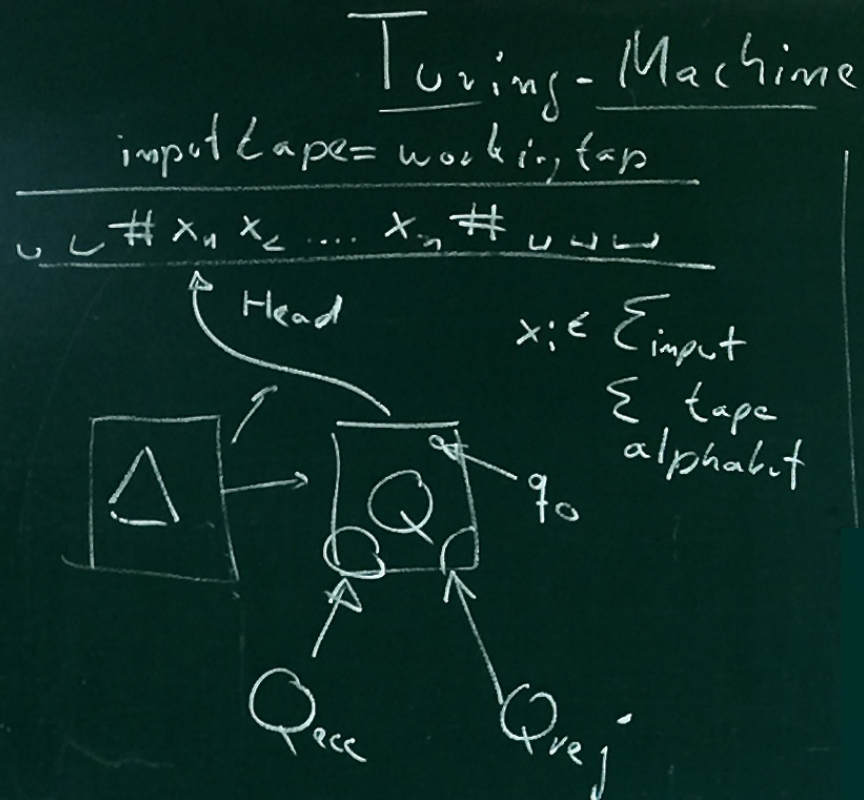


3 Universal TM, Linear Speedup & Time(T)



$$\Delta(q, b) = (q', b', \text{dir})$$

$$b', b \in \Sigma, q, q' \in Q$$

$$\text{dir} \in \{\text{left}, \text{right}\}$$

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Configuration
global state of TM
- state $\in Q$
- tape contents $\in \Sigma^*$
- head position $\in \mathbb{N}$

initial configuration -

$$C_{init}(x) := (q_0, x, 1)$$

$$C_{acc} \subseteq Q_{acc} \times \Sigma^* \times \mathbb{N}$$

$$C_{rej} \subseteq Q_{rej} \times \Sigma^* \times \mathbb{N}$$

$$|w| = n-1, w \in \Sigma^{n-1}, b \in \Sigma$$

$$(q, wby, n) \vdash (q', \underbrace{wb'y}_{w'}, n')$$

$$\text{if } \Delta(q, b) = (q', b', \text{dir})$$

$$\text{if dir} = \text{right} : n' = n+1$$

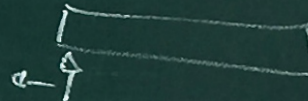
$$\text{if dir} = \text{left} : n' = n-1$$

and $n > 1$

$$\text{if dir} = \text{left} : n' = 1$$

$n = 1$

$$w' = \sqcup b'y$$



$$L(M) = \{w \in \Sigma^* \mid \exists C_1, C_2, \dots, C_t, C_{t+1}, C_{t+2}, \dots, C_{t+k} \in C_{acc},$$

$$C_{init}(w) \vdash C_1 \vdash C_2 \vdash \dots \vdash C_t \in C_{acc} \}$$

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$$L(M) := \{w \in \Sigma^* \mid \exists c_1, c_2, \dots, c_n, c_{n+1} \in C_{acc}, \\ c_{init}(w) \vdash c_1 \vdash c_2 \vdash \dots \vdash c_n \in C_{acc} \}$$

