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Algorithms for Radio Networks

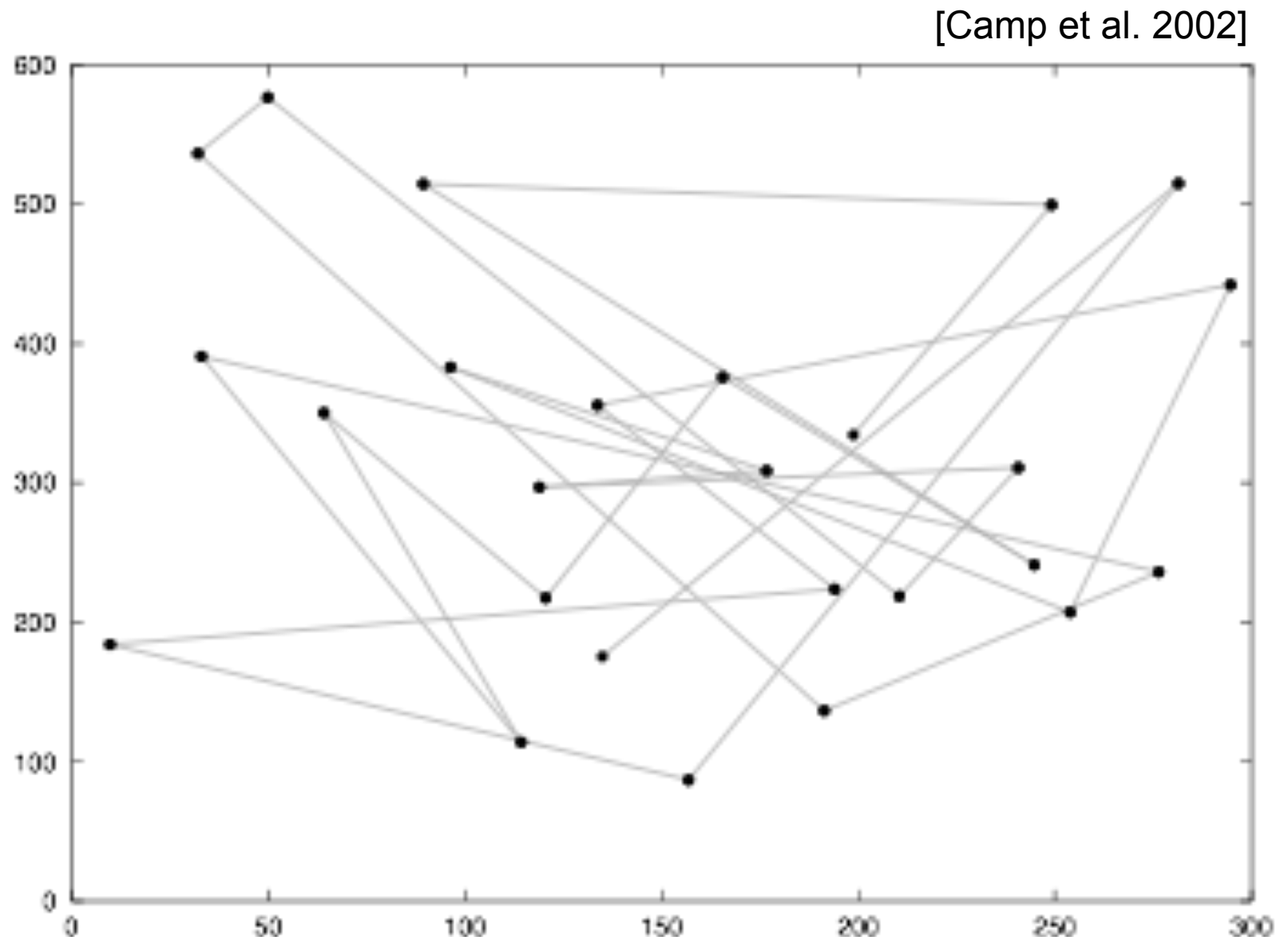
Random Waypoint Considered Harmful

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Random Waypoint Mobility Model

- ▶ Move directly to a randomly chosen destination
- ▶ Choose speed uniformly from $[V_{\min}, V_{\max}]$
- ▶ Stay at the destination for a predefined pause time
- ▶ Repeat from the beginning



