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Algorithms for Radio Networks

Public Key Cryptography and Byzantine Generals Problems

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Asymmetric Encryption

- **E.g. RSA, Ronald Rivest, Adi Shamir, Lenard Adleman, 1977**
  - Diffie-Hellman, PGP
- **Secret key: sk**
  - Only the receivers of the message know the secret key
- **Public key: pk**
  - All participants know this key
- **Generated by**
  - keygen(sk) = pk
- **Encryption function f and decryption function g**
  - Known to everybody
- **Encryption**
  - f(pk,text) = code
  - everybody can generate code
- **Decryption**
  - g(sk,code) = code
  - only possibly by receiver

Example: RSA

- **R. Rivest, A. Shamir, L. Adleman**
  - On Digital Signatures and Public Key Cryptosystems, Communication of the ACM
- **Algorithm is based on the computational complexity of integer factorization**
- **1st example**
  - 15 =   ? * ?
  - 15 = 3 * 5
- **2nd example**
  - 386581864584112731912956727734835955 744479041028993358648355204744 3 = 123456789012345678901234567890020 9 * 313131313131313131313131313130022 7

- **To this day no efficient integer factorization algorithm is known**
  - Yet, multiplication can be done efficiently
  - Prime numbers can be found efficiently
    - Since prime numbers occur frequently
    - Efficient randomized prime number tests are available

RSA

- **Generation of keys**
  - Choose two random prime numbers p, q with k bits ($k \geq 500$).
  - $n = p \cdot q$
  - e is a number relatively prime to $(p - 1) \cdot (q - 1)$.
  - $d = e^{-1} \bmod (p - 1)(q - 1)$
    - i.e. $d \cdot e \equiv 1 \bmod (p - 1)(q - 1)$
- **Public key pk = (e, n)**
- **Secret key sk = (d, n)**
- **Encoding**
  - Partition message in block sizes of 2k bits
  - Interprete block M as number $0 \leq M < 2^{2k}$
  - Code: $P(M) = M^e \bmod n$
- **Decoding**
  - $S(C) = C^d \bmod n$
- **Correctness follow from the little theorem of Fermat**

