CoNe Freiburg

5 Closures and Canonical Computation Trees

Closure Properties

1. Complement $C \subseteq 2^{\Sigma^*}$

$$co\text{-}C := \{ L \mid \overline{L} \in C \}$$

$$\chi_L(x) = \begin{cases} 0, & x \notin L \\ 1, & x \in L \end{cases}$$

$$\chi_{\overline{L}}(x) = \neg \chi(x) = \begin{cases} 1, & x \notin L \\ 0, & x \in L \end{cases}$$

C is closed under complement if $C = co\text{-}C$ ($\Leftrightarrow L \in C \Rightarrow \overline{L} \in C$)

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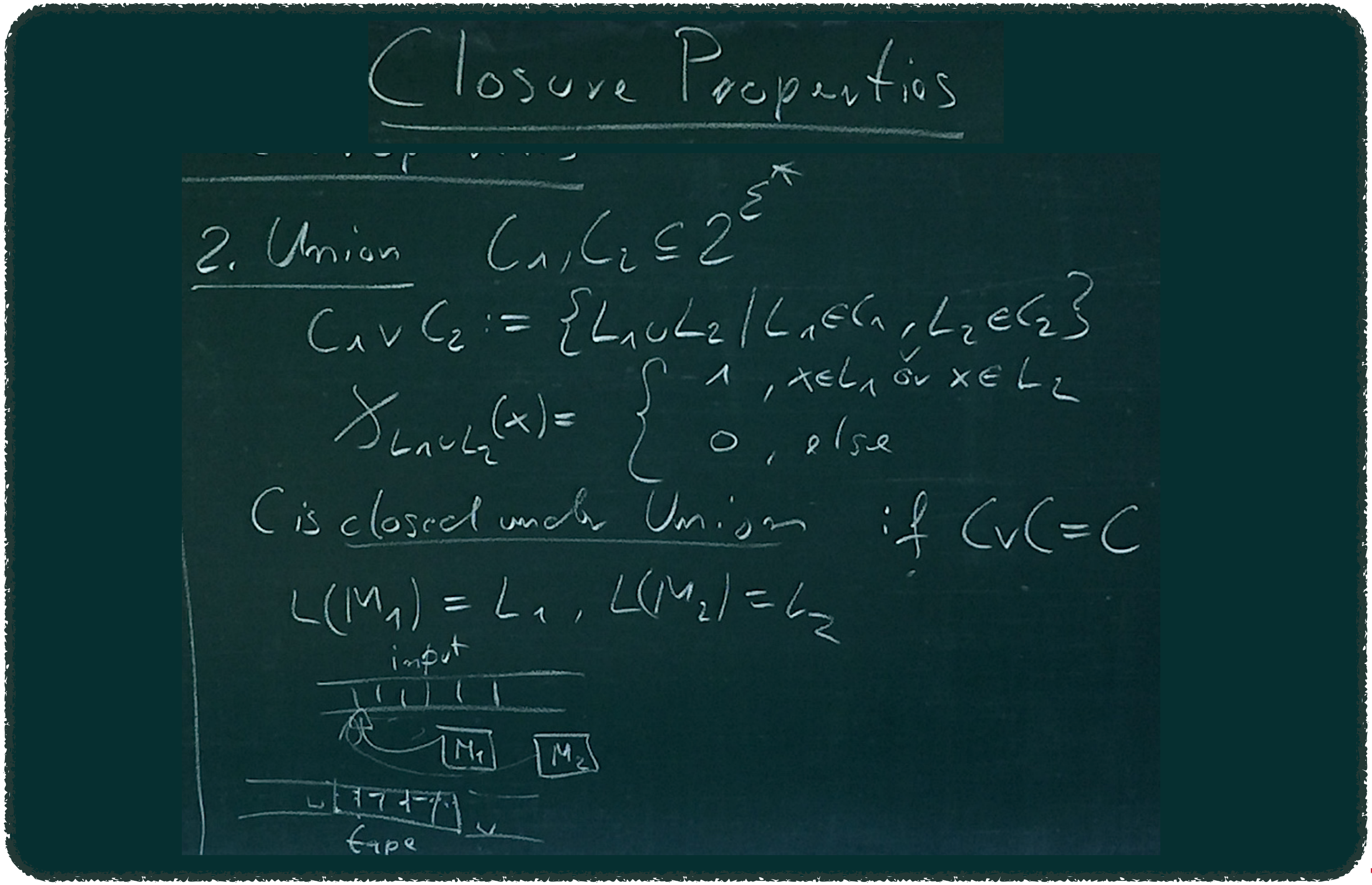

