# 19 IP = PSPACE, Average-P

## $IP \supseteq PSPACE$

given:

QBF build IP-system

**Idea:** Arithmetization

of $\underbrace{\exists}_{Q_1} x_1 \underbrace{\forall}_{Q_2} x_2 \exists x_2 \forall x_3 \overbrace{(x_1 \lor x_2) \land (\bar{x}_1 \lor x_3)}^{\varphi}$

$\Rightarrow F; \quad f() \Rightarrow f(x_1) \Rightarrow f(x_1, x_2) \ldots$

$f(x_1 \ldots x_n) = \varphi(x_1 \ldots x_n)$

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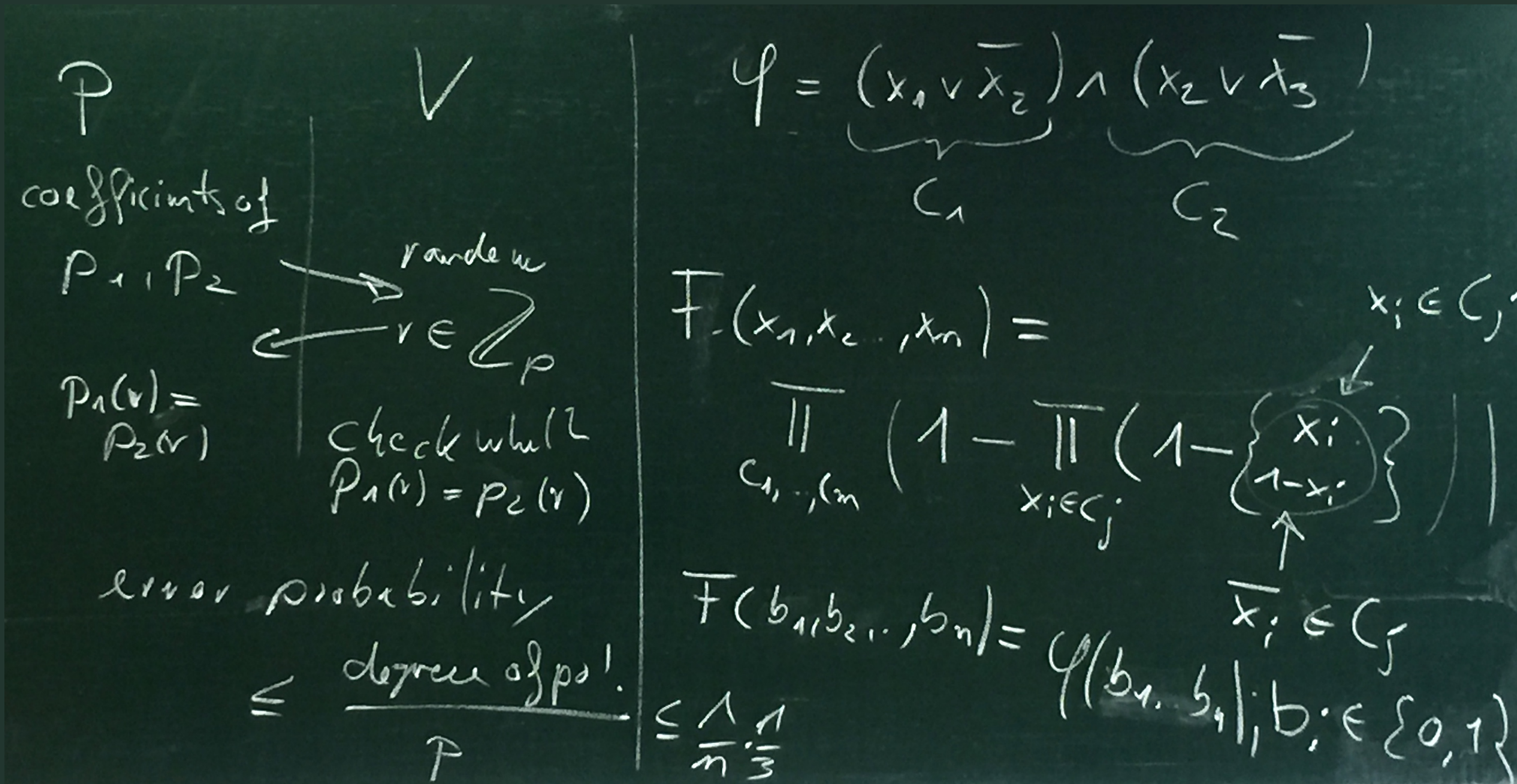