Broch, J; Maltz DA, Johnson DB, Hu Y-C, and Jetcheva J (1998). "A performance comparison of multi-hop wireless ad hoc network routing protocols" in *Proceedings of the Fourth Annual ACM/IEEE International Conference on Mobile Computing and Networking (Mobicom98)*, ACM, October 1998

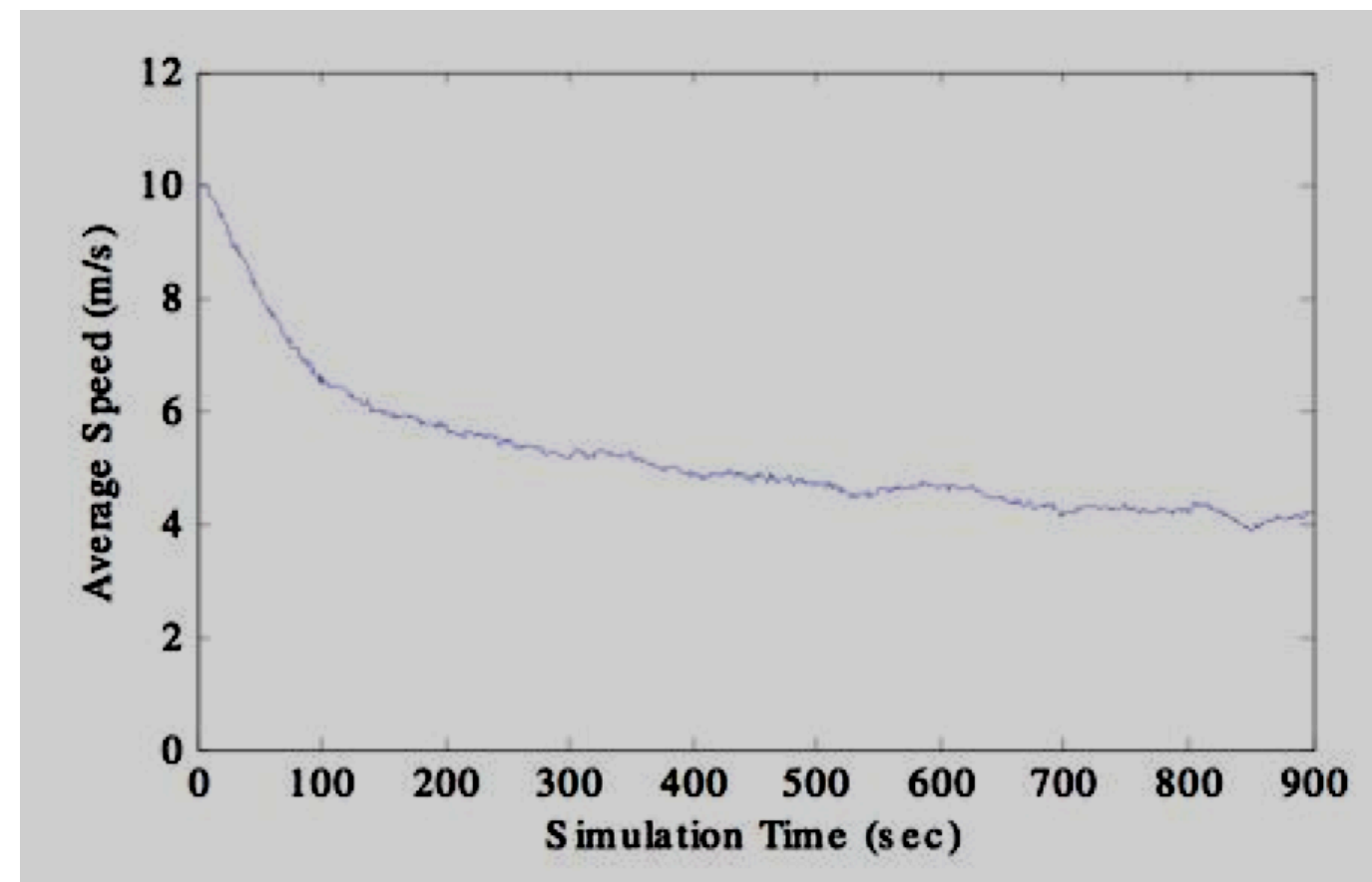
Random Waypoint Considered Harmful

► Yoon, Liu, Noble

- Random Waypoint Considered Harmful, INFOCOM 2003, S. 1312-1321

► Problem:

