ALBERT-LUDWIGS-
UNIVERSITÄT FREIBURG

Algorithms for Radio Networks

Frequency Assignment

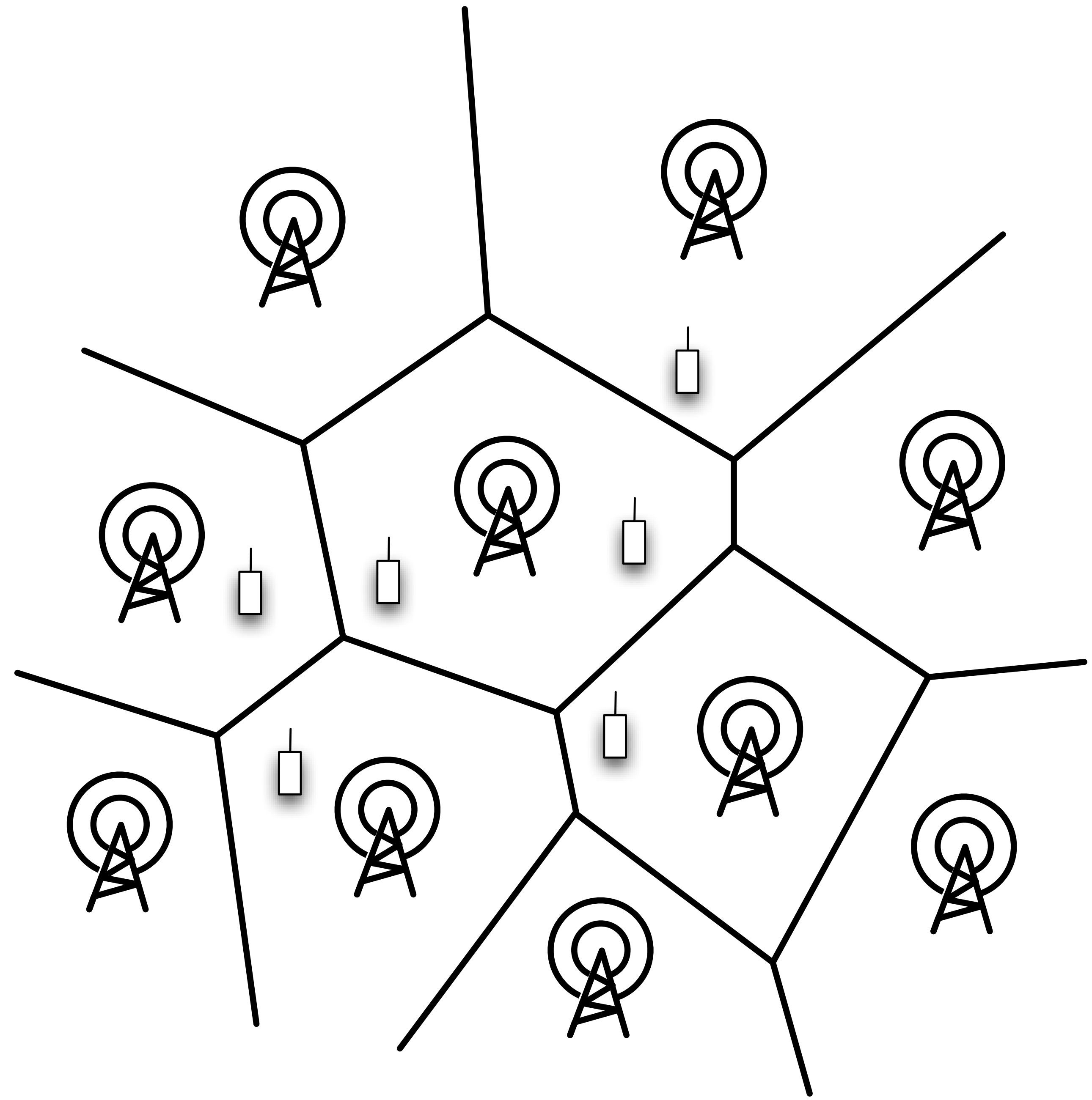
University of Freiburg
Technical Faculty
Computer Networks and Telematics
Prof. Christian Schindelhauer

