Algorithms for Radio Networks

Frequency Assignment

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Cellular Networks

- **Original problem**
  - Rigid frequency multiplexing for a given set of base stations

- **Given**
  - positions of base stations

- **Output**
  - frequency assignment which minimizes the number of interferences

- **How to model acceptable frequency assignments?**
Frequency Assignment

- **Given:**
  - set of points $V \subseteq \mathbb{R}^2$ of $n$ base stations $B_1, \ldots, B_n$
  - each base station covers an area

- **Output:**
  - function $f: V \rightarrow \mathbb{N}$, which maps each base station to a frequency respecting frequency and distance conditions

- **Sample restraints**
  - minimize the number of given frequencies
  - minimize the width of the frequency range
  - minimize the number of interferences
Frequency Assignment: Models

- Interference graph $G_{\text{Int}}$:
  - nodes are base stations
  - edges describe possible interferences between base stations
Graph Coloring

- **node k-coloring**
  - Given undirected graph $G=(V,E)$
  - A mapping $f:V \rightarrow F$ is a $k$-node coloring
    - if $f(u) \neq f(v)$ for $\{u,v\} \in E$ and $|F|=k$.

- **chromatic number $\chi(G)$**
  - is the minimum $k$ to color graph $G$

- **clique number $\omega(G)$**
  - is the largest number of nodes which form a complete subgraph (clique) in $G$

- **Relationship of $\omega(G)$, $\chi(G)$ and the degree of the graph $\Delta(G)$**
  - $\omega(G) \leq \chi(G) \leq \Delta(G) + 1$
Computational Complexity

- The degree can be easily seen from the graph description
- Clique number
  - Computation $\omega(G)$ is NP-hard
  - Can be computed in time $O(n^{\omega(G)})$
- Chromatic Number
  - k-Coloring of a graph is NP-complete (if $k \geq 3$)
  - Computation of the chromatic number is NP-hard
  - Can be computed in Zeit $O(\chi(G)^n)$
Approximation Algorithms

- Let $P(I)$ be the solution of an optimization problem for instance $I$
  - $I = G$ [given undirected graph]
  - $P(I) = \chi(G)$ [chromatic number of $G$]

- **Definition:**
  - $P$ can be absolutely approximated with additive term $f(n)$, if there is a polynomial time bound bounde algorithm $A$ such that for all instances $I$ of size $n$
    $$| P(I) - A(I) | \leq f(n)$$
  - $P$ can be relatively approximated with factor $g(n)$, if there is a polynomial time bound bounded algorithm $A$ such that for all instances $I$ of size $n$
    $$\max \left\{ \frac{P(I)}{A(I)}, \frac{A(I)}{P(I)} \right\} \leq g(n)$$
Results for Graph Coloring

- Graph Coloring is NP-hard
  - cannot be approximated by a factor of \( n^\varepsilon \) für \( \varepsilon > 0 \) unless \( \text{NP} \neq \text{P} \).

- „Can a given planar graph be colored with three colors“
  - is NP-complete

- But:
  - Every planar graph can be colored with four colors in polynomial time
  - Every graph can be colored (if possible) with two colors in polynomial time
  - There is an absolute approximation algorithm with quality \( O(n/\log n) \) for the general coloring problem
Approximation Algorithm for Node Coloring

- Independent Set Problem (NP complete):
  - Let $G=(V,E)$ be a graph and $U \subseteq V$.
    - $U$ is independent, if: $\{u,v\} \notin E$ für alle $u,v \in U$
  - Independent set problem
    - compute a maximum set
Approximation Algorithm for Node Coloring

- **Algorithmus GreedyIS:**
  \[ U = \emptyset, \ G = (V, E) \]

  **while** \( V \) not empty **do**
  
  Create graph with nodes \( V \)
  
  Choose nodes \( u \) with minimal degree
  
  Erase \( u \) and all neighbors of \( u \) in \( G \) from \( V \)
  
  Insert \( u \) into \( U \)
  
  **od**

  Return \( U \)

- **GreedyIS**
  - computes a maximal (non extendable) independent set
  - run-time \( O(|V|+|E|) \)
Approximation Algorithm for Node Coloring

- **Algorithm GreedyCol:**
  
  \[ G = (V, E), \text{Color} = 1; \]

  **while** \( V \) not empty do
  
  Create \( G \) from \( V \) and determine \( U \) with \( \text{GreedyIS}(G) \)
  
  Color all nodes in \( U \) with Color
  
  Remove \( U \) from \( V \) and increment Color

  **od**

  Return node coloring

- **GreedyCol computes in polynomial time a node coloring with \( O(n/\log n) \) colors**

  - There are better approximation algorithms
Models

- **Color model**
  - Neighbored cells have different frequencies
  - Leads to node coloring of the interference graph

- **Advantage**
  - Simple model

- **Disadvantage**
  - No efficient algorithms are known for Coloring
  - Not an adequate model
    - relationship of received signal strength and influence of neighbored frequencies is not reflected by the model
Labeling versus Coloring

- **Coloring**
  - Use of reusable frequencies
  - Minimize the total number of colors = frequencies available with minimum frequency distances

- **Labelling**
  - Each frequency is assigned only once
  - Frequency distances must be complied
  - Minimize used spectrum

- **Set-(Coloring/Labeling)**
  - A set of frequencies is assigned to a station instead of a single frequency

- **Distance function d of the interference graph**
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