- If $v_{\min}=0$ then the average speed decays over the simulation time



Random Waypoint Considered Harmful

► Random Waypoint ($V_{\min}, V_{\max}, T_{\text{wait}}$)-Model

- All participants start with random position (x,y) in $[0,1] \times [0,1]$
- For all participants $i \in \{1, \dots, n\}$ repeat forever:
 - Uniformly choose next position (x',y') in $[0,1] \times [0,1]$
 - Uniformly choose speed v_i from $(V_{\min}, V_{\max}]$
 - Go from (x,y) to (x',y') with speed v_i
 - Wait at (x',y') for time T_{wait} .
 - $(x,y) \leftarrow (x',y')$

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► What one might expect

- The average speed is $(V_{\min} + V_{\max})/2$
- Each point is visited with same probability
- The system stabilizes very quickly

► All these expectations are wrong!!!

► Reality

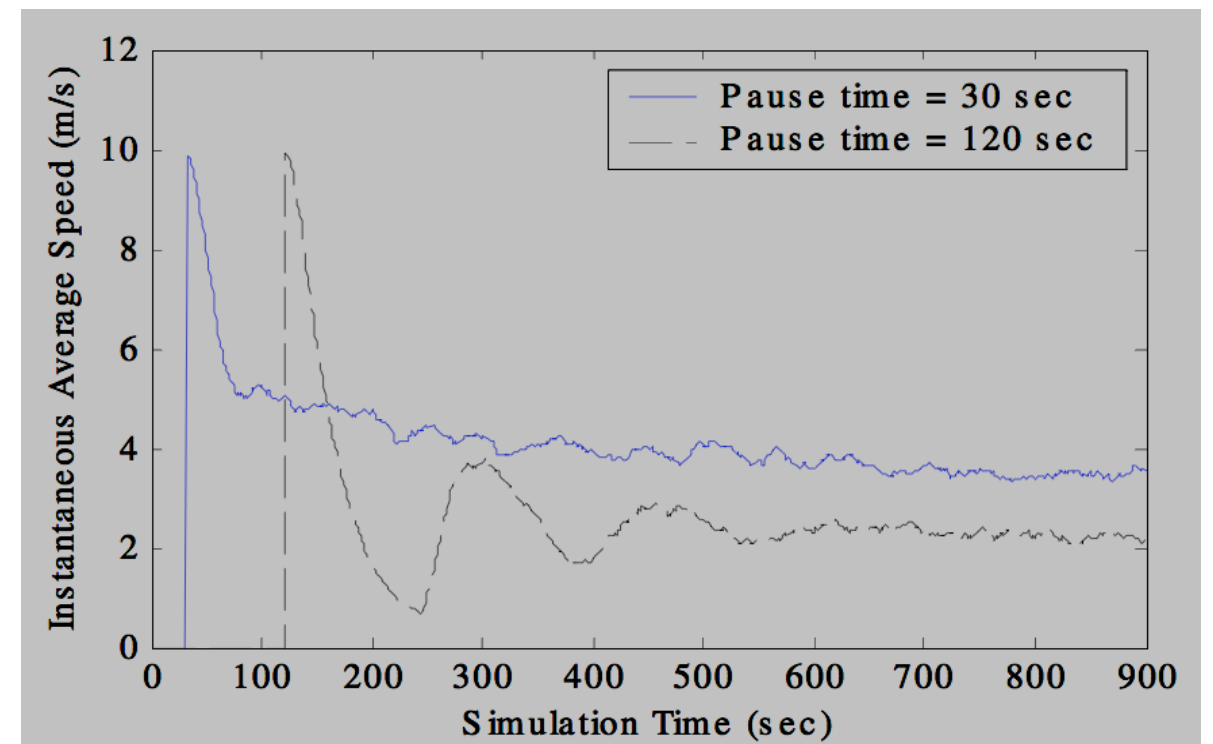
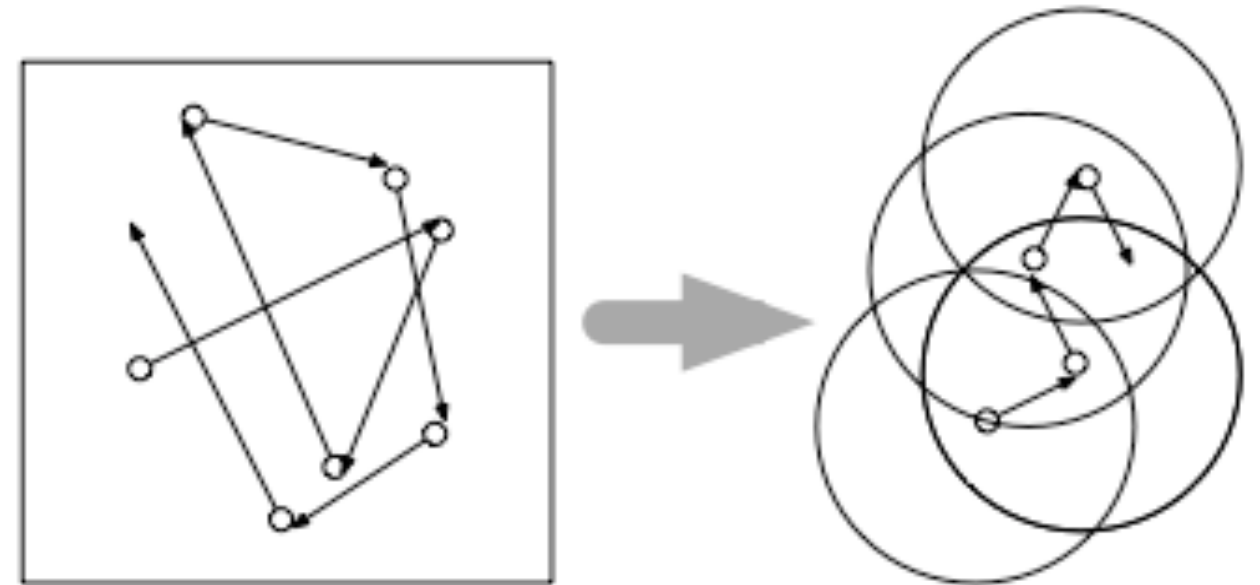
- The average speed is much smaller
- Average speed tends to 0 for $V_{\min} = 0$
- The location probability distribution is highly skewed
- The system stabilizes very slow
- For $V_{\min} = 0$ it never stabilizes

