ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG

# Algorithms for Radio Networks

## Wireless Sensor Networks: Sensor Coverage

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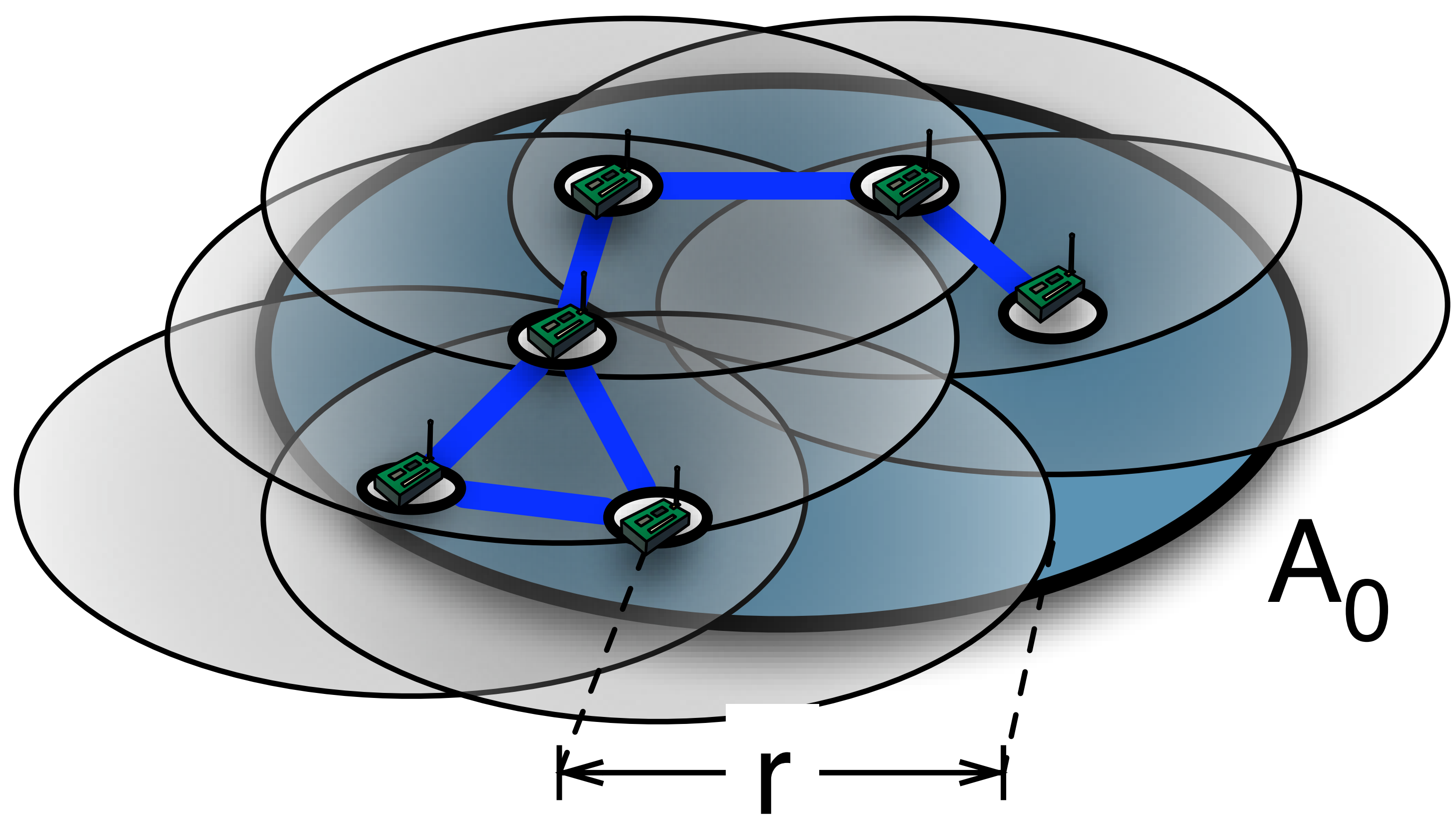
# Literature

- **Handbook on Theoretical and Algorithmic Aspects of Sensor, Ad Hoc Wireless and Peer-to-Peer-Networks (Editor: Jie Wu)**
  - Chapter 27: Models and Algorithms for Coverage Problems in Wireless Sensor Networks