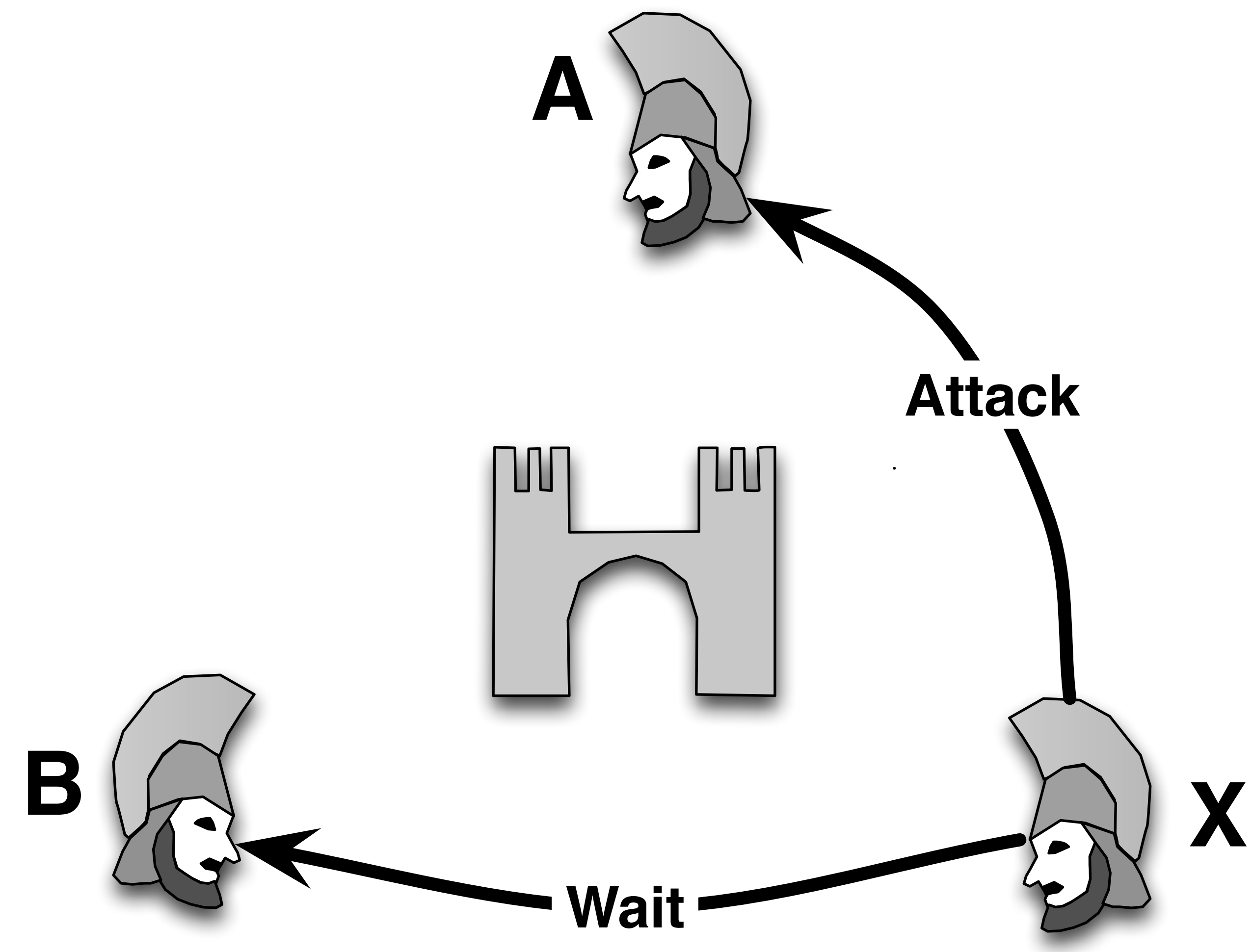
Digital Signatures

- **Digital Signatures**
  - signer has a secret key **sk**
  - document will be signed with the secret key
  - and can be verified with a public key **pk**
  - public key is known to all

- **Example of a signature scheme**
  - m: message
  - Signer
    - computes **h(text)** with cryptographic hash function **h**
    - and publishes m and **signature = g (sk, h (text))**, **g** is the decryption function
  - Checker
    - computes h(text)
    - and verifies **f (pk, signature) = h (text)** for the asymmetric encryption function **f**