► Why?

The Average Speed is much Smaller

► Assumption to simplify the analysis:

- Replace the rectangular area by an unbounded plane
- Choose the next position uniformly within a disk of radius R_{\max} with the current position as center
- Set the pause time to 0:
 $T_{\text{wait}} = 0$
- This increases the average speed
 - supports our argument



The Average Speed is much Smaller

- ▶ The probability density function of speed of each node

$$V_{\min} \leq v \leq V_{\max}$$

- ▶ Given by

$$f_V(v) = \frac{1}{V_{\max} - V_{\min}}$$

- ▶ since $f_V(v)$ is constant and

$$\int_{v=V_{\min}}^{V_{\max}} f_V(v) dv = 1$$

The Average Speed is much Smaller

- ▶ The Probability Density Function (pdf) of travel distance R:

$$f_R(r) = \frac{2r}{R_{\max}^2} \quad \text{for } 0 \leq r \leq R_{\max}$$

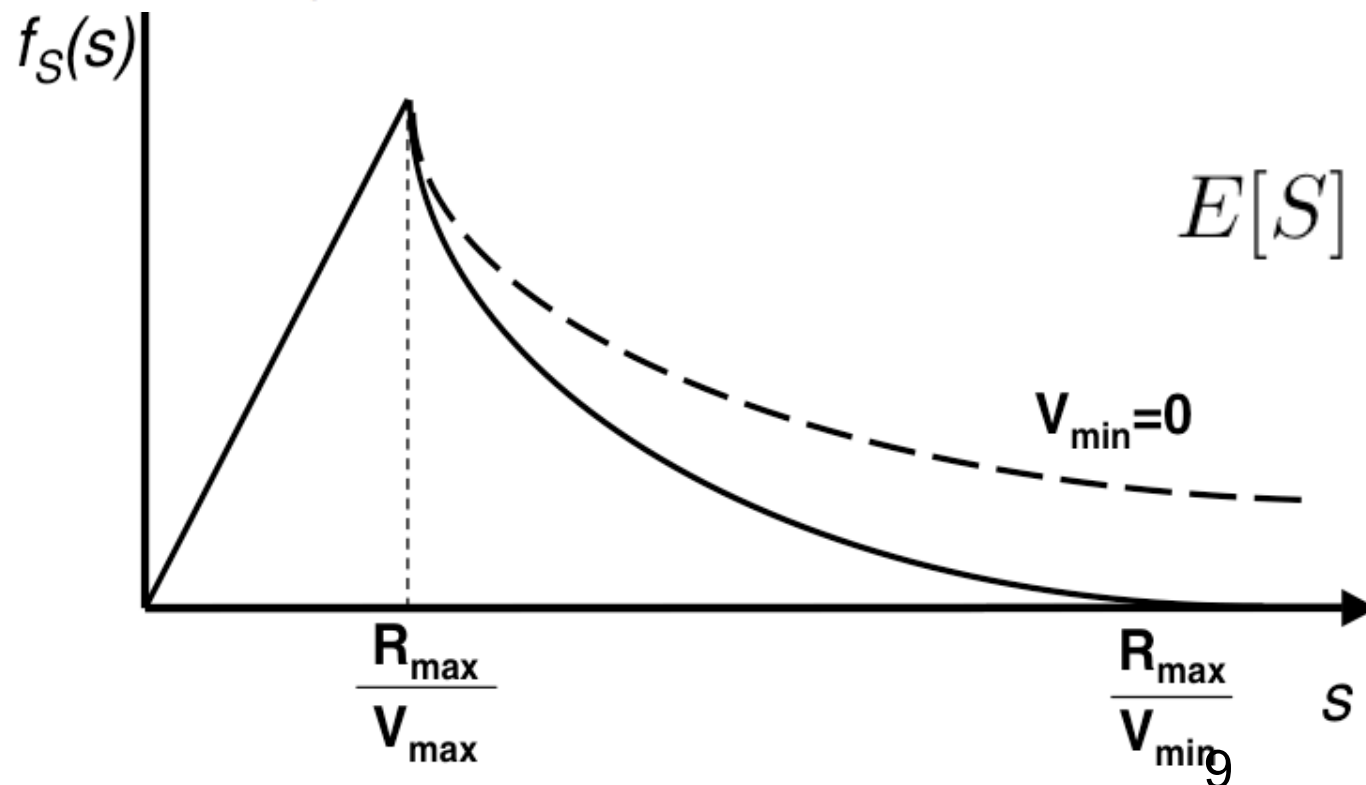
- ▶ The Probability Density Function (pdf) of travel time:

$$f_S(s) = \begin{cases} \frac{2s}{3R_{\max}^2} (V_{\max}^2 + V_{\min}^2 + V_{\max} V_{\min}) , & 0 \leq s \leq \frac{R_{\max}}{V_{\max}} \\ \frac{2R_{\max}}{3(V_{\max} - V_{\min})} \frac{1}{s^2} - \frac{2V_{\min}^3}{3R_{\max}^2 (V_{\max} - V_{\min})} s , & \frac{R_{\max}}{V_{\max}} \leq s \leq \frac{R_{\max}}{V_{\min}} \\ 0 & s \geq \frac{R_{\max}}{V_{\min}} . \end{cases}$$

The Average Speed is much Smaller

- The Probability Density Function (pdf) of travel time:

$$f_S(s) = \begin{cases} \frac{2s}{3R_{\max}^2} (V_{\max}^2 + V_{\min}^2 + V_{\max} V_{\min}) , & 0 \leq s \leq \frac{R_{\max}}{V_{\max}} \\ \frac{2R_{\max}}{3(V_{\max} - V_{\min})} \frac{1}{s^2} - \frac{2V_{\min}^3}{3R_{\max}^2(V_{\max} - V_{\min})} s , & \frac{R_{\max}}{V_{\max}} \leq s \leq \frac{R_{\max}}{V_{\min}} \\ 0 & s \geq \frac{R_{\max}}{V_{\min}} \end{cases}$$



$$E[S] = \frac{2R_{\max}}{3(V_{\max} - V_{\min})} \ln \left(\frac{V_{\max}}{V_{\min}} \right)$$