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$$f_0() = Q_1 x_1 \ldots Q_n x_n \, \varphi(x_1 \ldots x_n)$$

$$f_i(x_1 \ldots, x_i) = Q_{i+1} x_{i+1} \, Q_n x_n \, \varphi(x_1 \ldots x_n)$$

$Q_{i+1} = $ „$\forall$" $\quad f_i(x_1 \ldots, x_i) = \prod_{b \in \{0,1\}} f_{i+1}(x_1 \ldots x_i, b)$

$Q_{i+1} = $ „$\exists$" : $\quad f_i(x_1, \ldots x_i) = 1 - \prod_{b \in \{0,1\}} \left(1 - f_{i+1}(x_1 \ldots x_i, b)\right)$

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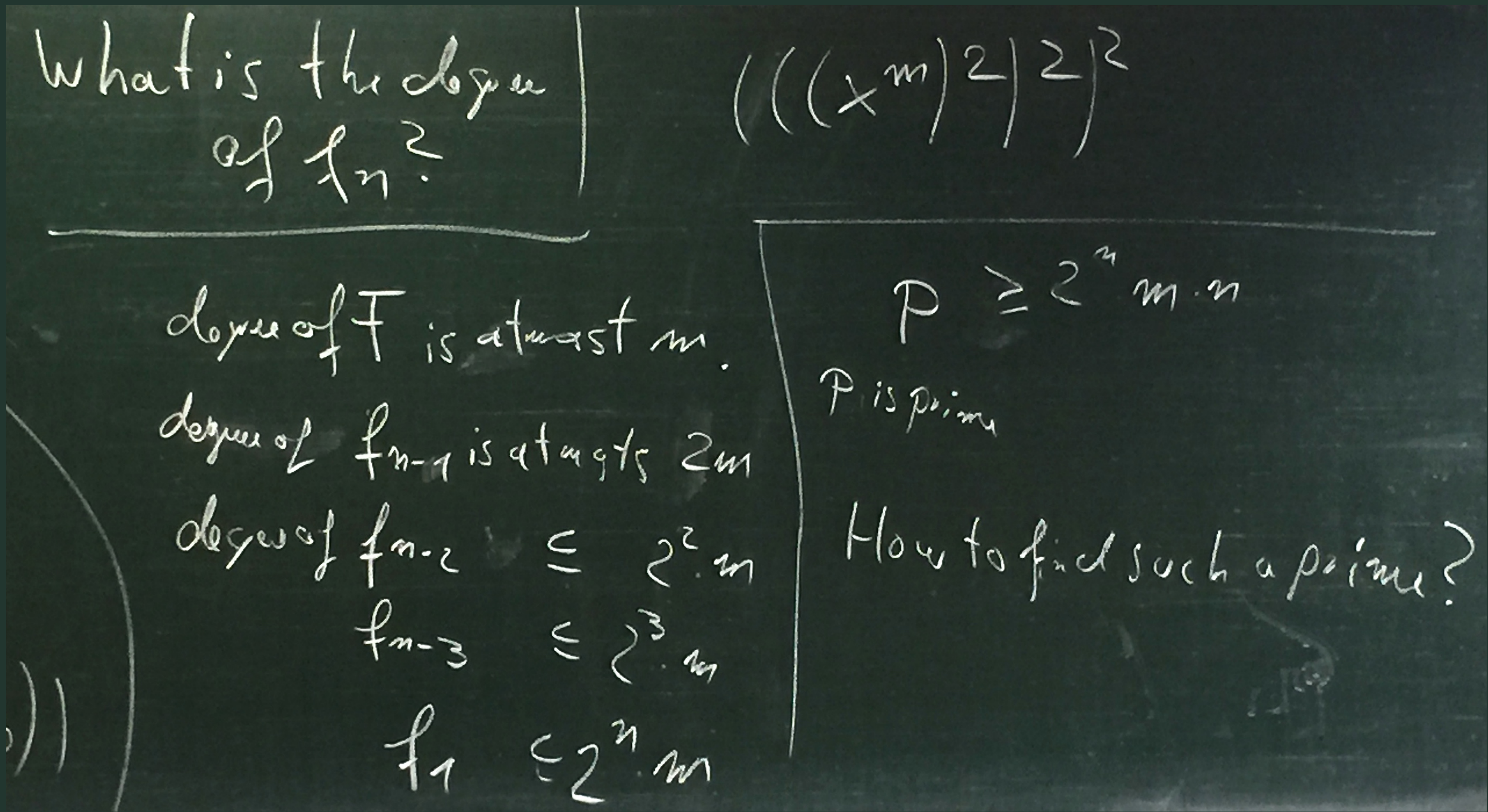