# Sensor Coverage

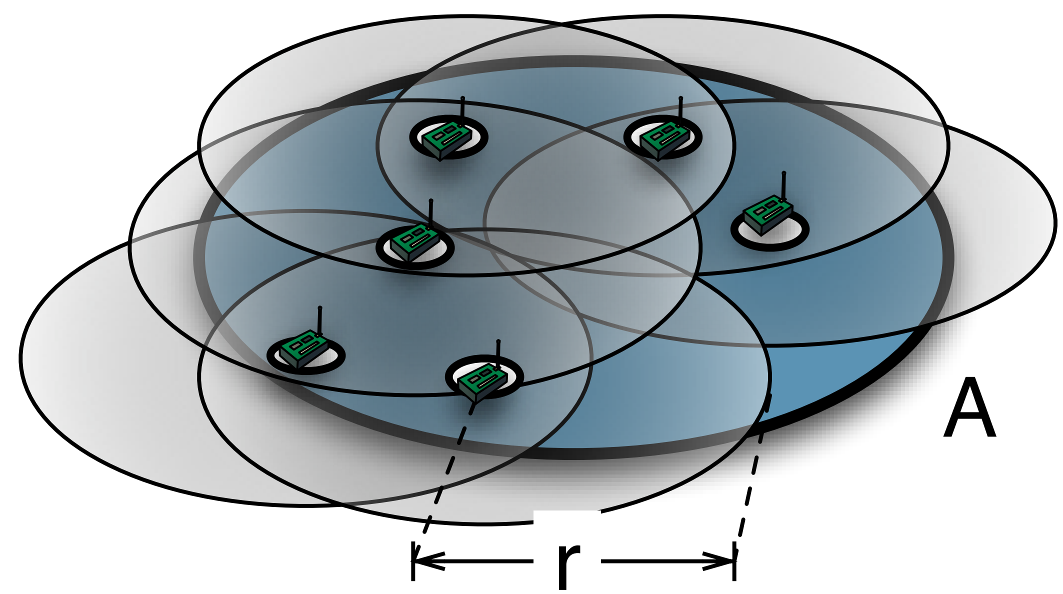


# Sensor Coverage

- **Problem**
  - Given an area
  - Cover the area with the smallest possible number of sensor nodes
- **Variants**
  - Circle Covering
    - 2-dimensional surface, sensor coverage is given by circles
  - Art Gallery Problem
    - Angled rooms: Sensor coverage and line of sight angle
      - e.g. camera surveillance
  - Arbitrarily complexer variants conceivable

# Random Circle Covering

- **Naive approach**
  - Given a square of area A
  - How many randomly positioned the sensors with unit disk cover the square?
- **Naive calculation**
  - Area of the unit circle: $r^2\pi$
  - Number of sensors required: $n = A / (r^2\pi)$
- **Intuition**
  - $O(A/r^2)$ should be sufficient
- **But: intuition is wrong!**



# Random Circle Covering

- **Naive approach**
  - Given a square of area A
  - How many randomly positioned sensors with unit disk cover the square?
- **Theorem**
  - Let $n = A / (r^2\pi)$
    - where A denotes the area of the square
    - and r denotes the sensor radius
  - To cover such a square of $\Theta$ (n log n) randomly placed sensors are necessary and sufficient

# Random Circle Covering

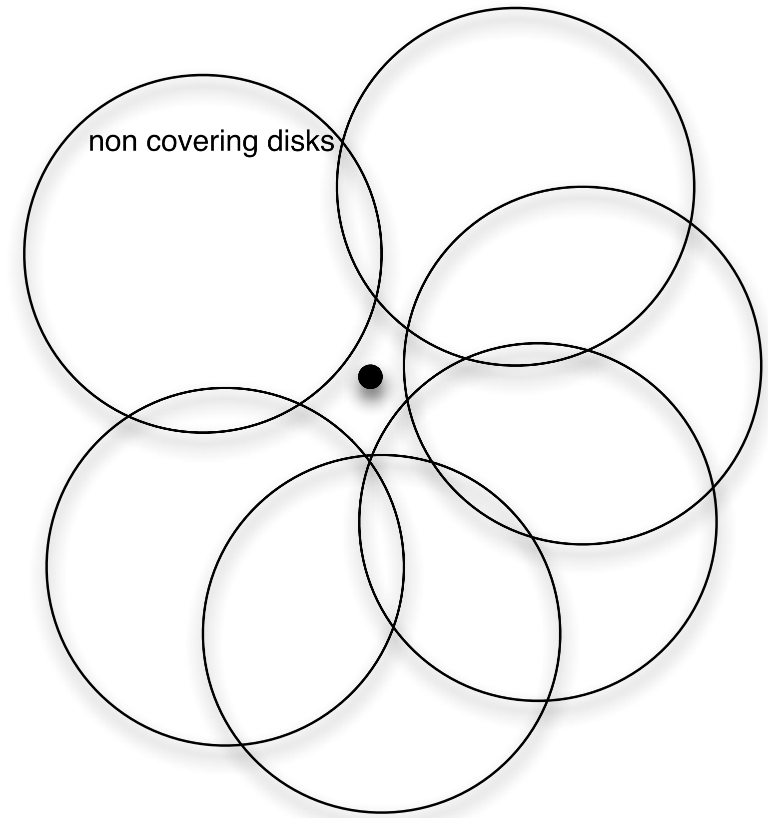
- **Theorem**
  - Let $n = A / (r^2\pi)$
    - where A denotes the area of the square
    - and r denotes the sensor radius
  - To cover such a square of $\Theta$ (n log n) randomly placed sensors are necessary and sufficient
- **Proof sketch (lower bound):**
  - The probability that a given point is not covered by a sensor is at least