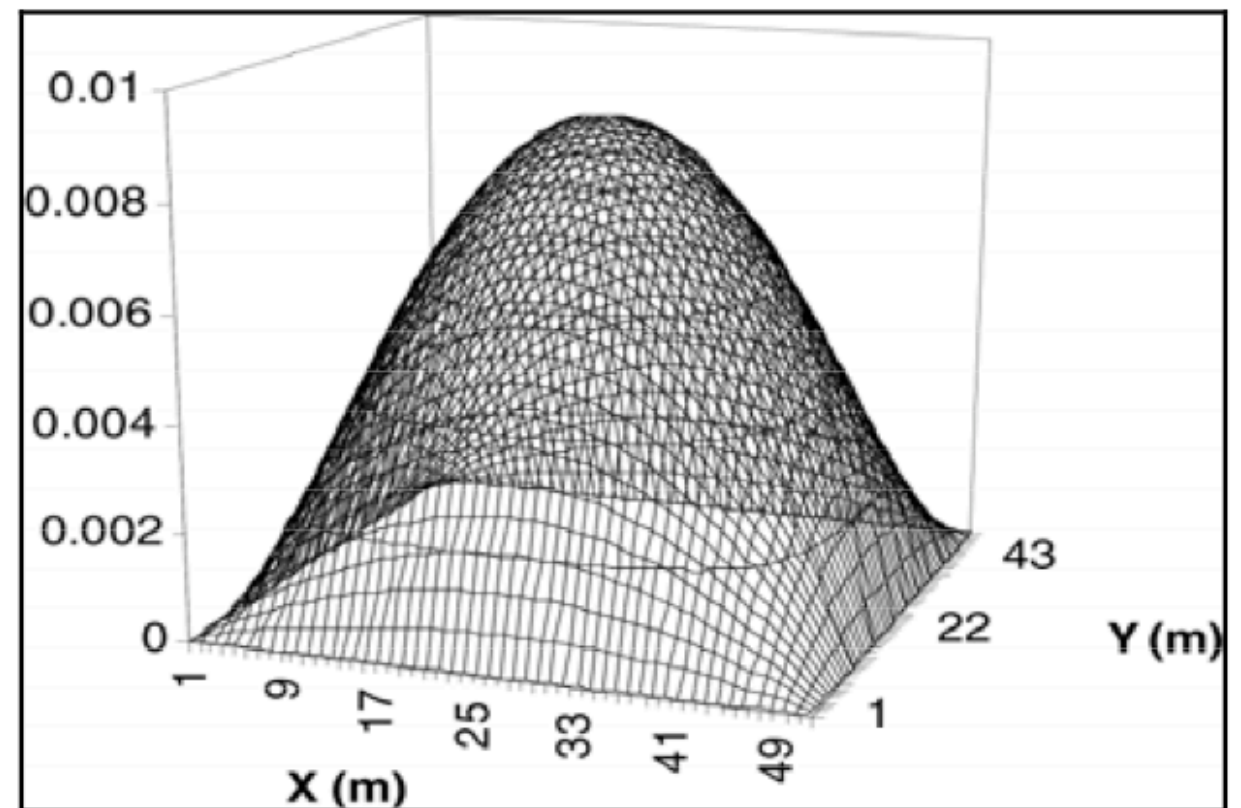
The Average Speed is much Smaller

► Average speed of a node

$$\begin{aligned}\bar{V} &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T v(t) dt \\ &= \lim_{T \rightarrow \infty} \frac{\sum_{k=1}^{K(T)} r_k}{\sum_{k=1}^{K(T)} s_k} \\ &= \lim_{T \rightarrow \infty} \frac{\frac{1}{K(T)} \sum_{k=1}^{K(T)} r_k}{\frac{1}{K(T)} \sum_{k=1}^{K(T)} s_k} \\ &= \frac{E[R]}{E[S]} = \frac{V_{max} - V_{min}}{\ln \left(\frac{V_{max}}{V_{min}} \right)}.\end{aligned}$$

Problems of Random Waypoint

- ▶ In the limit not all positions occur with the same probability
- ▶ If the start positions are uniformly at random
 - then the transient nature of the probability space changes the simulation results
- ▶ **Solution:**
 - Start according the final spatial probability distribution





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