33	4.12 +/- 0.73	929.19 +/- 210.77
6	2	3.03	1.52	5.00 +/- 0.32	34.85 +/- 4.30	919.84 +/- 188.72
6	3	4.20	1.40	4.00 +/- 0.33	117.10 +/- 10.99	1051.85 +/- 159.39
8	1	1.49	1.49	12.13 +/- 2.03	3.15 +/- 0.61	582.44 +/- 280.00
8	2	2.69	1.35	5.10 +/- 0.45	30.89 +/- 1.97	884.97 +/- 136.52
8	3	3.81	1.27	4.05 +/- 0.22	108.05 +/- 7.54	943.38 +/- 101.19
10	1	1.36	1.36	12.09 +/- 1.51	2.83 +/- 0.56	663.70 +/- 456.18
10	2	2.52	1.26	5.15 +/- 0.59	28.26 +/- 2.60	857.53 +/- 217.14
10	3	3.61	1.21	4.00 +/- 0.00	102.35 +/- 6.53	946.29 +/- 62.57
UDG r=25m				9.84 +/- 1.61	116.39 +/- 5.47	5216.28 +/- 552.05

Table 3: Dilation, unit energy and flow energy of the HL graph with 100 nodes (average values of 20 node sets with standard error)