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New Gadget : Degree Reducer

$$R_x \quad f'(x_1,x_2,\dots,x_n,x) = (1-x)\cdot f(x_1,\dots,x_n,0) + x\cdot f(x_1,\dots,x_n,1)$$

$x=0$ : $f'(x_1,\dots,x_n,x) = f(x_1,\dots,x_n,0)$ ✓

$x=1$ : $f'(x_1,\dots,x_n,x) = f(x_1,\dots,x_n,1)$ ✓

$\Rightarrow$ correct arithmetization!

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Properties of $R_x f(x_1 \ldots x_n, x)$

- Same for $x_1 \ldots x_n \in \mathbb{Z}_p$, $x \in \{0, 1\}$
- degree in $x$ is $\leq 1$
- degree in $x_i$ for constant $\{x_1 \ldots x_n, x\} \setminus \{x_i\}$ is the same
- There exists a IP to show correctness.

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$$\exists x_1 \forall x_2 \exists x_3 \underbrace{(x_1 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_3)}_{F(x)}$$

$$Q x_1 R x_1 Q x_2 \ldots Q x_{n-1} R_1 x_1 \ldots R_{n-1} x_{n-1} Q x_n R_1 x_1 R_2 x_2 \ldots R_n x_n F(x_1, \ldots, x_n)$$