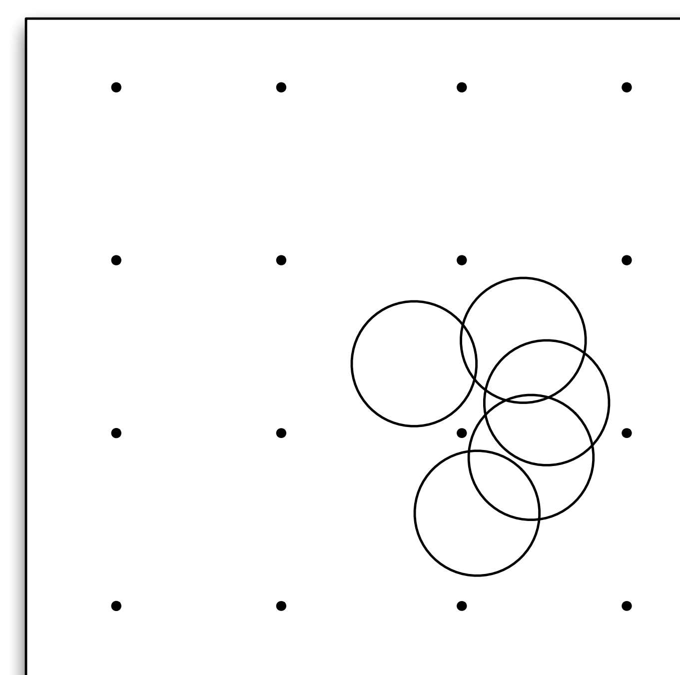
    $$1-r^2\pi/A = 1- 1/n$$

  - Consider n such points with distance r
  - The probability that at least 1/ n log n sensors do **not** cover one of these point is therefore

$$\left(1-\frac{1}{n}\right)^{\frac{1}{2}n\log n} \geq \left(\frac{1}{4}\right)^{\frac{1}{2}\log n} = \frac{1}{n}$$

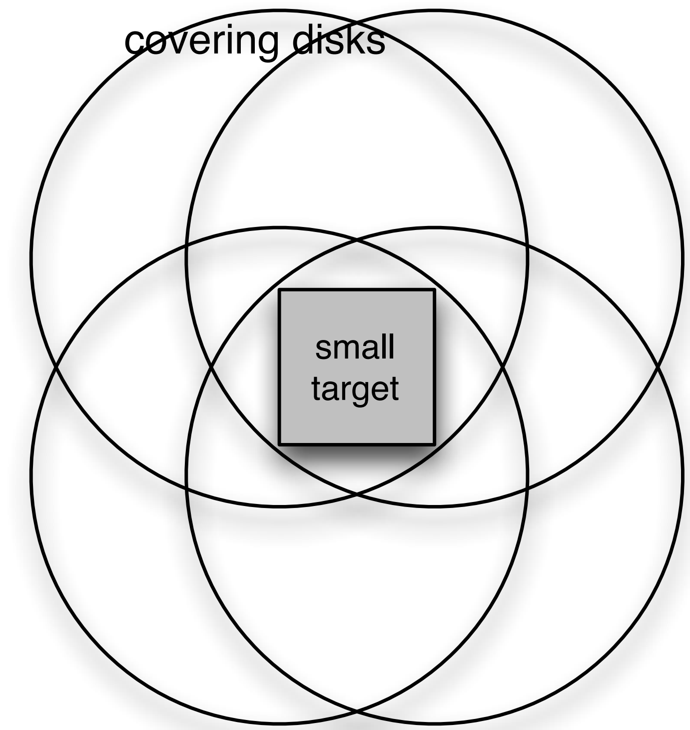
  - Hence, the expected number of uncovered points is 1

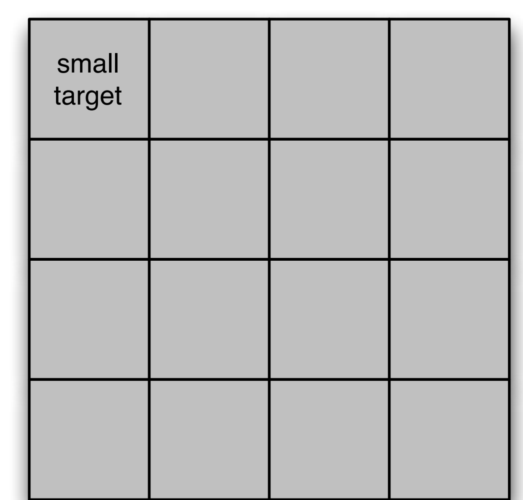


# Random Circle Covering

- **Theorem**
  - Let $n = A / (r^2\pi)$
    - where A denotes the area of the square
    - and r denotes the sensor radius
  - To cover such a square of $\Theta(n \log n)$ randomly placed sensors are necessary and sufficient
- **Proof sketch (upper bound):**
  - By $c\, n \log n$ random sensors every square of size $r/3 \times r/3$ is covered with probability $1-n^{-k}$
    - where k grows linear with c
  - Then the whole square is covered with probability $1-n^{-k-1}$