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3. Intersection

$$C_1 \wedge C_2 := \{ L_1 \cap L_2 \mid L_1 \in C_1, L_2 \in C_2 \}$$

$$\chi_{L_1 \cap L_2} = \begin{cases} 1, & x \in L_1 \wedge x \in L_2 \\ 0, & \text{else} \end{cases}$$

closed under Intersection: $C \wedge C = C$

De Morgan: $\text{co-}(\text{co-}C \vee \text{co-}C) = C \wedge C$

$$\neg(\overline{L_1} \vee \overline{L_2}) = L_1 \wedge L_2$$

4. Concatenation

$$L_1 \circ L_2 := \{ w_1 \circ w_2 \mid w_1 \in L_1, w_2 \in L_2 \}$$

C is closed under concatenation if

$$\forall L_1, L_2 \in C \Rightarrow L_1 \circ L_2 \in C$$

regular expression

$$R = \begin{cases} \varepsilon & \\ a & , a \in \Sigma \\ R_1 \vee R_2 & , R_1, R_2 \\ R_1 \circ R_2 & \\ R_1^* & \end{cases}$$

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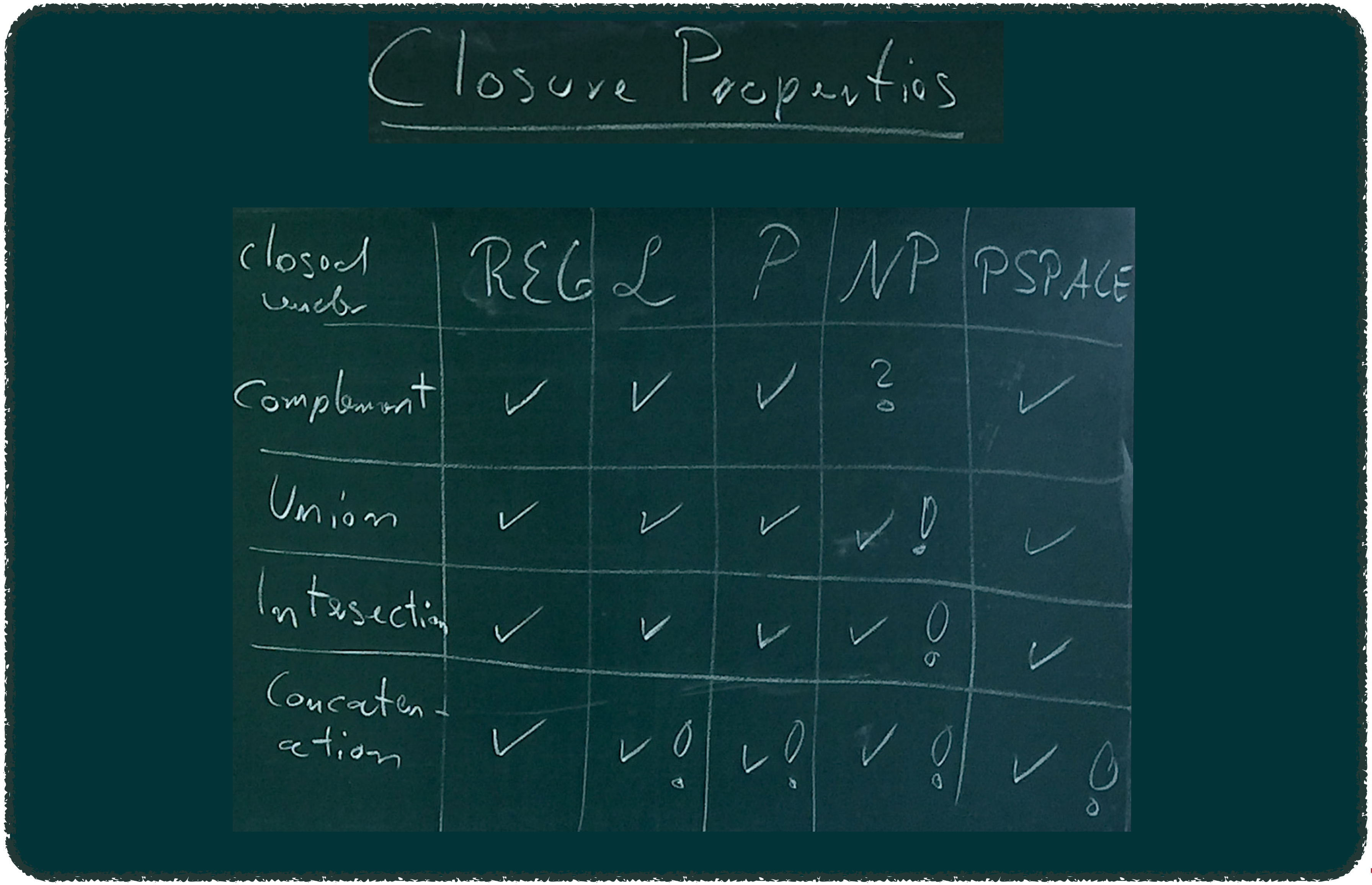
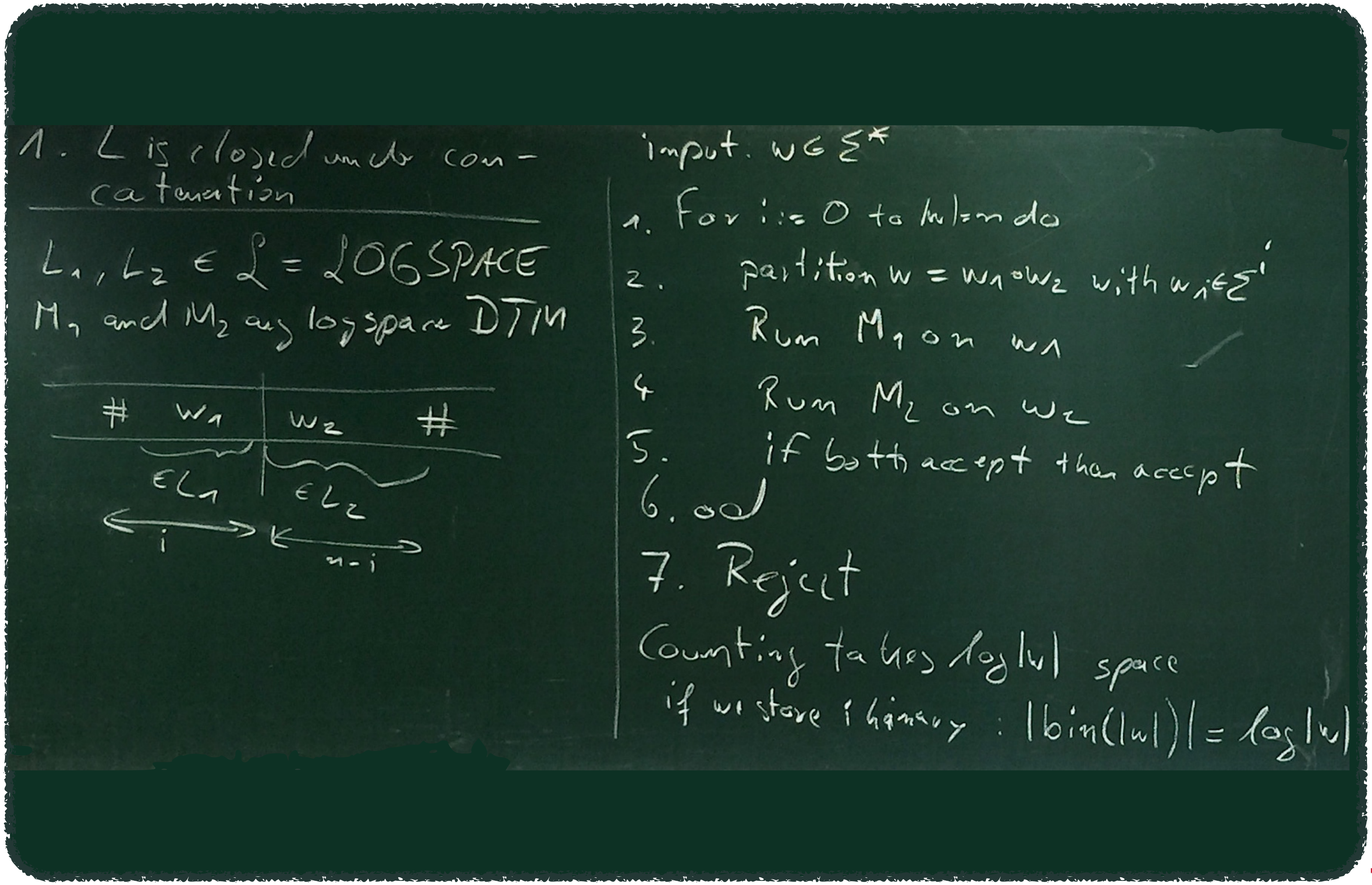


