

16 NL = co-NL

$$\underline{NL = co-NL}$$

$$NL := \{L \subseteq \Sigma^* \mid \exists \text{NTM } M$$

$$X_M(x) = M(x) \text{ and}$$

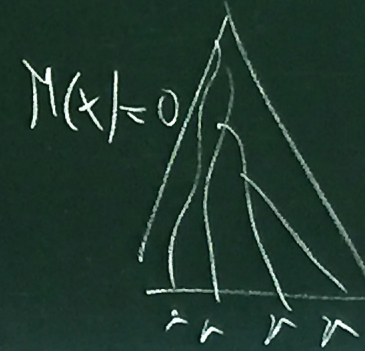
$$\forall x \in \Sigma^*: \text{space}_M(x) \leq O(\log |x|)\}$$

$$co-NL = \{L \subseteq \Sigma^* \mid \overline{L} \in NL\}$$

$$\overline{L} = \Sigma^* \setminus L$$



$$M(x) = 1$$



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$$X_L(x) = \begin{cases} 1 & x \in L \\ 0 & x \in \Sigma^* \setminus L \end{cases}$$

$$X_L^-(x) = \begin{cases} 1 & x \in \Sigma^* \setminus L \\ 0 & x \in L \end{cases}$$

What can we do in NL?

- Store constant no. of nodes
- a lot of guessing
- conf space is pol.

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$PATH = \{ (G, s, t) \mid \text{is there a path from } s \text{ to } t \text{ in } G? \}$

$PATH \in NL$

$\boxed{\text{if } PATH \in co-NL \Leftrightarrow NL = co-NL}$

$\Leftrightarrow PATH \in NL$

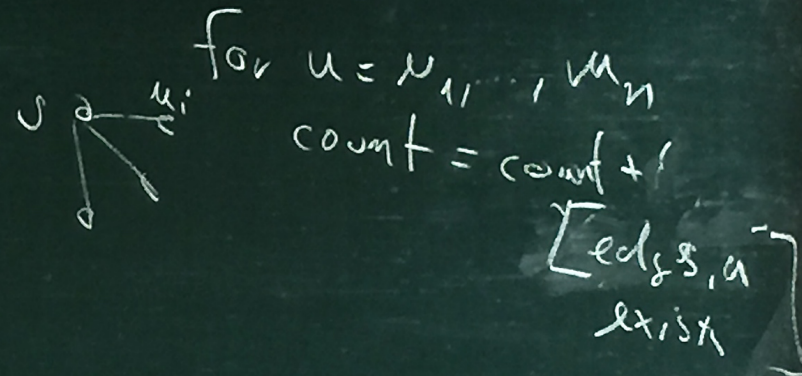
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$\overline{\text{PATH}} = \{ x \mid x \text{ is not in the form } (G, s, t) \text{ } \checkmark$
 $\text{there is no path from station } G \}$

given: G, s, t, P

compute: is t not a direct neighbor of s ?

is the number of direct neighbors = p ?



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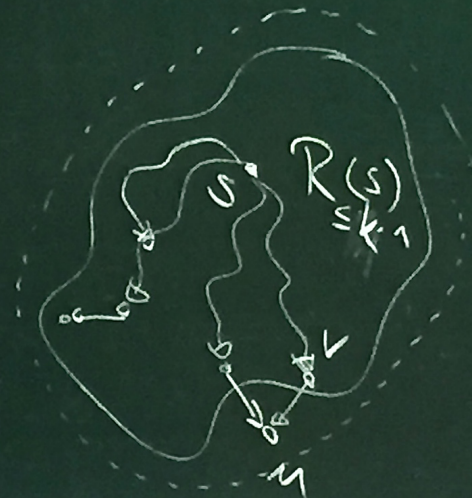
given G, s, t

compute # nodes in G
in distance $\leq k$

$t=0 \checkmark \quad t=1 \checkmark$

Assume $m_k = \# \text{ nodes with}$
distance $\leq k$
from s

given $m_{k-1} = |R_{k-1}(s)|$



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count = 0
For  $u = u_1, \dots, u_n$ 
     $V = \{u_1, \dots, u_n\}$ 
    count previos = 0
    for  $v = u_1, \dots, u_n$ 
        guess a path of length  $k-1$ 
        if path existthen count previos++
        do check whether  $(v, u) \in E$ 
        if count previos =  $m_{k-1}$  and  $(v, u) \in E$ 
            then count = count + 1
    od  $m_k = \text{count}$ 
  
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$m_0 = 1$
 for $k = 1, \dots, n$ do
 calculate m_k from m_{k-1}
 od

given m_n

count = 0
 for $u = \{u_1, \dots, u_n\} \setminus t$ do
 guess a path from s to u
 if a path exists then
 count := count + 1
 od

t is not

reachable

$\Leftarrow$ if count = m_n then accept

$$16 \text{ NL} = \text{co-NL}$$