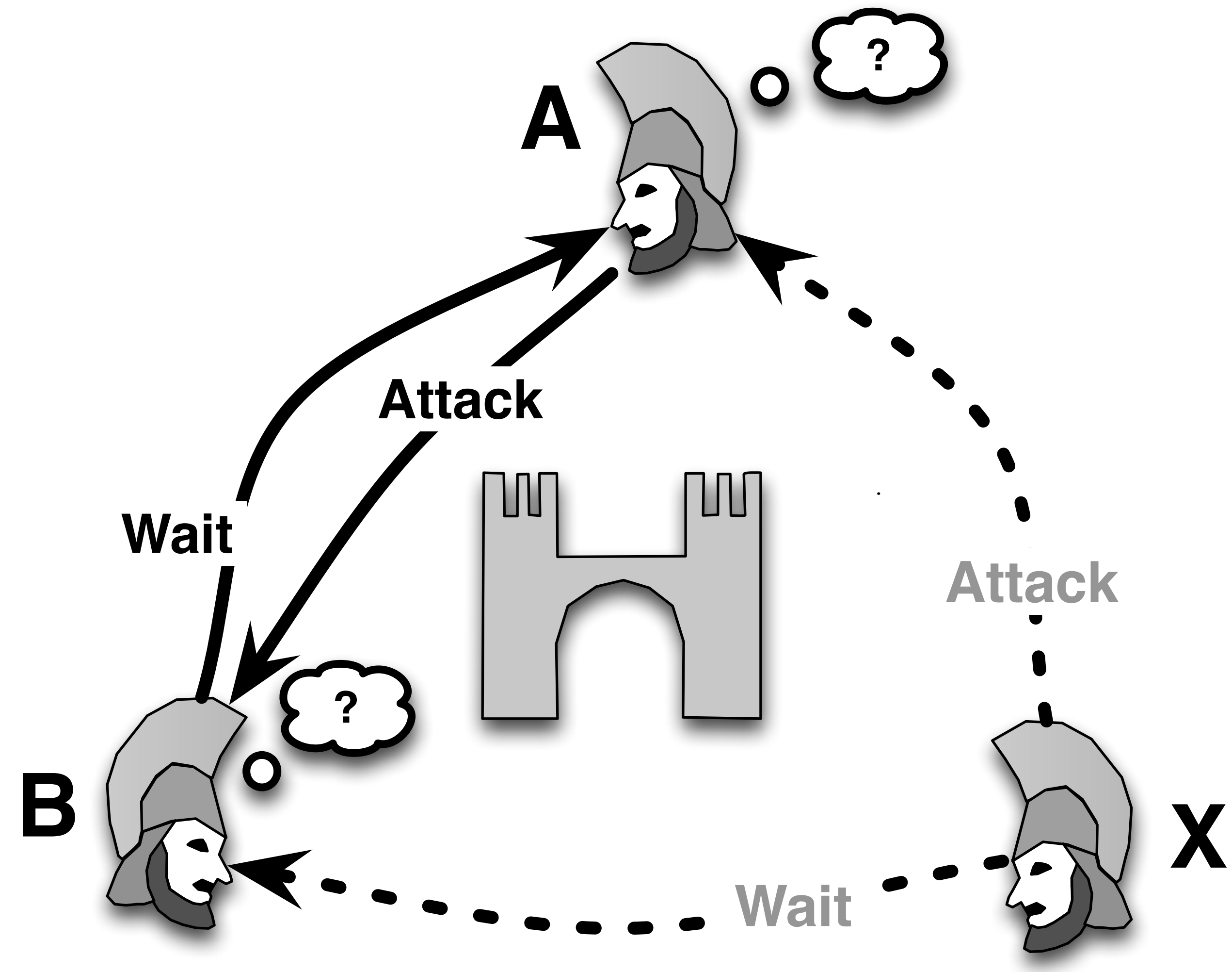
Problem of Byzantine Generals

- **After the hijacking of a node in a network it can cause malicious actions on the network**
  - This problem is known as the Byzantine Generals problem
- **3 armies are ready to conquer the enemy castle**
  - These are separated and communicate via messengers
  - If only army attacks then all will loose
  - If two armies atteck, they will win
  - If no army attacks, they will win
    - (because the defenders will starve out)
- **But one general is an evil traitor**
  - you do not know who ...