CoNe Freiburg

IIF INSTITUT FÜR INFORMATIK FREIBURG

Cellular Networks

- **Original problem**
  - Rigid frequency multiplexing for a given set of base stations
- **Given**
  - positions of base stations
- **Output**
  - frequency assignment which minimizes the number of interferences
- **How to model acceptable frequency assignments?**

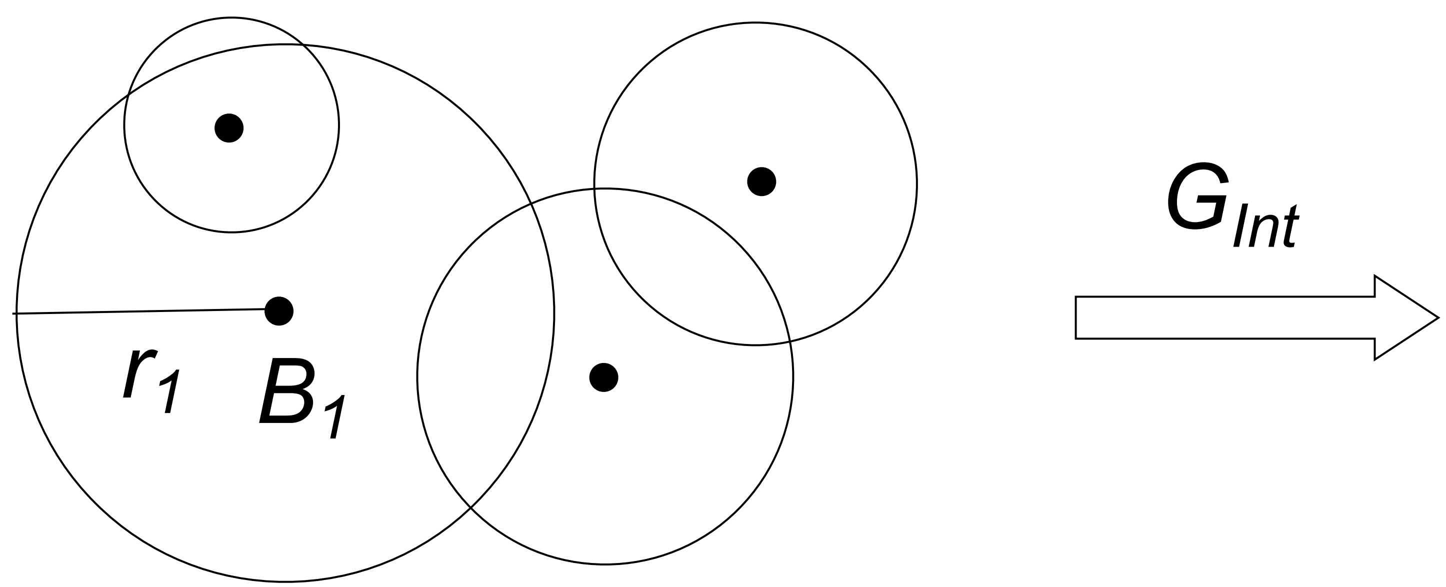


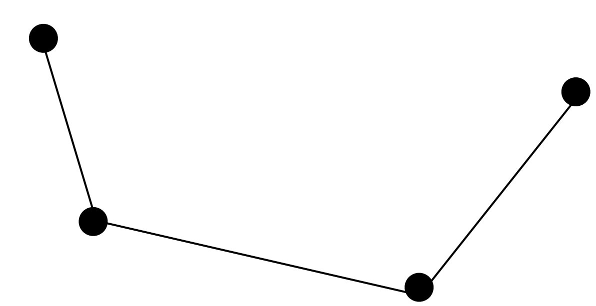
Frequency Assignment

- **Given:**
  - set of points $V \subseteq \mathrm{IR}^2$ of n base stations $B_1, \ldots, B_n$
  - each base station covers an area
- **Output:**
  - function f:$V \rightarrow \mathrm{IN}$, which maps each base station to a frequency respecting frequency and distance conditions
- **Sample restraints**
  - minimize the number of given frequencies
  - minimize the width of the frequency range
  - minimize the number of interferences

Frequency Assignment: Models

- **Interference graph** G_{Int}**:**
  - nodes are base stations
  - edges describe possible interferences between base stations