closed under	REG	L	P	NP	PSPACE
Complement	✓	✓	✓	?	✓
Union	✓	✓	✓	✓ !	✓
Intersection	✓	✓	✓	✓ !	✓
Concatenation	✓	✓ !	✓ !	✓ !	✓ !


1. L is closed under concatenation
$L_1, L_2 \in \mathcal{L} = LOGSPACE$
M_1 and M_2 are logspace DTM
w_1 w_2
$\in L_1$ $\in L_2$
i n-i
input: $w \in \Sigma^*$
1. For i := 0 to |w| = n do
2. partition $w = w_1 \circ w_2$ with $w_1 \in \Sigma^i$
3. Run M_1 on w_1
4 Run M_2 on w_2
5. if both accept then accept
6. od
7. Reject
Counting takes log |w| space
if we store i binary : |bin(|w|)| = log |w|

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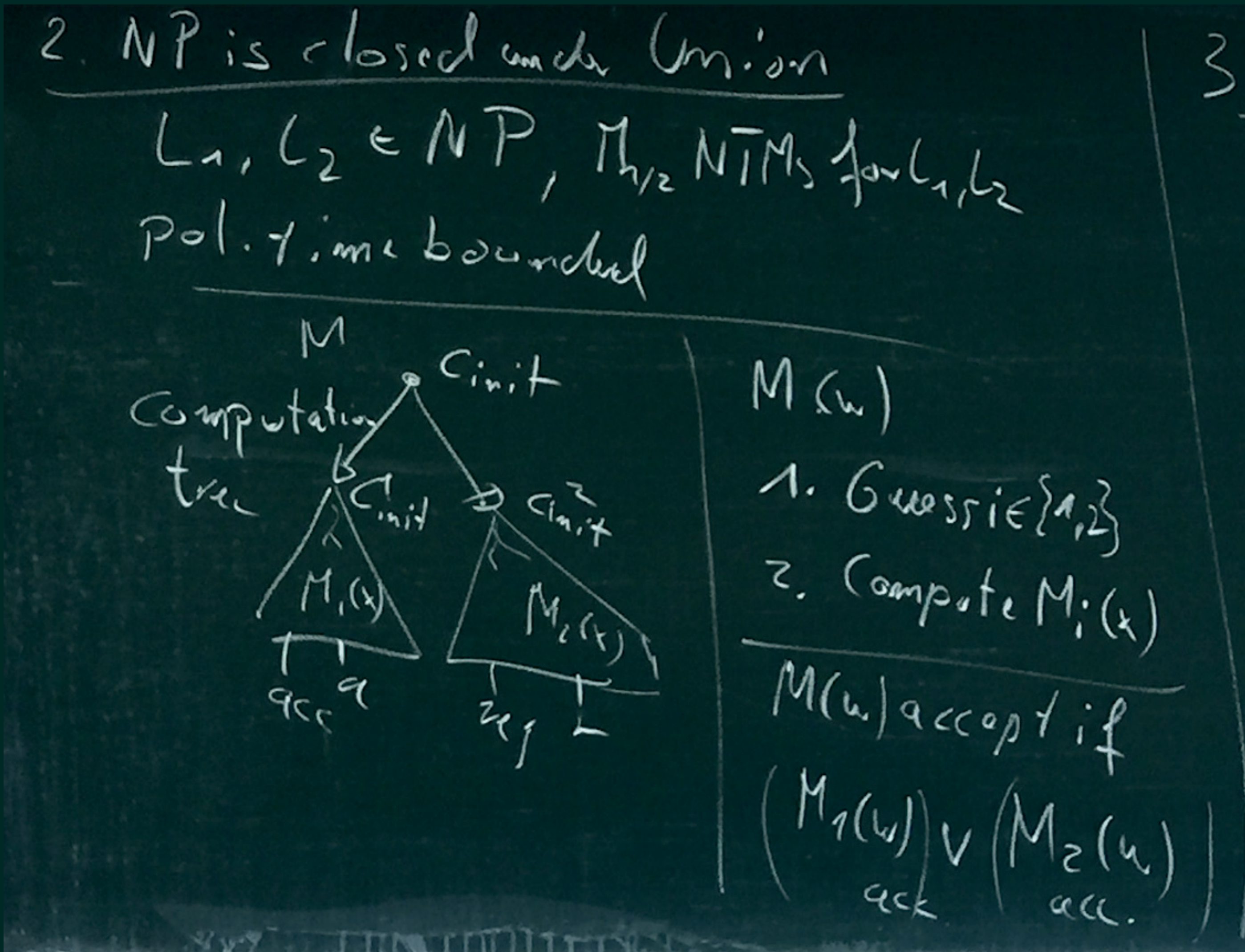

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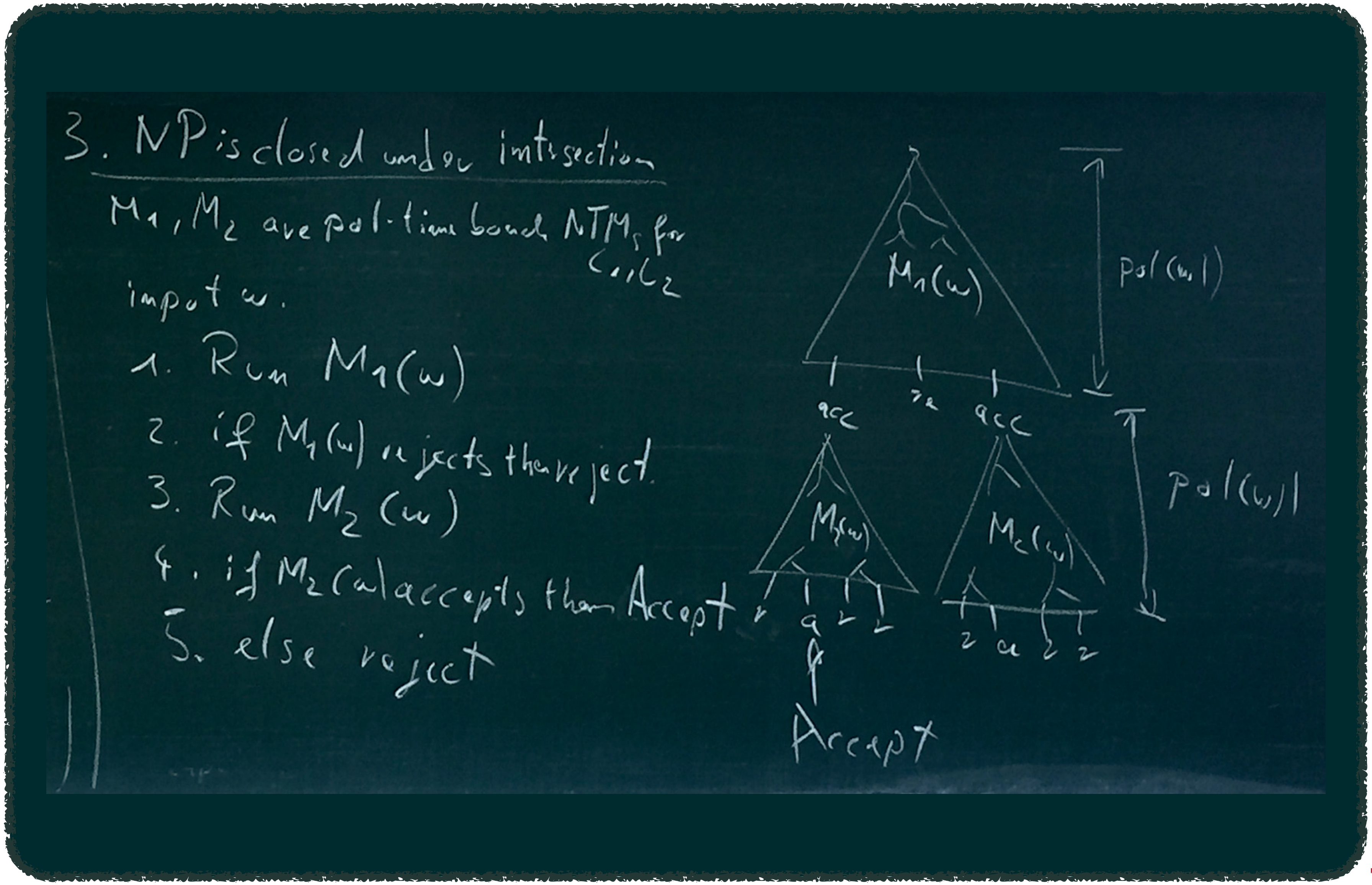

