Algorithms for Radio Networks

Public Key Cryptography and Byzantine Generals Problems
Asymmetric Encryption

- E.g. RSA, Ronald Rivest, Adi Shamir, Lenard Adleman, 1977
  - Diffie-Hellman, PGP
- Secret key: sk
  - Only the receivers of the message know the secret key
- Public key: pk
  - All participants know this key
- Generated by
  - keygen(sk) = pk

- Encryption function f and decryption function g
  - Known to everybody
  - Encryption
    - $f(pk, text) = code$
    - everybody can generate code
  - Decryption
    - $g(sk, code) = code$
    - only possibly by receiver
Example: RSA

- R. Rivest, A. Shamir, L. Adleman
  - On Digital Signatures and Public Key Cryptosystems, Communication of the ACM

- Algorithm is based on the computational complexity of integer factorization

- 1st example
  - $15 = ? \times ?$
  - $15 = 3 \times 5$

- 2nd example
  - $386581864584112731912956727734835955$
    $7444790410289933586483552047443 = 1234567890123456789012345678900209 \times$
    $3131313131313131313131313131313131300227$

- To this day no efficient integer factorization algorithm is known
  - Yet, multiplication can be done efficiently
  - Prime numbers can be found efficiently
    - Since prime numbers occur frequently
    - Efficient randomized prime number tests are available
### RSA

**Generation of keys**
- Choose two random prime numbers $p$, $q$ with $k$ bits ($k \geq 500$).
- $n = p \cdot q$
- $e$ is a number relatively prime to $(p - 1) \cdot (q - 1)$.
- $d = e^{-1} \mod (p - 1)(q - 1)$
  - i.e. $d \cdot e \equiv 1 \mod (p - 1)(q - 1)$

**Public key pk = (e, n)**

**Secret key sk = (d, n)**

**Encoding**
- Partition message in block sizes of $2^k$ bits
- Interpret block $M$ as number $0 \leq M < 2^{2k}$
- Code: $P(M) = M^e \mod n$

**Decoding**
- $S(C) = C^d \mod n$

**Correctness follow from the little theorem of Fermat**
Digital Signatures

- Digital Signatures
  - signer has a secret key \( sk \)
  - document will be signed with the secret key
  - and can be verified with a public key \( pk \)
  - public key is known to all

- Example of a signature scheme
  - \( m \): message
  - Signer
    - computes \( h(\text{text}) \) with cryptographic hash function \( h \)
    - and publishes \( m \) and signature = \( g(sk, h(\text{text})) \), \( g \) is the decryption function
  - Checker
    - computes \( h(\text{text}) \)
    - and verifies \( f(pk, \text{signature}) = h(\text{text}) \)
      for the asymmetric encryption function \( f \)
Problem of Byzantine Generals

- After the hijacking of a node in a network it can cause malicious actions on the network
  - This problem is known as the Byzantine Generals problem
- 3 armies are ready to conquer the enemy castle
  - These are separated and communicate via messengers
  - If only army attacks then all will loose
  - If two armies attack, they will win
  - If no army attacks, they will win
    - (because the defenders will starve out)
- But one general is an evil traitor
  - you do not know who ...
Problem of Byzantine Generals

- The traitorous general X tries to
  - persuade A to attack
  - persuade B to wait
- A tells B about the command
- B tells A about the command
  - Something is wrong
  - But nobody can tell who is cheating
  - Even after further communication
Byzantine Agreement

- **Theorem**
  - The problem of the three Byzantine generals cannot be solved*.
- **For four generals, the problem is solvable**.

* if all participants have no computing limitations.
Byzantine Agreement

- For four generals, the problem is solvable:
  - 1 general, 3 officers problem
  - consider a (loyal) general and three officers.
  - Disseminate information to all officers of the loyal generals

- Algorithm
  - General A sends his command to all others
    - A follows his own command
  - Any other office sends that its received order to all others
  - Each officer calculates the majority decision of the orders of B, .., D
Byzantine Agreement
What if General A is a Traitor

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Solution of the Byzantine General Problem

- **Theorem**
  - If m generals are traitors, then at least 2m + 1 generals must be honest such that the problem of the Byzantine Generals is solvable.

- **This barrier is tight if we do not allow cryptography**
  - i.e. if you have powerful computers which can break into every encryption

- **Theorem**
  - If a digital signature scheme is available, then any number of false generals can be dealt with

- **Solution:**
  - Every general signs his commands
  - In each round every general forwards all commands and signatures to all others
  - Each inconsistent command or false forwarding can be immediately detected and proved
  - False silence or changed commands can be detected
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