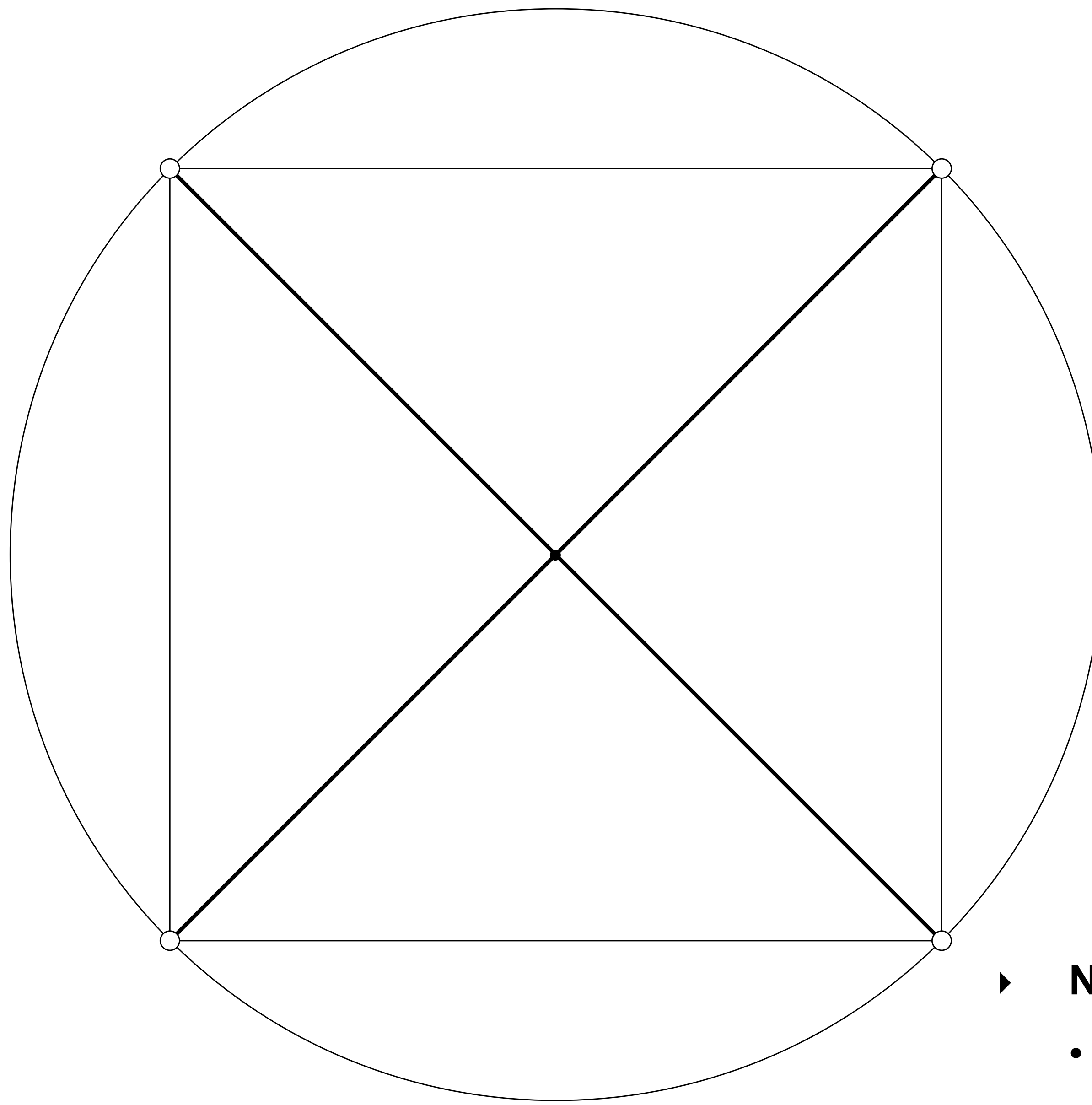


# Optimal Deterministic Bound

- **Nurmela, Östergard**
  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteorian laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)
- **How many circles can cover a square?**
  - A closed form solution is unknown
  - However for a small number of circles the problem can be solved by exhaustive search

