

4 Space and Non-Determinism

Space

$space_M(w) :=$ number of cells
 on working tapes
 a TM M steps into

$$SPACE(S) := \{L \subseteq \Sigma^* \mid$$

$$\exists \text{ TM } M: L(M) = L$$

$$\text{and } \forall w \in \Sigma^* \text{ space}_M(w) \in S(|w|)\}$$

$$space_M(x) \leq time_M(x)$$

$$SPACE(S) \supseteq TIME(S)$$

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$$SPACE(S) \supseteq TIME(S)$$

$$PSPACE := SPACE(n^{O(1)})$$

$$\rightarrow L = SPACE(\log n)$$

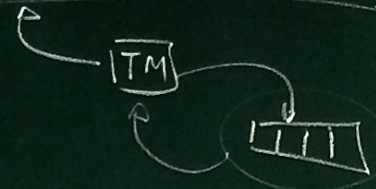
input tape is read only, does not count

$$REG = SPACE(O(n))$$

storing in the finite memory

READ ONLY

a a a a b b b b b b



4 Space and Non-Determinism

$C_{init} \vdash C_1 \vdash C_2 \vdash \dots \vdash C_i \vdash \dots \vdash C_j$
 \Rightarrow TM does not halt

IF TM halt \Rightarrow within #conf.s steps
 \Rightarrow SPACE(S) \subseteq TIME($2^{O(S)}$)

#configurations $\leq |Q| \cdot S \cdot k \cdot |\Sigma|^k$ (Space used)

 $= |Q| \cdot 2^{\log S} \cdot 2^{k \cdot S \cdot \log |\Sigma|}$ (states)

 $O(1) \cdot 2^{O(S + \log S)} = 2^{O(S)}$

$\{x \in \Sigma^n\}$
 $REG \neq L \subseteq P \subseteq PSPACE \subseteq EXPTIME$

$2^{O(\log n)} = 2^{(\log n) \cdot O(1)} = n^{O(1)}$

 $2^{O(n^c)} = 2^{n^{O(1)}}$

4 Space and Non-Determinism

Take a TM and replace Δ with a relation
 transition relation

Non-Determinism

1-tape TM

$\$ 0 1 0 0 1 0 1 0 \$$

$$\Delta = \left\{ \begin{array}{l} (q_0, 0, q_0, 0, \text{right}) \\ (q_0, 1, q_0, 1, \text{right}) \\ (q_0, 1, q_1, 0, \text{left}) \\ \vdots \end{array} \right\}$$

$$\begin{array}{l} \Delta(q_0, 0) = (q_0, 0, "r") \\ \Delta(q_0, 1) = \left\{ \begin{array}{l} (q_0, 1, "r") \\ (q_1, 0, "l") \end{array} \right\} \\ \Delta(q_1, 0) = (q_1, 0, \text{left}) \\ \Delta(q_1, \$) = (q_{\text{acc}}, \dots) \\ \Delta(q_0, \$) = (q_{\text{rej}}, \dots) \end{array}$$

4 Space and Non-Determinism

$N\bar{T}M$. accepts if at least one branch accepts.

$$\bar{N}TIME(T) \quad NP = \bar{N}TIME(POL)$$

$co-N\bar{T}M$: accepts if all branches accept

$$co-\bar{N}TIME(T) \quad co-NP = co-\bar{N}TIME(POL)$$

$\oplus-TM$. accepts if the number of accepting branches is odd

$$\oplus P = \oplus TIME(POL)$$

Graph-Isomorphism $\in \oplus P$