$$Q \in \{\forall, \exists\}$$

$$\Downarrow$$

$$S y_1 S y_2 S y_3 \ldots \ldots S y_n F(x_1, \ldots, x_n)$$

$S \in \{\forall, \exists, R\}$ → degree is at most $m$

$\blacksquare$

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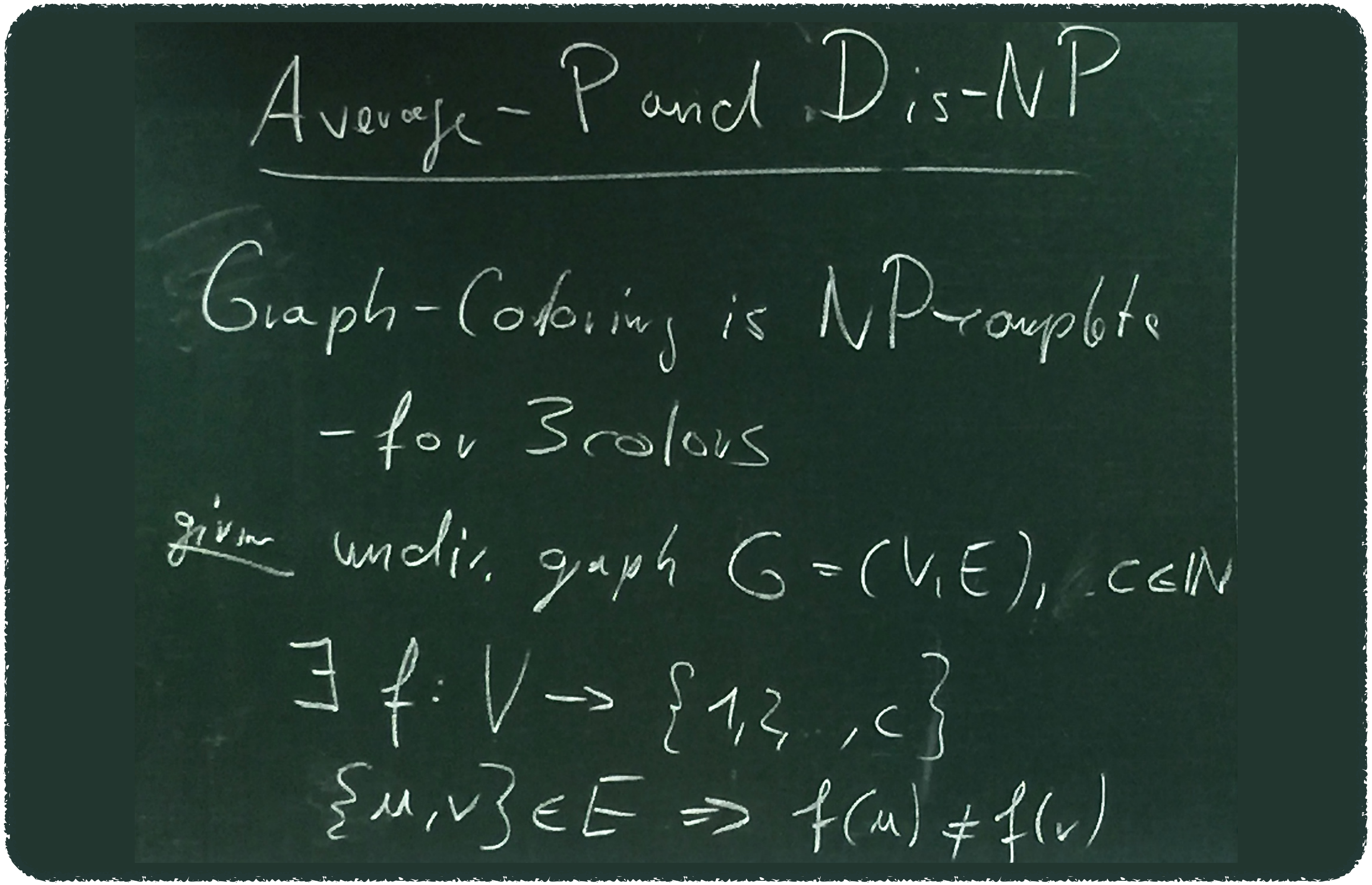

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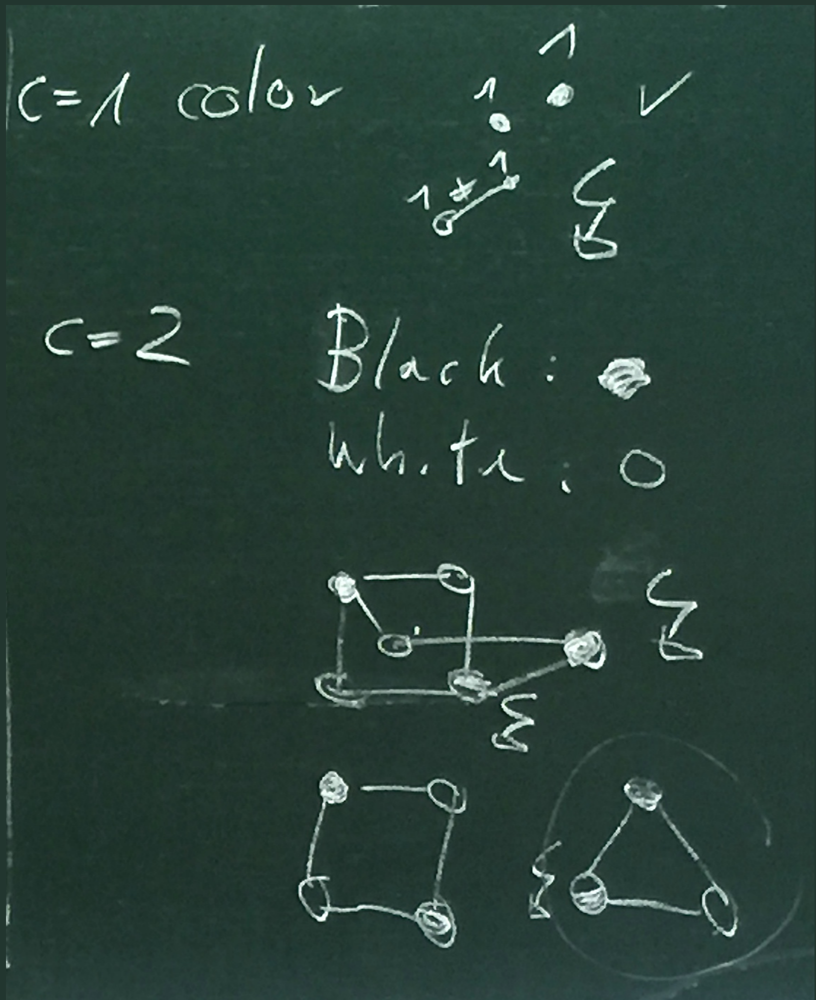