Problem of Byzantine Generals

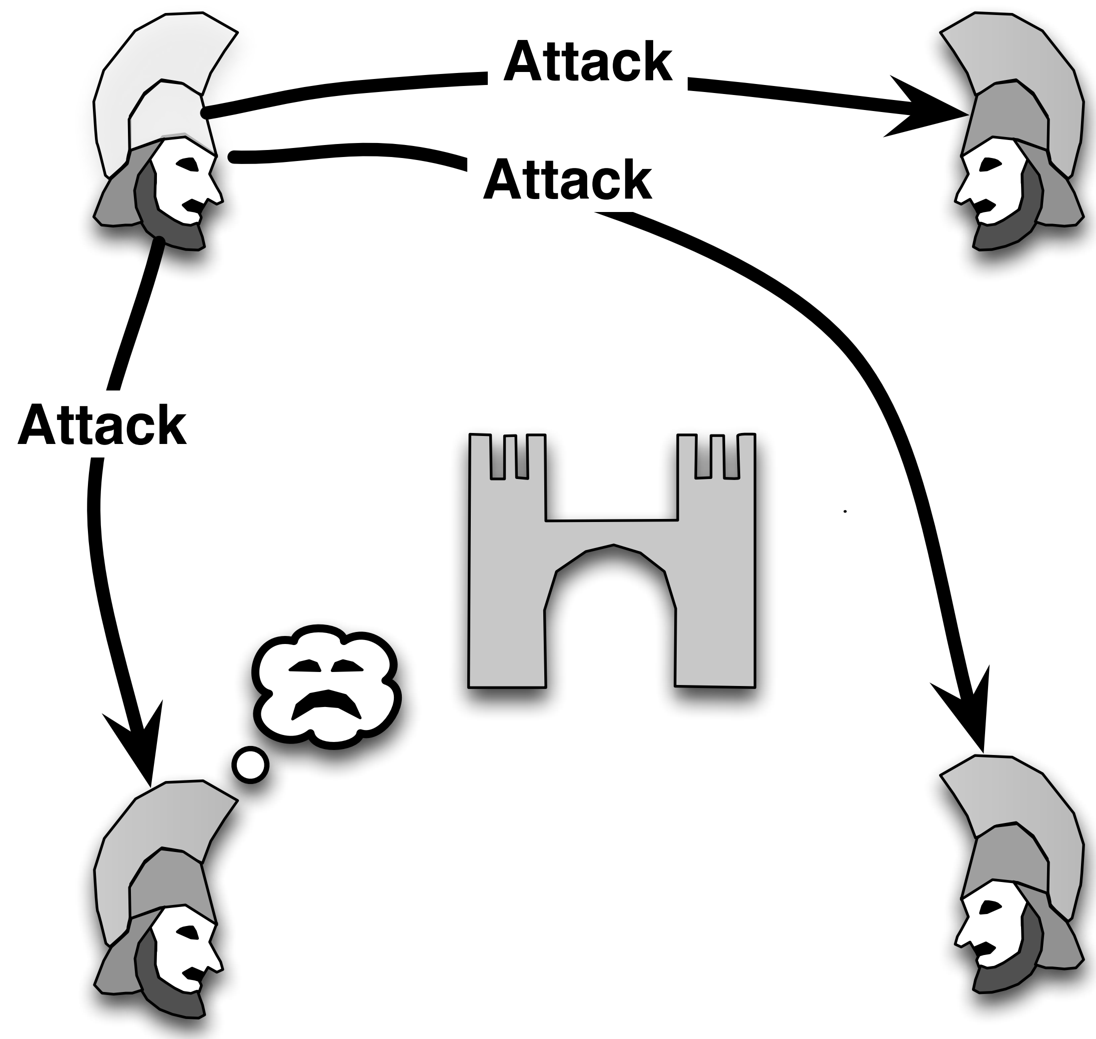
- **The traitorous general X tries to**
  - persuade A to attack
  - persuade Be to wait
- **A tells B about the command**
- **B tells A about the command**
  - Something is wrong
  - But nobody can tell who is cheating
    - Even after further communication