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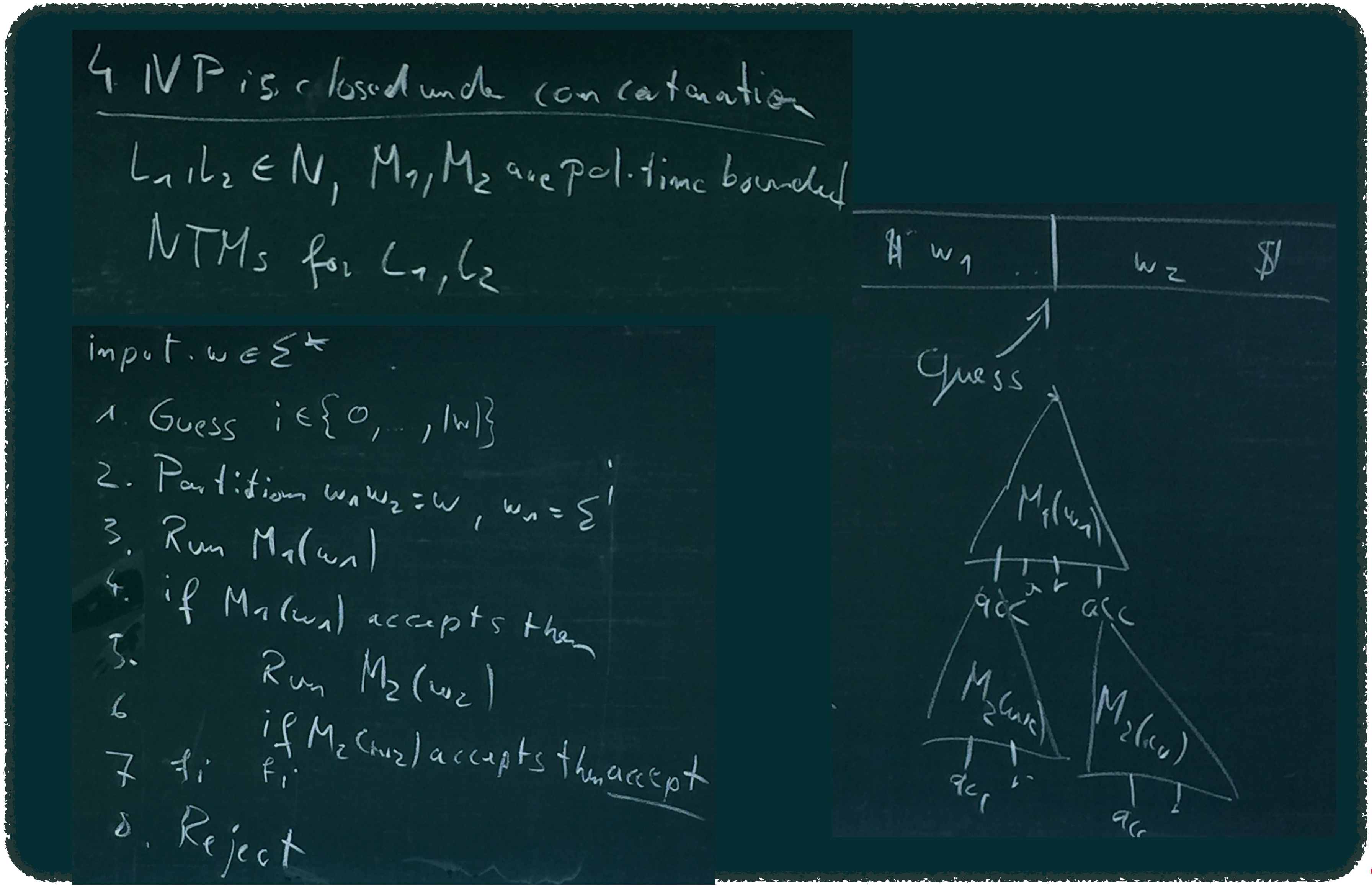