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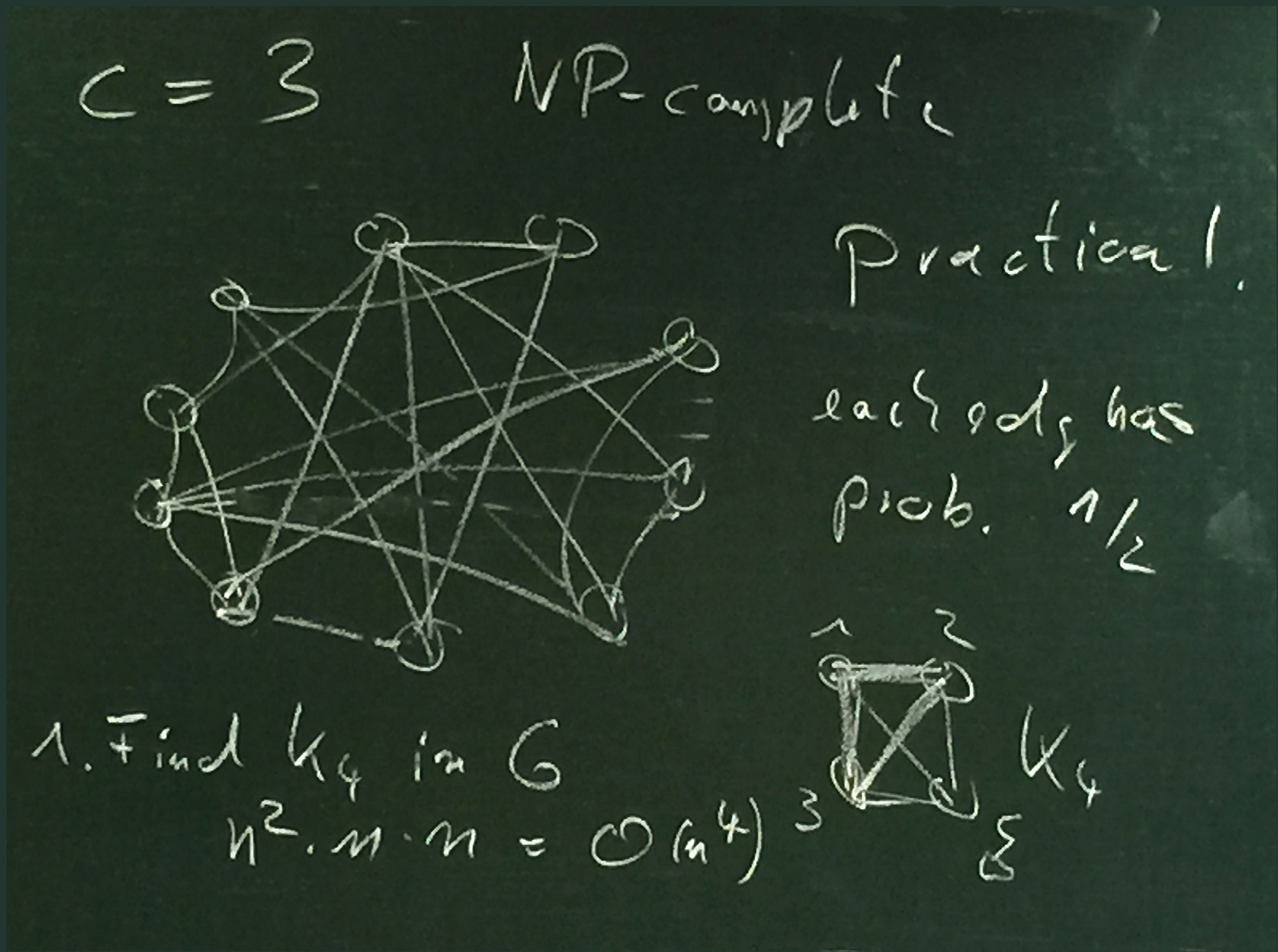

