Algorithms for Radio Networks

Orthogonal Frequency Division Multiplexing

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Repetition

- **Multiplexed**
  - Spatial Multiplexing
  - Frequency division multiplexing
  - Time division multiplexing
  - Code division multiplexing
  - Multiple-input multiple-output (next lecture)

- **Modulation**
  - Amplitude modulation
  - Phase modulation
  - Frequency modulation
Principle of OFDM

- OFDM (Orthogonal Frequency Division Multiplex)
  - Signals are divided into parallel signal streams
  - Parallel signals are modulated on carrier waves of different frequencies, phase / amplitude
  - e.g. 16-QAM
  - The carrier signals are combined and transmitted simultaneously

- Special form of frequency-division multiplexing
- The carrier waves using orthogonal frequency:
  - frequencies f, 2f, 3f, 4f, 5f, ...
Repetition: Complex Numbers

- i: imaginary number with
  - $i^2 = -1$

- A complex number is a linear combination of a real part $a$ and imaginary $b$
  - $z = a + bi$

- Calculation rules:
  - $(a+bi)+(c+di) = (a+c) + (b+d) i$
  - $(a+bi)(c+di) = (ac - bd) + (ad + bc) i$
  - $1/(a+bi) = (a-bi)/(a^2+b^2)$

- Complex conjugate
  - $(a+bi)^* = (a - bi)$
Exponentiation of Complex Numbers

- Important equation
  - $e^{i\pi} = -1$
  - $e^{i\varphi} = \cos \varphi + i \sin \varphi$

- Exponentiation of a complex number
  - $e^{a+bi} = e^a e^{bi} = e^a (\cos b + i \sin b)$

- Therefore
  - real part $e^{i\varphi}$: $\text{Re}(e^{i\varphi}) = \cos \varphi$
  - imaginary of $e^{i\varphi}$: $\text{Im}(e^{i\varphi}) = \sin \varphi$
Equivalent Representations of the FFT

- **Real number representation**
  - Sine and cosine functions of different frequencies

  \[ g(x) = \sum_{k=0}^{N-1} a_k \cos \frac{2\pi kt}{T} + b_k \sin \frac{2\pi kt}{T} \]

- **Computation of the inverse by cosine/sine integral product**

  \[ a_k = \frac{2}{T} \int_{0}^{T} g(t) \cos(2\pi n ft) dt \]
  \[ b_k = \frac{2}{T} \int_{0}^{T} g(t) \sin(2\pi n ft) dt \]

- **Complex representation**
  - real part of the exponential function of different frequencies

  \[ f(x) = \sum_{k=0}^{N-1} z_k e^{i2\pi kt/T} \]

- **Computation of the inverse by the integral over the product with the complex conjugated carrier wave**

  \[ z_k = \frac{1}{T} \int_{0}^{T} \left( e^{i2\pi kt/T} \right)^* f(x) dt \]
Advantage of the Complex Representation

- Each of the QAM symbols can be represented directly as a complex number

\[ f(x) = \sum_{k=0}^{N-1} z_k e^{i2\pi kt/T} \]
Application OFDM

- **Wired**
  - Broadband Internet (ADSL, VDSL)
  - Powerline communications networks (power line communication)

- **Wireless**
  - WLAN: 802.11 a,g,n
  - Terrestrial digital television DVB-T
  - Mobile communication
    - 802.16 WiMAX (Worldwide Interoperability for Microwave Access)
  - WPAN 802.15.3a
Pros and Cons

‣ Pro
  • High bandwidth at low SINR
  • Simple and efficient method
  • Proven technology
  • Robust to Multiple Path Fading
  • Efficient use of frequency bands

‣ Contra
  • Susceptible to Doppler effect
  • High power consumption
  • Synchronization reduces efficiency
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