# Disk Coverage of a Square



$$n = 1$$

- **Nurmela, Östergard**
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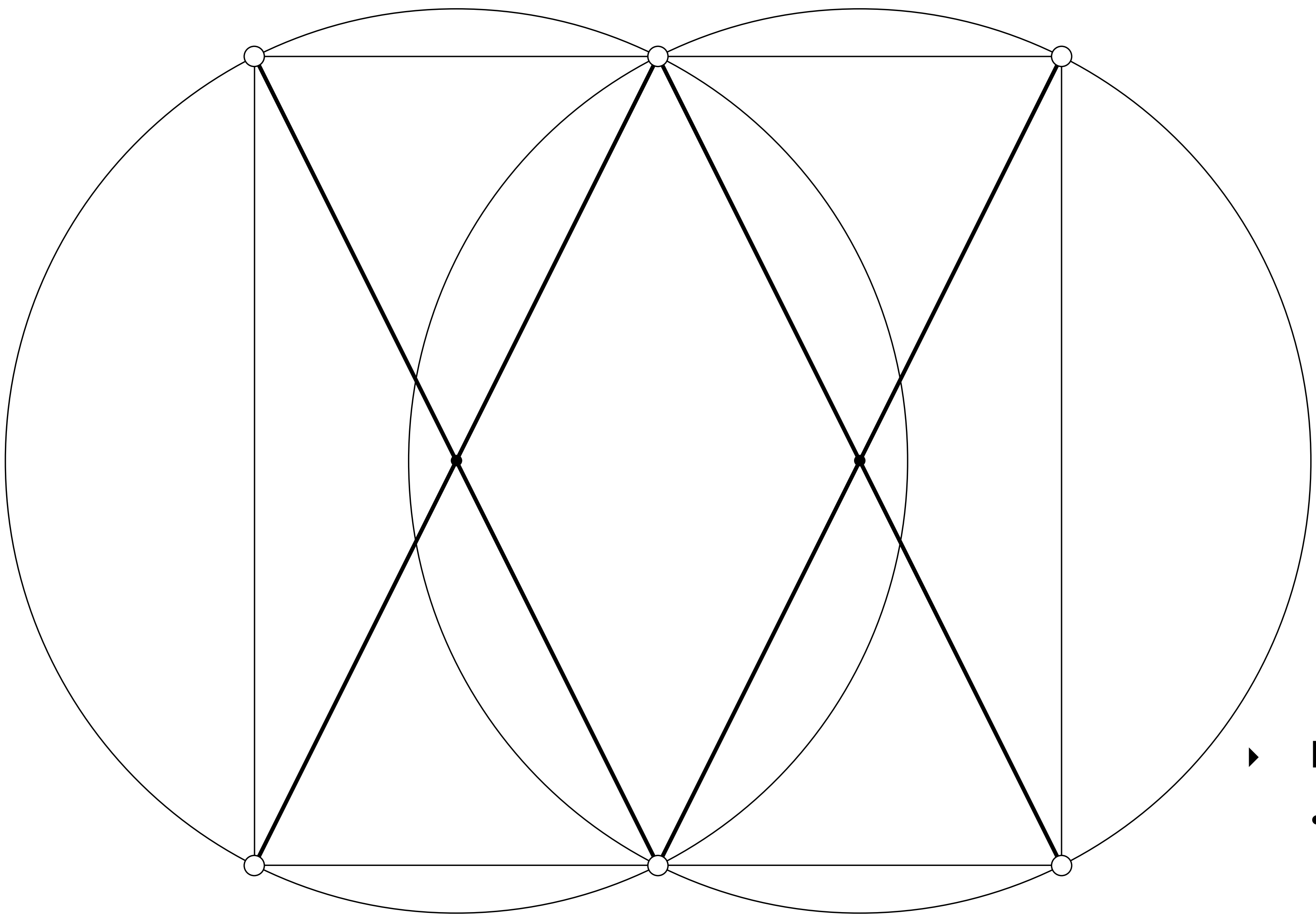

$$n = 2$$

- **Nurmela, Östergard**
  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteoria laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