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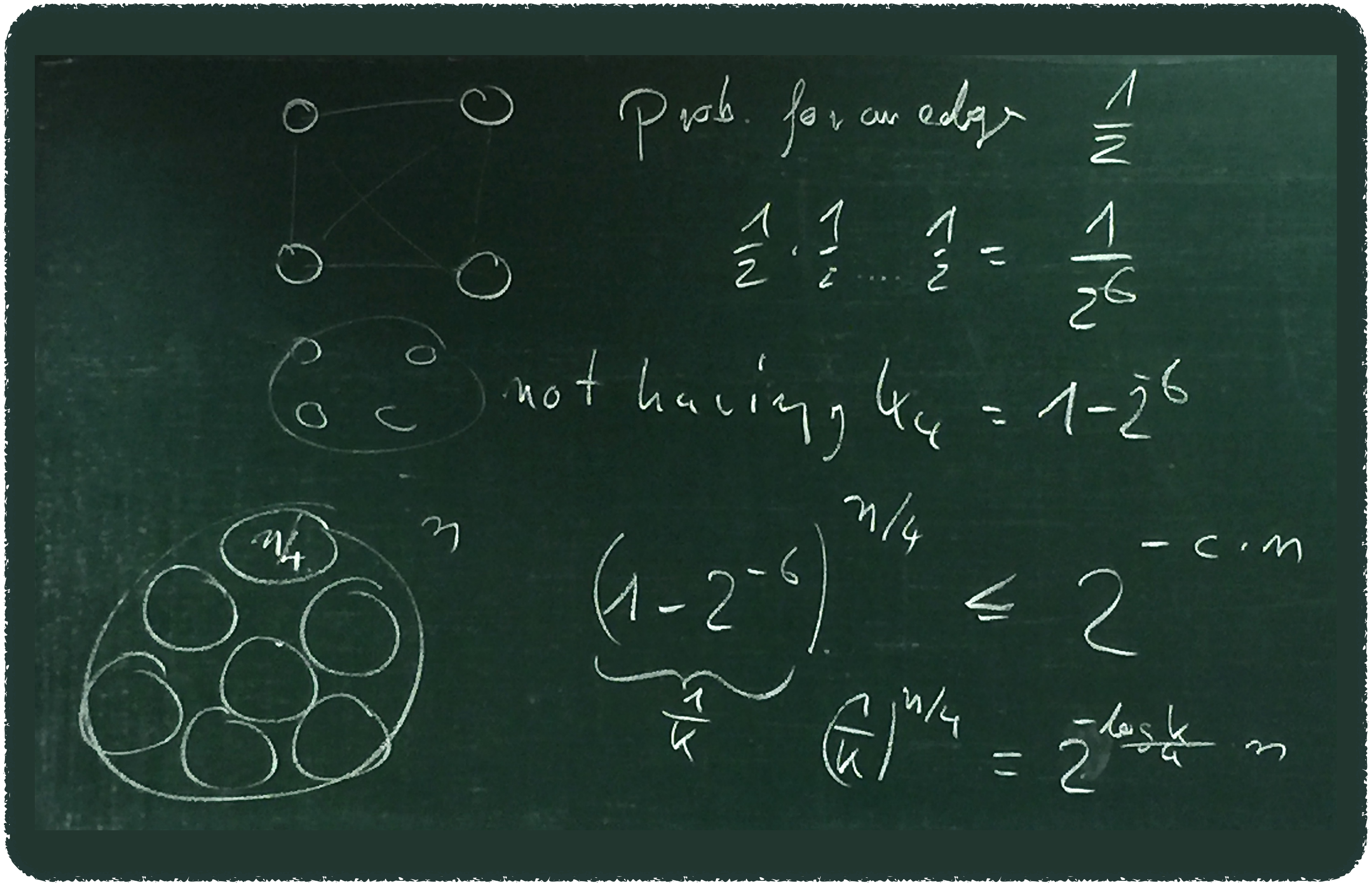