Graph-Färbungsproblem

Graph Coloring

- **node k-coloring**
  - Given undirected graph $G=(V,E)$
  - A mapping $f:V \rightarrow F$ is a k-node coloring
    - if $f(u) \neq f(v)$ for $\{u,v\} \in E$ and $|F|=k$.
- **chromatic number $\chi(G)$**
  - is the minimum k to color graph G
- **clique number $\omega(G)$**
  - is the largest number of nodes which form a complete sub-graph (clique) in G
- **Relationship of $\omega(G)$, $\chi(G)$ and the degree of the graph $\Delta(G)$**
  - $\omega(G) \leq \chi(G) \leq \Delta(G) + 1$

Computational Complexity

- **The degree can be easily seen from the graph description**
- **Clique number**
  - Computation $\omega(G)$ is NP-hard
  - Can be computed in time $O(n^{\omega(G)})$
- **Chromatic Number**
  - k-Coloring of a graph is NP-complete (if $k \geq 3$)
  - computation of the chromatic number is NP-hard
  - Can be computed in Zeit $O(\chi(G)^n)$

Approximation Algorithms

- **Let P(I) be the solution of an optimization problem for instance I**
  - I=G [given undirected graph]
  - P(I) = χ(G) [chromatic number of G]
- **Definition:**
  - P can be absolutely approximated with additive term f(n), if there is a polynomial time bound bounde algorithm A such that for allinstances I of size n

    $$| P(I) - A(I) | \leq f(n)$$

  - P can be relatively approximated with factor g(n), if there is a polynomial time bound bounded algorithm A such that for all instances I of size n

    $$\max\left\{\frac{P(I)}{A(I)}, \frac{A(I)}{P(I)}\right\} \leq g(n)$$

Results for Graph Coloring

- **Graph Coloring is NP-hard**
  - cannot be approximated by a factor of n^{ε} für $\varepsilon>0$ unless NP≠P.
- **„Can a given planar graph be colored with three colors“**
  - is NP-complete
- **But:**
  - Every planar graph can be colored with four colors in polynomial time
  - Every graph can be colored (if possible) with two colors in polynomial time
  - There is an absolute approximation algorithm with quality O(n/log n) for the general coloring problem

Approximation Algorithm for Node Coloring

- **Independent Set Problem (NP complete):**
  - Let $G=(V,E)$ be a graph and $U \subseteq V$.
    - U is **independent**, if: $\{u,v\} \notin E$ für alle $u,v \in U$
  - Independent set problem
    - compute a maximum set

Approximation Algorithm for Node Coloring

- **Algorithmus GreedyIS:**

  $U=\emptyset$, $G=(V,E)$

  while *V* not empty **do**

  Create graph with nodes *V*

  Choose nodes *u* with minimal degree

  Erase *u* and all neighbors of *u* in G from *V*

  Insert *u* into *U*

  od

  Return *U*

- **GreedyIS**
  - computes a maximal (non extendable) independent set
  - run-time $O(|V|+|E|)$

Approximation Algorithm for Node Coloring

- **Algorithm GreedyCol:**

  G=(V,E), Color =1;

  while V not empty **do**

      Create G from V and determine U with GreedyIS(G)

      Color all nodes in U with Color

      Remove U from V and increment Color

  od

  Return node coloring

- **GreedyCol computes in polynomial time a node coloring with O(n/log n) colors**
  - There are better approximation algorithms

Models

- **Color model**
  - Neighbored cells have different frequencies
  - Leads to node coloring of the interference graph
- **Advantage**
  - Simple model
- **Disadvantage**
  - No efficient algorithms are known for Coloring
  - Not an adequate model
    - relationship of received signal strength and influence of neighbored frequencies is not reflected by the model

Labeling versus Coloring

- **Coloring**
  - Use of reusable frequencies
  - Minimize the total number of colors = frequencies available with minimum frequency distances
- **Labelling**
  - Each frequency is assigned only once
  - Frequency distances must be complied
  - Minimize used spectrum
- **Set-(Coloring/Labeling)**
  - A set of frequencies is assigned to a station instead of a single frequency
- **Distance function d of the interference graph**

ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG

Algorithms for Radio Networks

Frequency Assignment

University of Freiburg
Technical Faculty
Computer Networks and Telematics
Prof. Christian Schindelhauer

CoNe Freiburg

IIF INSTITUT FÜR INFORMATIK FREIBURG