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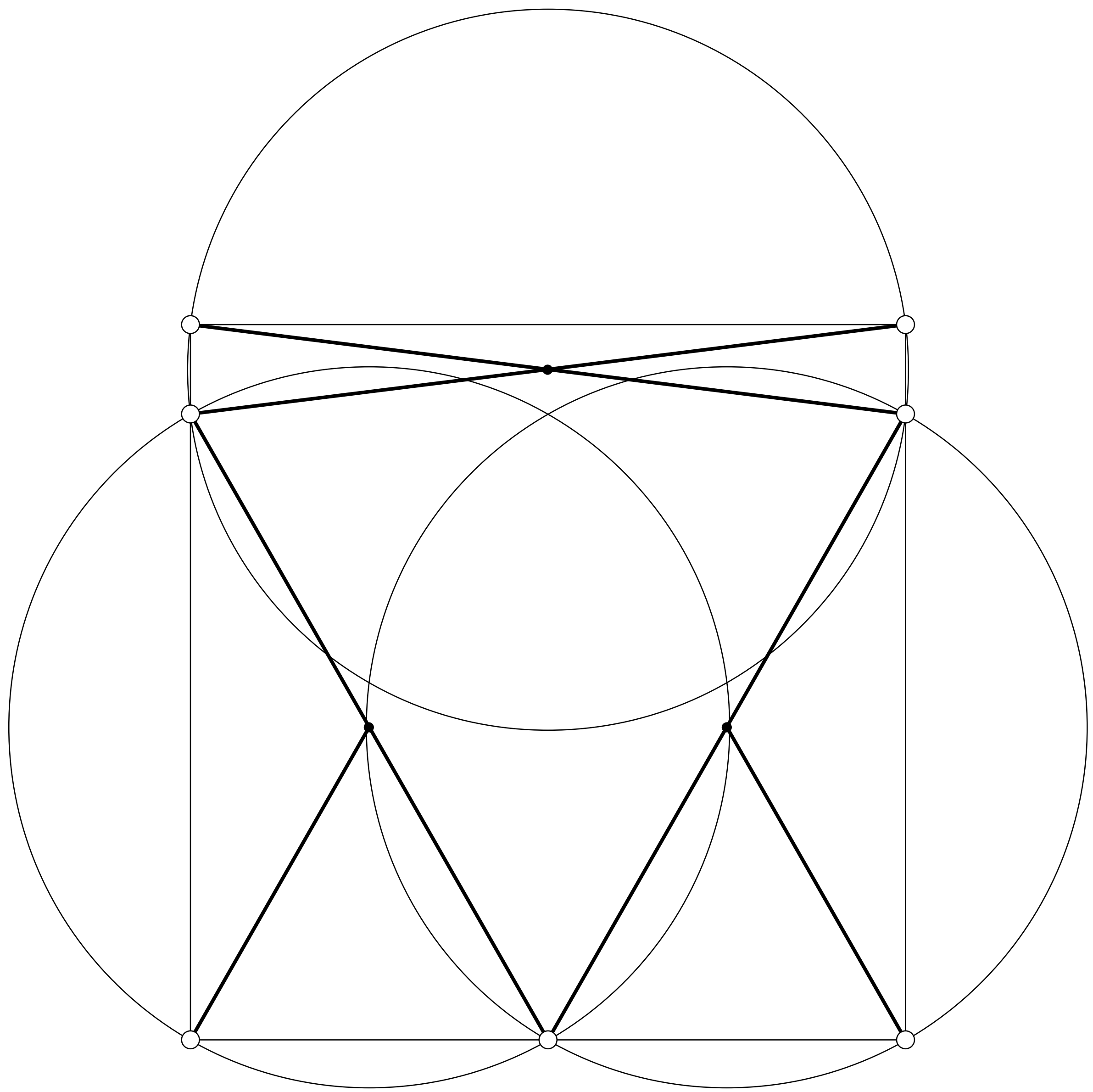
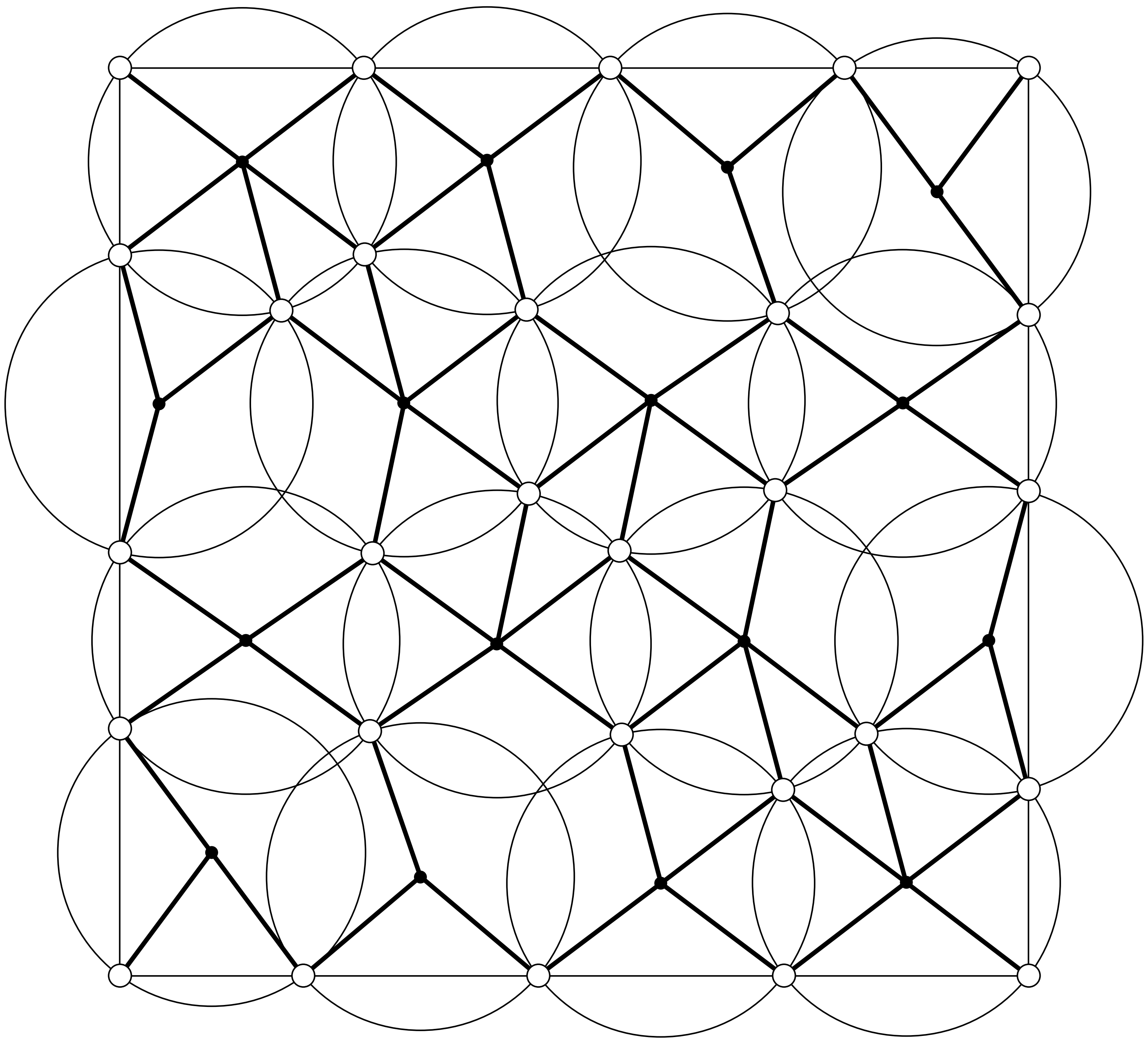


$$n = 3$$

- **Nurmela, Östergard**
  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteoria laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