Byzantine Agreement

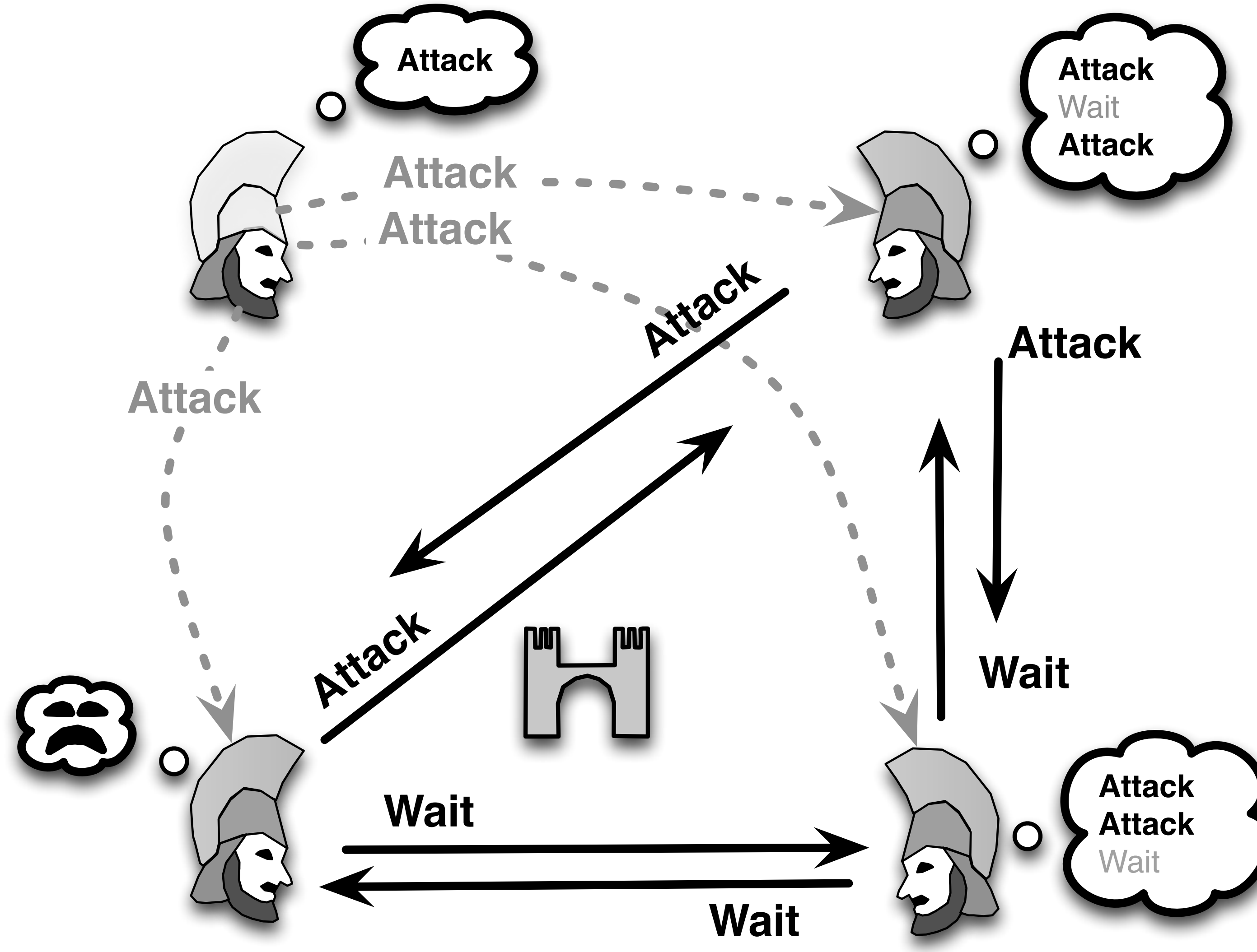
- **Theorem**
  - The problem of the three Byzantine generals cannot be solved*
- **For four generals, the problem is solvable**

* if all participants have no computing limitations



Byzantine Agreement

- **For four generals, the problem is solvable:**
  - 1 general, 3 officers problem
  - consider a (loyal) general and three officers.
  - Disseminate information to all officers of the loyal generals
- **Algorithm**
  - General A sends his command to all others
    - A follows his own command
  - Any other office sends that its received order to all others
  - Each officer calculates the majority decision of the orders of B, .., D