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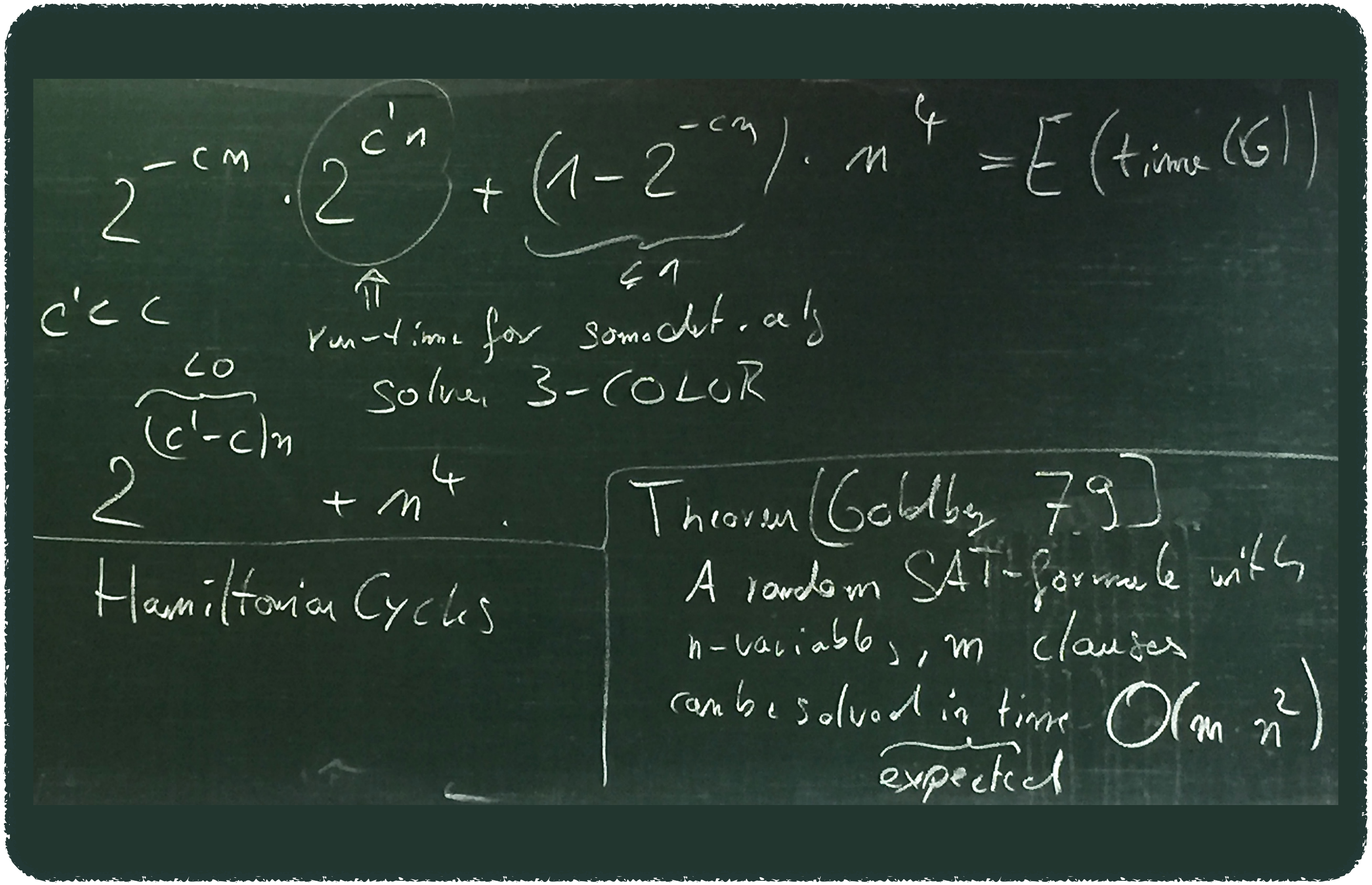

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Davis-Putnam-Alg ($\Phi \in$ CNF)

if $\Phi$ is empty then return 1
else if $\exists$ empty clause then return 0
else if $\exists$ clause with only one literal $x_i$ ($\bar{x}_i$)
then $x_i = 1$ (0)
DP($\varphi$ with $x_i = 1$ (0))
else if $x_i$ never occurs (always) negated
then $x_i = 1$ (0)
DP($\varphi$ with $x_i = 1$ (0))
else choose $x_i$ from $\varphi$
if DP($\varphi$, $x_i = 1$) = 1 then return 1
if DP($\varphi$, $x_i = 0$) = 1 then return 1
else return 0
fi fi