# Disk Coverage of a Square



$n = 16$

- **Nurmela, Östergard**
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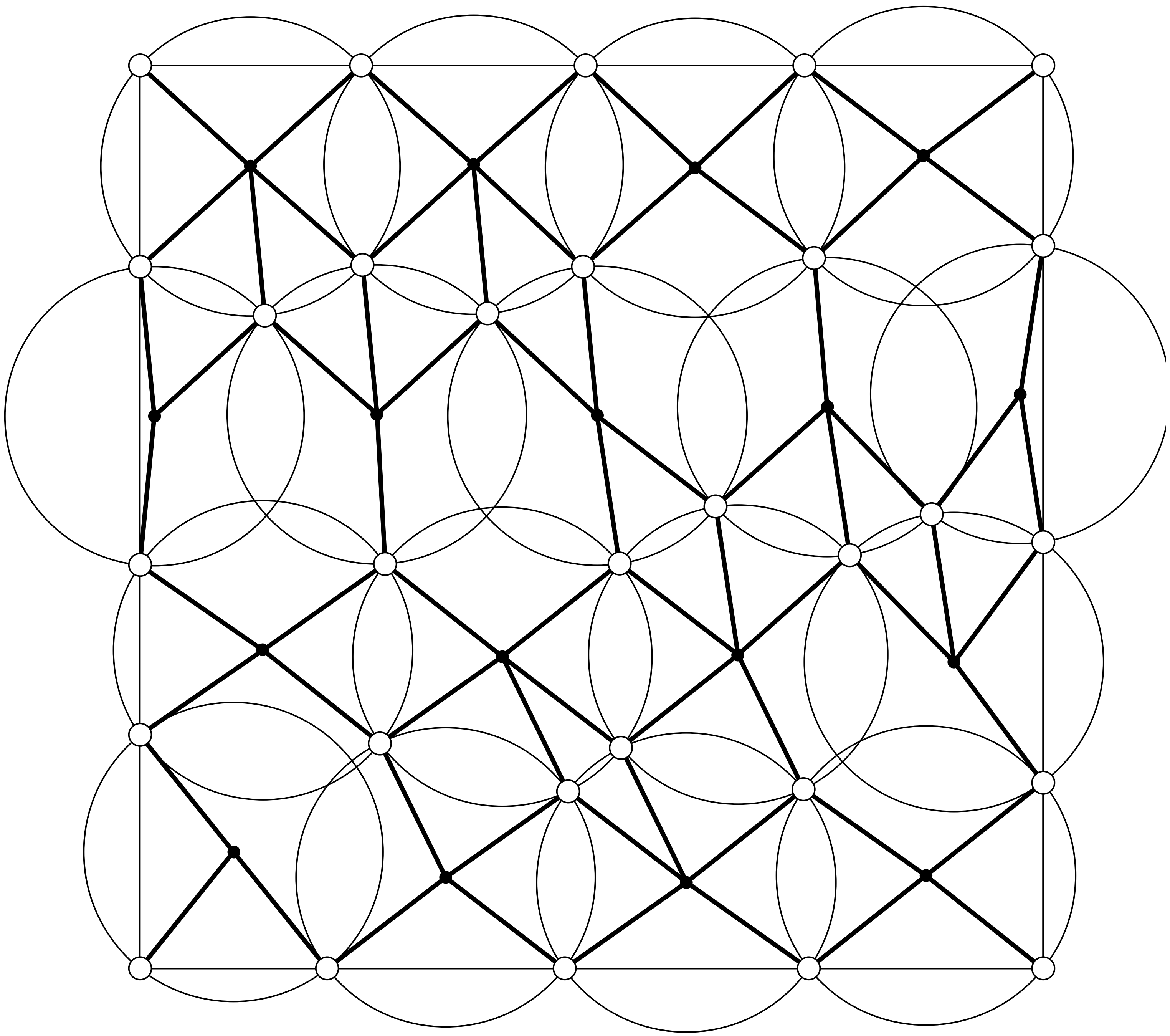
# Disk Coverage of a Square



$$n = 17$$

- **Nurmela, Östergard**
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# Disk Coverage of a Square

| $n$ | $r_n$ | $G$ | $n$ | $r_n$ | $G$ |
|---|---|---|---|---|---|
| 1 | 0.70710678118654752440... | $D_4$ | 16 | 0.16942705159811602395... | $C_2$ |
| 2 | 0.55901699437494742410... | $D_2$ | 17 | 0.16568092957077472538... | $C_1$ |
| 3 | 0.50389110926865935327... | $D_1$ | 18 | 0.16063966359715453523... | $D_1$ |
| 4 | 0.35355339059327376220... | $D_4$ | 19 | 0.15784198174667375675... | $C_1$ |
| 5 | 0.32616058400398728086... | $D_1$ | 20 | 0.15224681123338031005... | $D_1$ |
| 6 | 0.29872706223691915876... | $C_2$ | 21 | 0.14895378955109932188... | $C_1$ |
| 7 | 0.27429188517743176508... | $D_2$ | 22 | 0.14369317712168800049... | $D_2$ |
| 8 | 0.26030010588652494367... | $D_2$ | 23 | 0.14124482238793135951... | $D_2$ |
| 9 | 0.23063692781954790734... | $D_1$ | 24 | 0.13830288328269767697... | $C_1$ |
| 10 | 0.21823351279308384300... | $D_2$ | 25 | 0.13354870656077049693... | $D_1$ |
| 11 | 0.21251601649318384587... | $D_2$ | 26 | 0.13176487561482596463... | $C_1$ |
| 12 | 0.20227588920818008037... | $C_2$ | 27 | 0.12863353450309966807... | $D_2$ |
| 13 | 0.19431237143171902878... | $C_1$ | 28 | 0.12731755346561372147... | $D_2$ |
| 14 | 0.18551054726041864107... | $D_1$ | 29 | 0.12555350796411353317... | $C_1$ |
| 15 | 0.17966175993333219846... | $C_1$ | 30 | 0.12203686881944873607... | $C_2$ |