Byzantine Agreement
What if General A is a Traitor

- **For four generals, the problem is solvable:**
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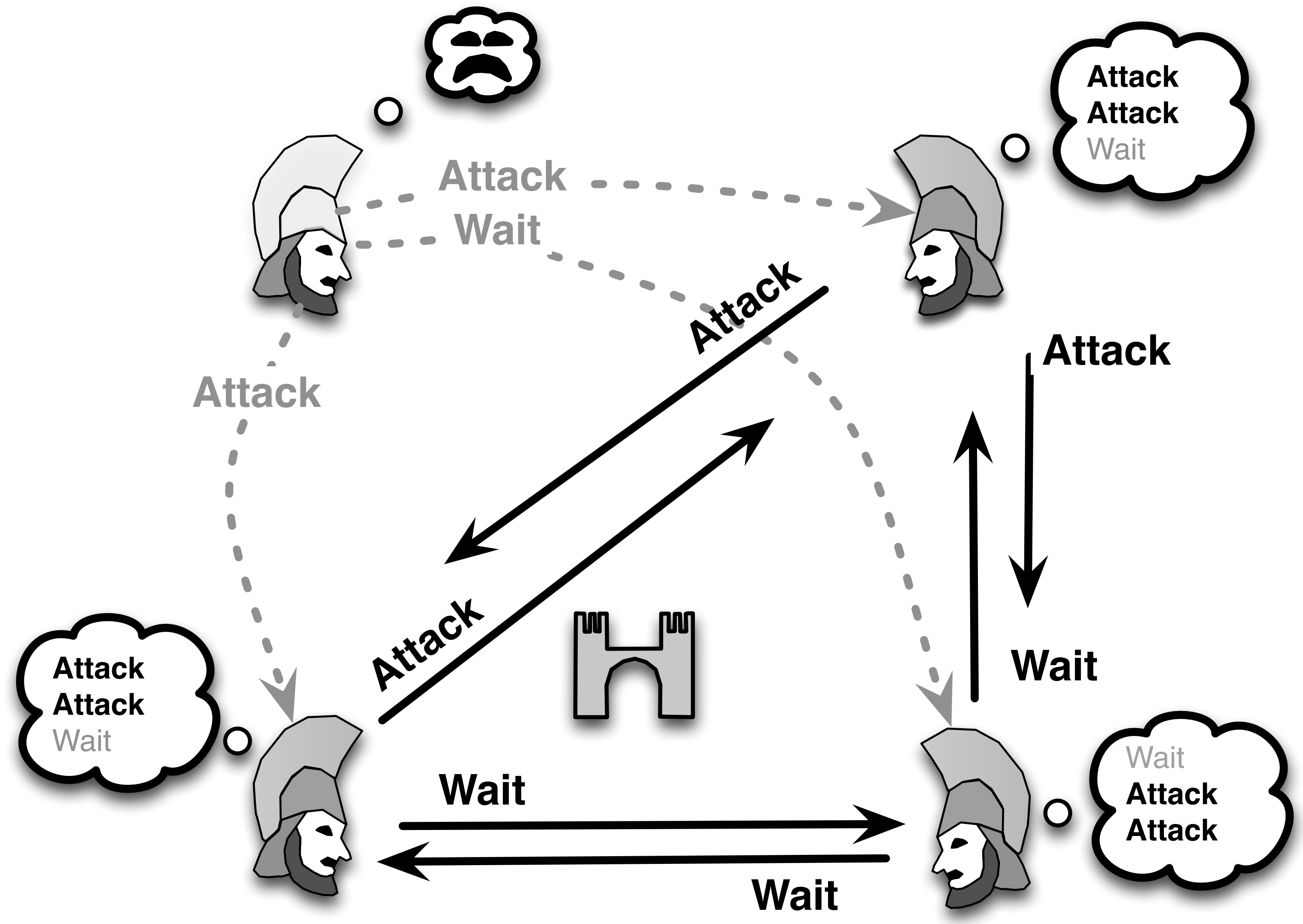

Solution of the Byzantine General Problem

- **Theorem**
  - If m generals are traitors, then at least 2m +1 generals must be honest such that the problem of the Byzantine Generals is solvable.
- **This barrier is tight if we do not allow cryptography**
  - i.e. if you have powerful computers which can break into every encryption
- **Theorem**
  - If a digital signature scheme is available, then any number of false generals can be dealt with
- **Solution:**
  - Every general signs his commands
  - In each round every general forwards all commands and signatures to all others
  - Each inconsistent command or false forwarding can be immediately detected and proved
  - False silence or changed commands can be detected

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