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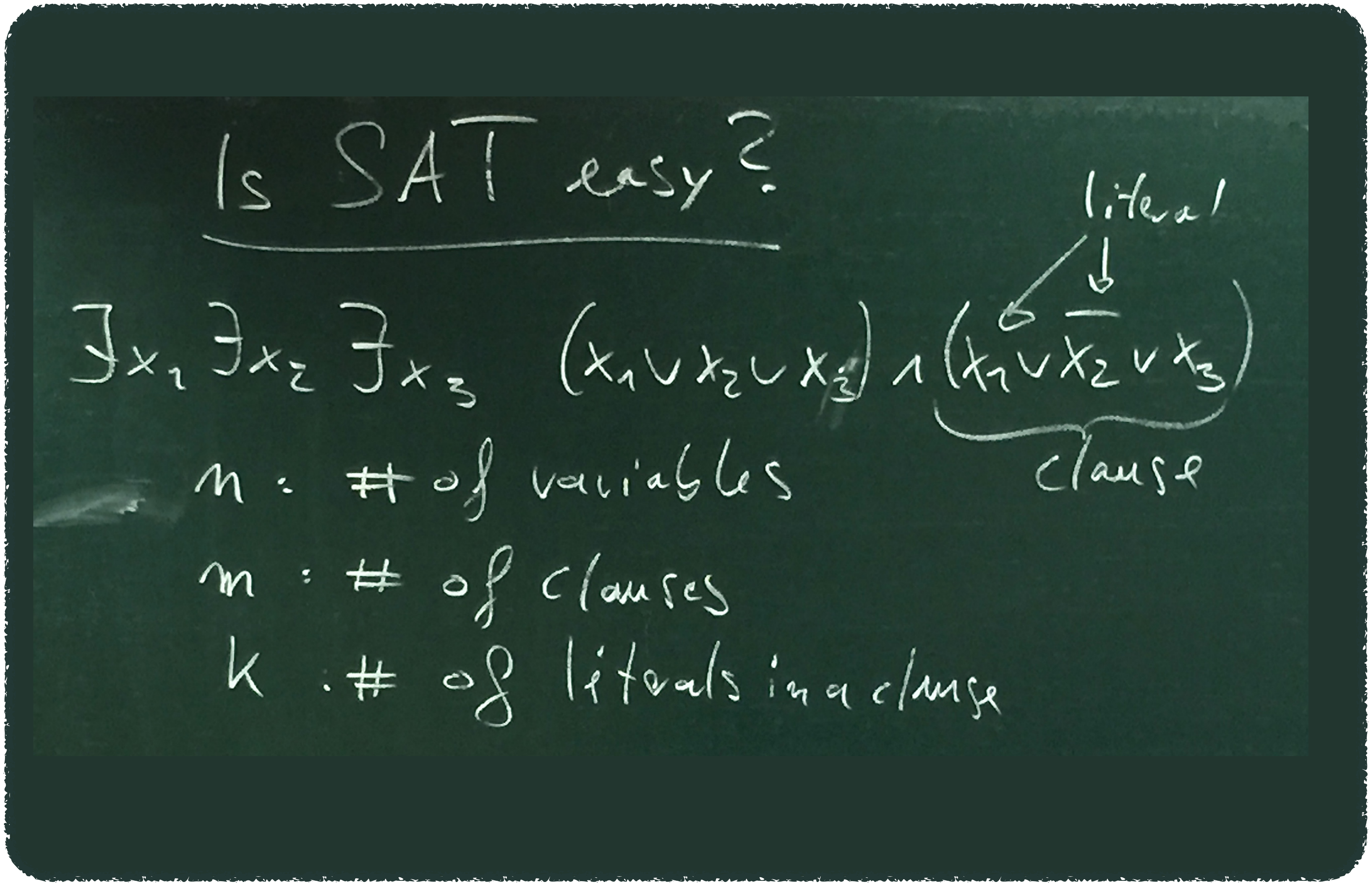

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