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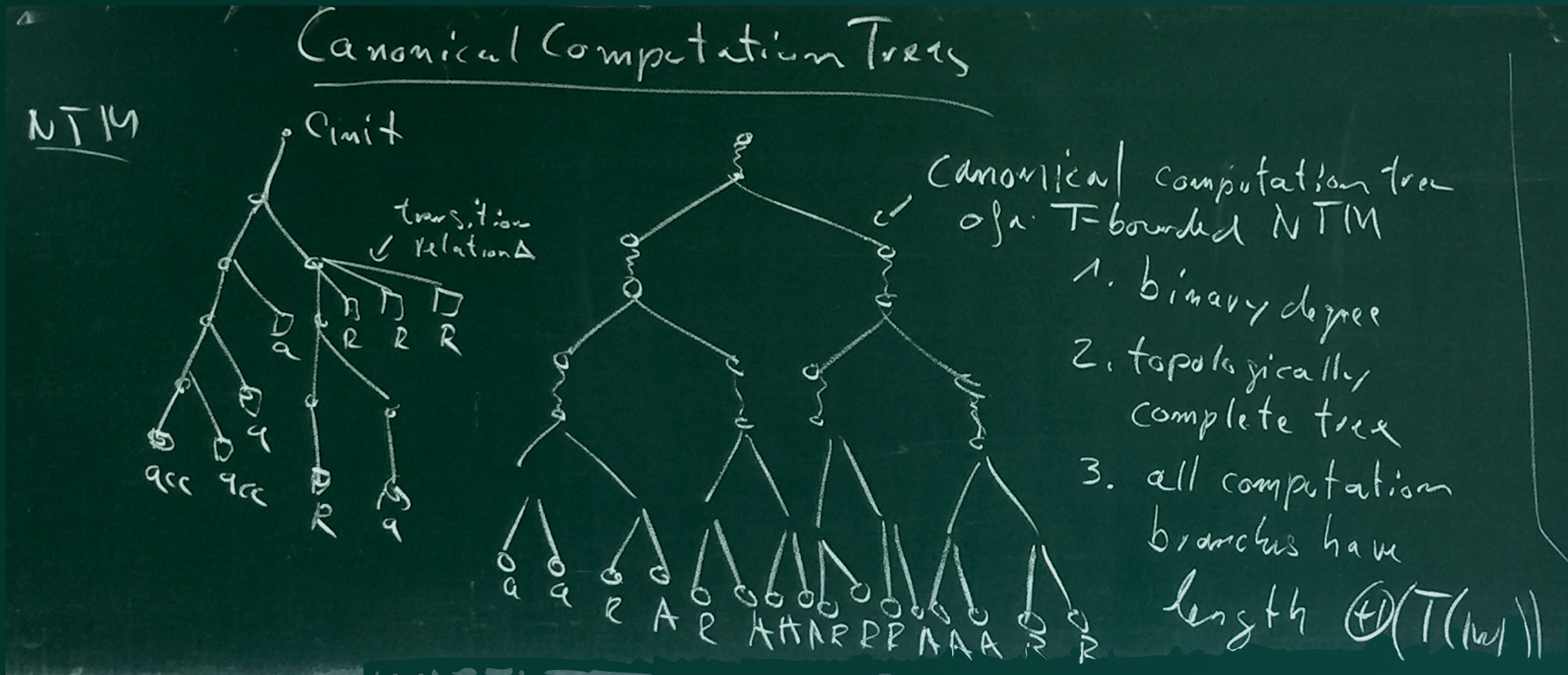