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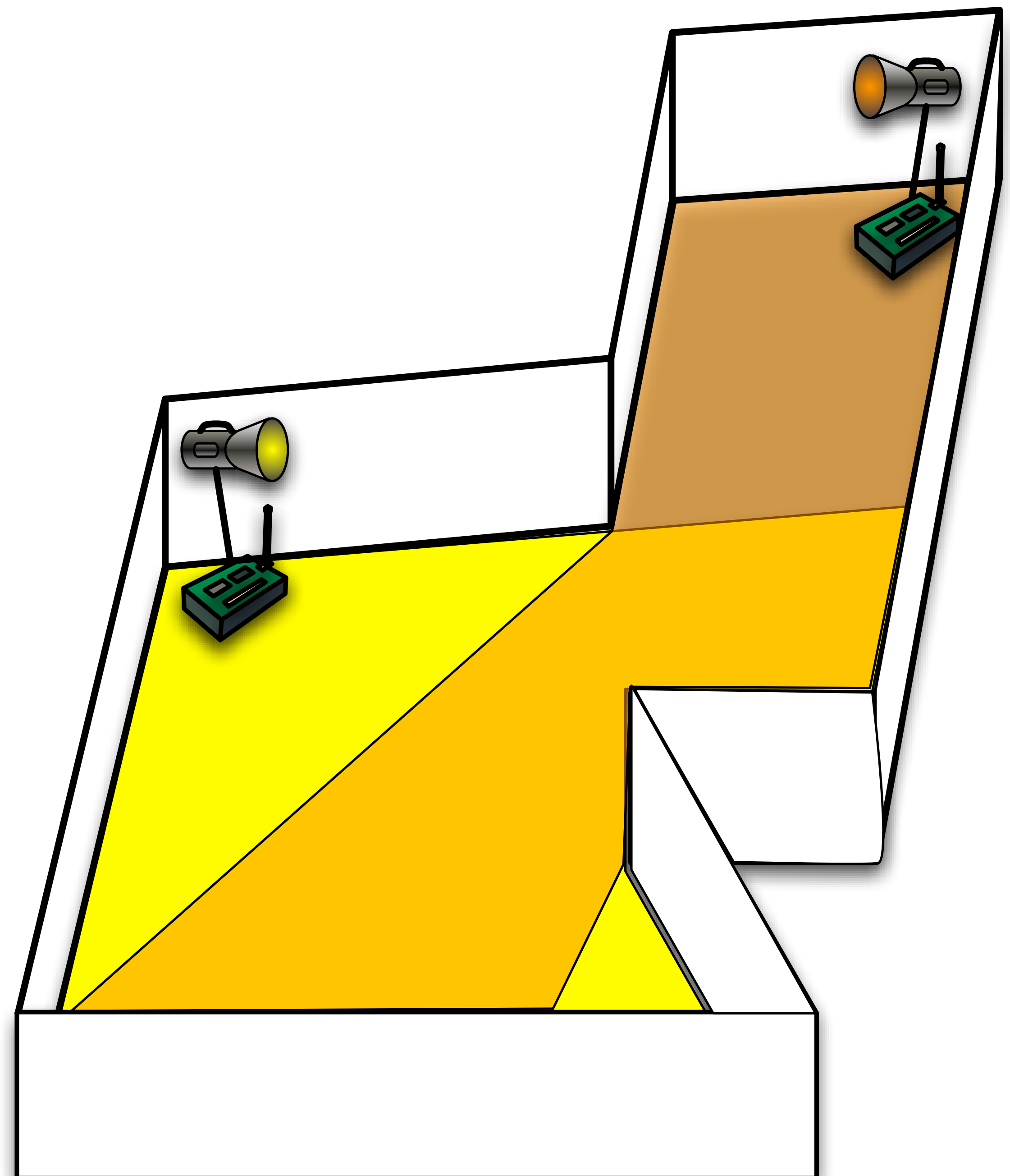
# Art Gallery Problem

- **Given**
  - a room (described by polygon)
- **Compute**
  - Minimum number of cameras and their placement
    - such that the entire space is covered
- **Results**
  - Every room with n edges can be monitored by at most n / 3
  - The exact solution is NP-hard
    - even in the two-dimensional case
  - Polynomial time approximation with a factor O (log n)



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