Set of partial recursive Languages

$$RE := \{L \subseteq \Sigma^* \mid \exists TM M: L(M) = L\}$$

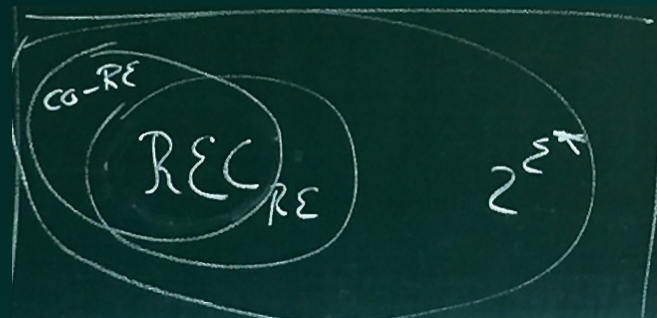
recursive enumerable

$$REC := \{L \subseteq \Sigma^* \mid \exists TM M: L(M) = L \\ \text{and } M \text{ halts on each input}\}$$

Recursive languages

$$co-RE := \{w \notin L \mid L \in RE\}$$

$$REC = RE \cap co-RE$$



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w	ε	0	1	00	01	10	11	000	...
w _ε	1	0	1	1	1	0	1	1	
L	w _ε								
	1	2	3	4	5	6	7	8	...

Isomorphism between $\mathbb{N}$ and S
then S is $\aleph_0$ -infinite

$S_1 = 2^{\mathbb{N}} = \{\{\}, \{1\}, \{1, 2\}, \{2, 3\}, \dots\}$
 is $\aleph_1$ -infinite

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	1	2	3	4	5	6	...
L_1	1	0	1	1	1	1	
L_2	0	1	1	1	0	0	
L_3	1	0	1	0	1	0	
$\vdots$							
$L_i = D$	0	1	0	...			

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$\text{time}_M(x) :=$ number of steps M
needs on input x to
reaching a halting state

$$\text{Time}(T) := \{L \subseteq \Sigma^* \mid \exists \text{TM } M$$

$$\bigcup_{c \in \mathbb{N}} \text{Time}(n^c) = P; \quad \text{EXP} = \bigcup_{c \in \mathbb{N}} \text{Time}(2^{n^c})$$

$L(M) = L \text{ and } \forall x \in \Sigma^n \text{ time}_M(x) \leq T(n)$

$\text{Time}(n^2) \quad \text{Time}(10^{10} \cdot n) \quad \text{Time}(n^{237})$

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Theorem

$\forall c \in \mathbb{N} \quad \forall \text{TM } M \text{ with } k \text{ tapes}$

$\exists \text{TM } M' \text{ with } k \text{ tapes.}$

$$\text{time}_{M'}(x) \leq \frac{1}{c} \cdot \text{time}_M(x) + |x|$$

$$\text{Time}_c(2n^2) = \text{Time}_c(n^2)$$

$$\text{Time}(O(n))$$

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$Q' = Q$, construct new Δ'
 $\Delta'(q, c) = (q', c', dir)$
 $\uparrow \quad \uparrow$
 $\Sigma^c \quad \Sigma^c$

tape $q_1, q_2, \dots, q_4$
 | a | b | c | d | b | a |
 ↑
 $q_1 \rightarrow q_4$
 | a b c | d b a |
 use tape compression
 - combine letters to a new letter

State transition diagram showing a state q with a self-loop and a transition to state q' on input c .

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almost everywhere $f(n) \leq g(n) \Leftrightarrow \exists n_0 \forall n \geq n_0. f(n) \leq g(n)$

infinitely often $f(n) \leq g(n) \Leftrightarrow \forall n_0 \exists n \geq n_0: f(n) \leq g(n)$

$$O(g) = \{ f \mid \exists k \in \mathbb{N} : f(n) \leq_{ae} k \cdot g(n) \}$$

$$\Omega(g) := \{ f \mid \exists k \in \mathbb{N} : f(n) \geq_{ae} \frac{1}{k} g(n) \}$$

$$\Theta(g) = O(g) \cap \Omega(g)$$

$$o(g) := \{ f \mid \forall k \in \mathbb{N} : f(n) \leq_{ae} \frac{1}{k} g(n) \}$$

$$\omega(g) := \{ f \mid \forall k \in \mathbb{N} : f(n) \geq_{ae} k \cdot g(n) \}$$

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