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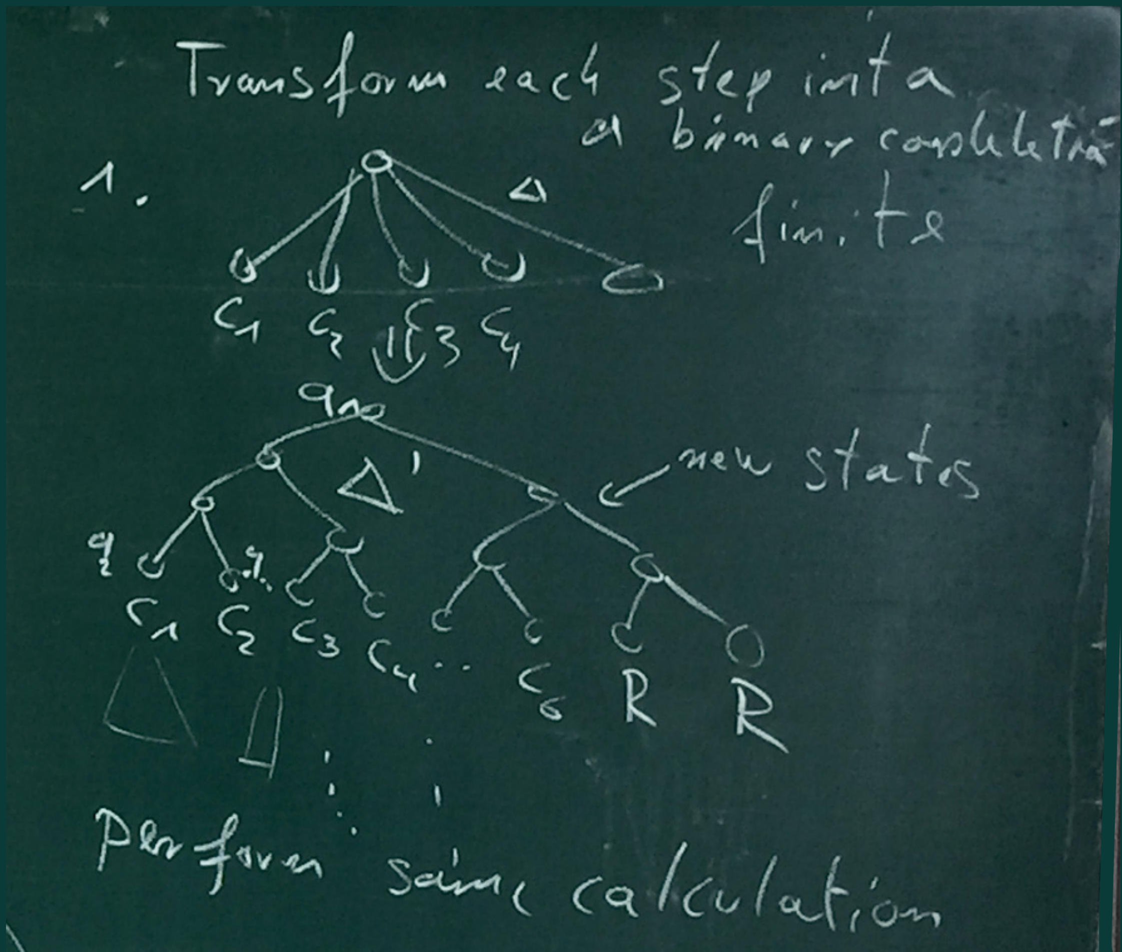

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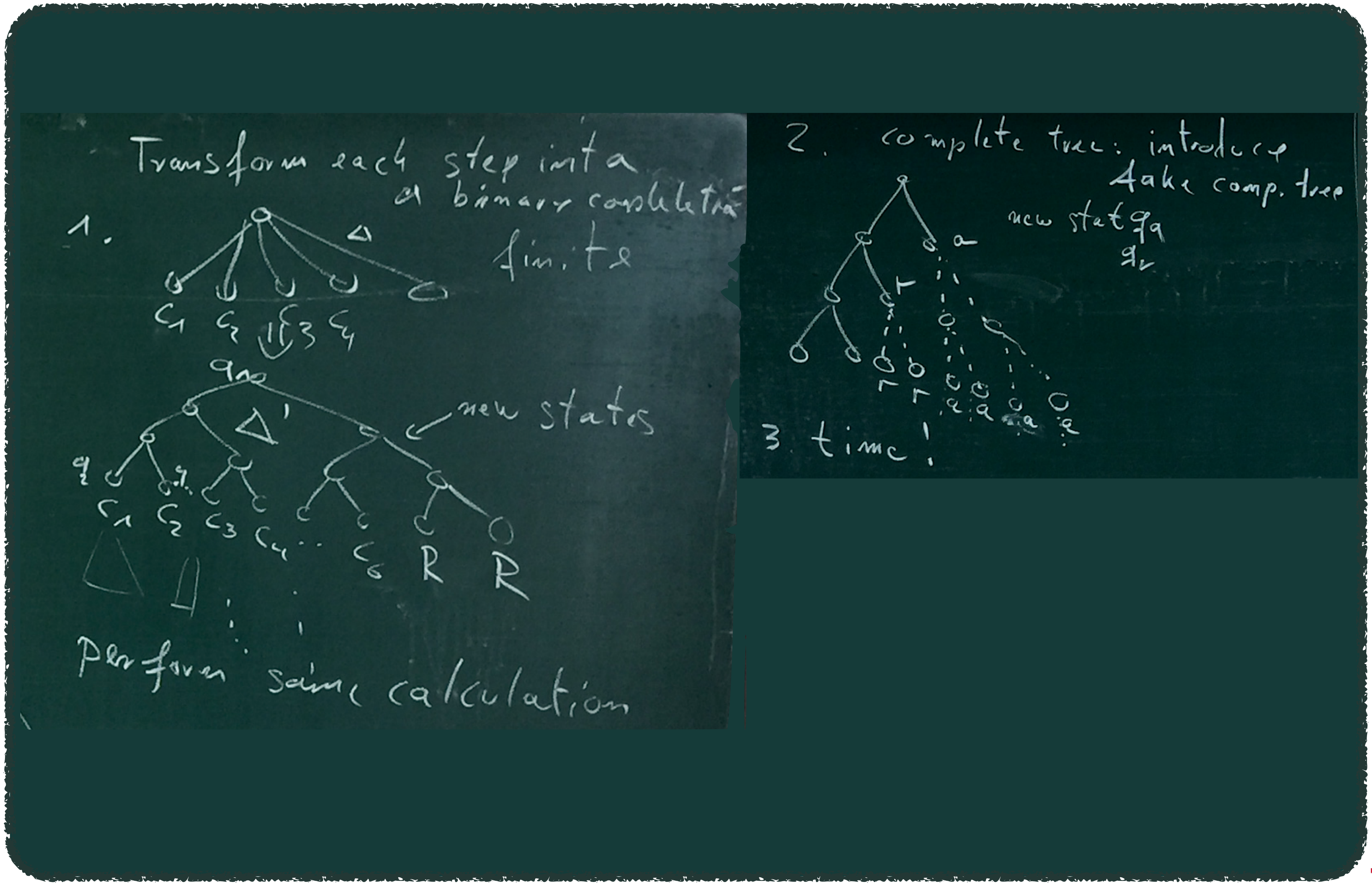

