Algorithms for Radio Networks
Routing as a Flow Problem

University of Freiburg
Technical Faculty
Computer Networks and Telematics
Prof. Christian Schindelhauer
Data Flows in Networks

- **Motivation**
  - Optimize data flow from source to target

- **Definition:**
  - (Single-commodity) maximum flow problem
  - Given
    - a graph $G=(V,E)$
    - a capacity function $w:E \rightarrow \mathbb{R}^+\_0$,
    - source set $S$ and target set $T$
  - Find a maximum flow from $S$ to $T$

- **A flow is a function** $f : E \rightarrow \mathbb{R}^+_0$ such that
  - for all $e \in E$: $f(e) \leq w(e)$
  - for all $e \not\in E$: $f(e) = 0$
  - for all $u, v \in V$: $f(u,v) \geq 0$

\[ \forall u \in V \setminus (S \cup T) \]
\[ \sum_{v \in V} f(v, u) = \sum_{v \in V} f(u, v) \]

- **Maximize flow**

\[ \sum_{u \in S} \sum_{v \in V} f(u, v) \]
Data Flows in Networks

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Dienstag, 31. Januar 12
Data Flows in Networks
Computation of the Maximum Flow

- Every natural pipe system solves the minimum maximum flow problem

Algorithms

- Linear Programming
  - for real numbers
  - the flow is described by equations of a linear optimization problem
  - Simplex algorithm (or Ellipsoid method) can solve any linear equation system
- Ford-Fulkerson
  - also for integers
  - as long as open paths exist, increase the flow on these paths
    * open path: path which increases the flow
- Edmonds-Karp
  - special case of Ford-Fulkerson
  - use BFS (breadth first search) to find open paths
Ford-Fulkerson

- Find a path from the source node to the target node
  - where the capacity is not fully utilized
  - or which reduces the existing flow
- Compute the maximum flow on this augmenting path
  - by the minimum of the flow that can be added on all paths
- Add the flow on the path to the existing flow
- Repeat this step until no flow can be added anymore
Edmunds-Karp

- Search path for Ford-Fulkerson algorithm
- Choose the shortest augmenting path
  - Computation by breadth-first-search
- leads to run-time $O(|V| |E|^2)$
  - whereas Ford-Fulkerson could have exponential run-time
Example
Example
Example
Example
Example
Minimum Cut in Networks

- **Motivation**
  - Find bottleneck in networks

- **Definition**
  - Min Cut problem
  - Given
    - graph G=(V,E)
    - capacity function w: E → R+0,
    - sources S and targets T
  - Find minimum cut between S and T

- **A cut C is a set of edges**
  - such that every path from a node of S to a node of T, contains an edge of C

- **The size of a cut is**
  \[ \sum_{e \in C} w(e) \]
Min-Cut-Max-Flow Theorem

- **Theorem**
  - The minimum cut equals the maximum flow

- **Algorithms for minimum cut**
  - can be obtained from the maximum flow algorithms
Multi-Commodity Flow Problem

- **Motivation**
  - theoretical model for point to point communication

- **Definition**
  - Multi-commodity flow problem
  - given
    - a graph $G=(V,E)$
    - a capacity function $w: E \rightarrow \mathbb{R}^+$
    - commodities $K_1, .., K_k$:
      * $K_i=(s_i,t_i,d_i)$ with
        * $s_i$: source node
        * $t_i$: target node
        * $d_i$: demand

- **Find flows $f_1, f_2, ..., f_k$ for all commodities such that**
  - capacities
    $$\sum_{i=1}^{k} f_i(u,v) \leq w(u,v)$$
  - flow property
    $$\forall v \not\in \{s_i, t_i\} : \sum_{u \in V} f_i(u,v) = \sum_{u \in V} f_i(v,u)$$
  - demand
    $$\sum_{v \in V} f_i(s_i,v) = \sum_{u \in V} f_i(u,t_i) = d_i$$
Solving the Multi-Commodity Flow Problem

- Multi-Commodity Flow Problem
- Optimize
  - sum of all flows or
  - maximize the worst ratio between commodity and the demand
- Problem can be solved in polynomial time
  - for real number
  - using linear programming
Complexity of the Multi Commodity Flow Problem

- Problem is NP-vollständig
  - for integers
    - e.g. packets
  - even for two commodities
    - Shai, Itai, Even, 1976

- Polynomial solution
  - with respect to the number of paths between sources and targets

- Approximation
  - good central and distributed approximation algorithms exist
    (polylogarithmic approximation factor)

- Weaker forms of the Min-Cut-Max-Flow-Theorems exist
Algorithms for Radio Networks

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