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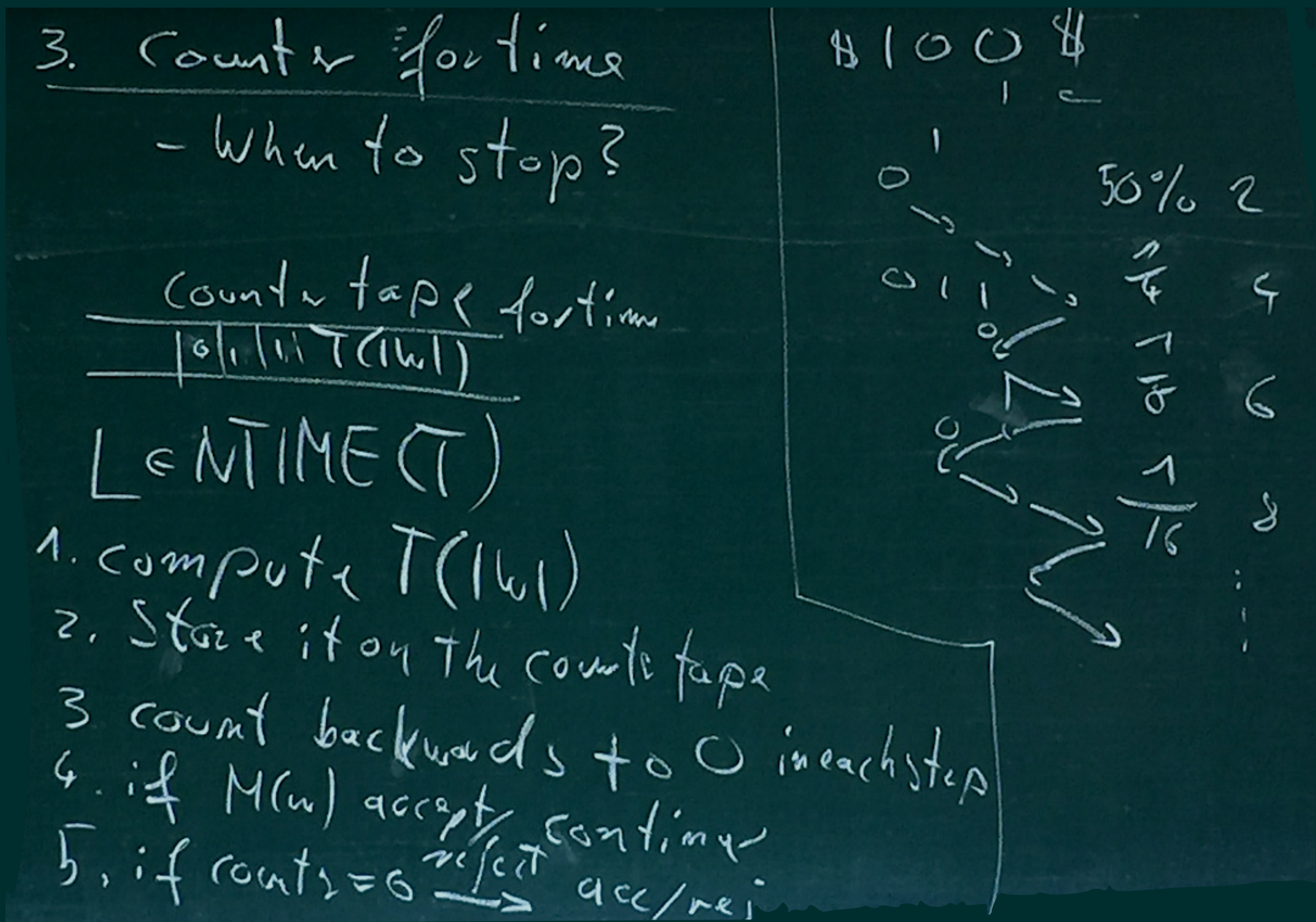

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