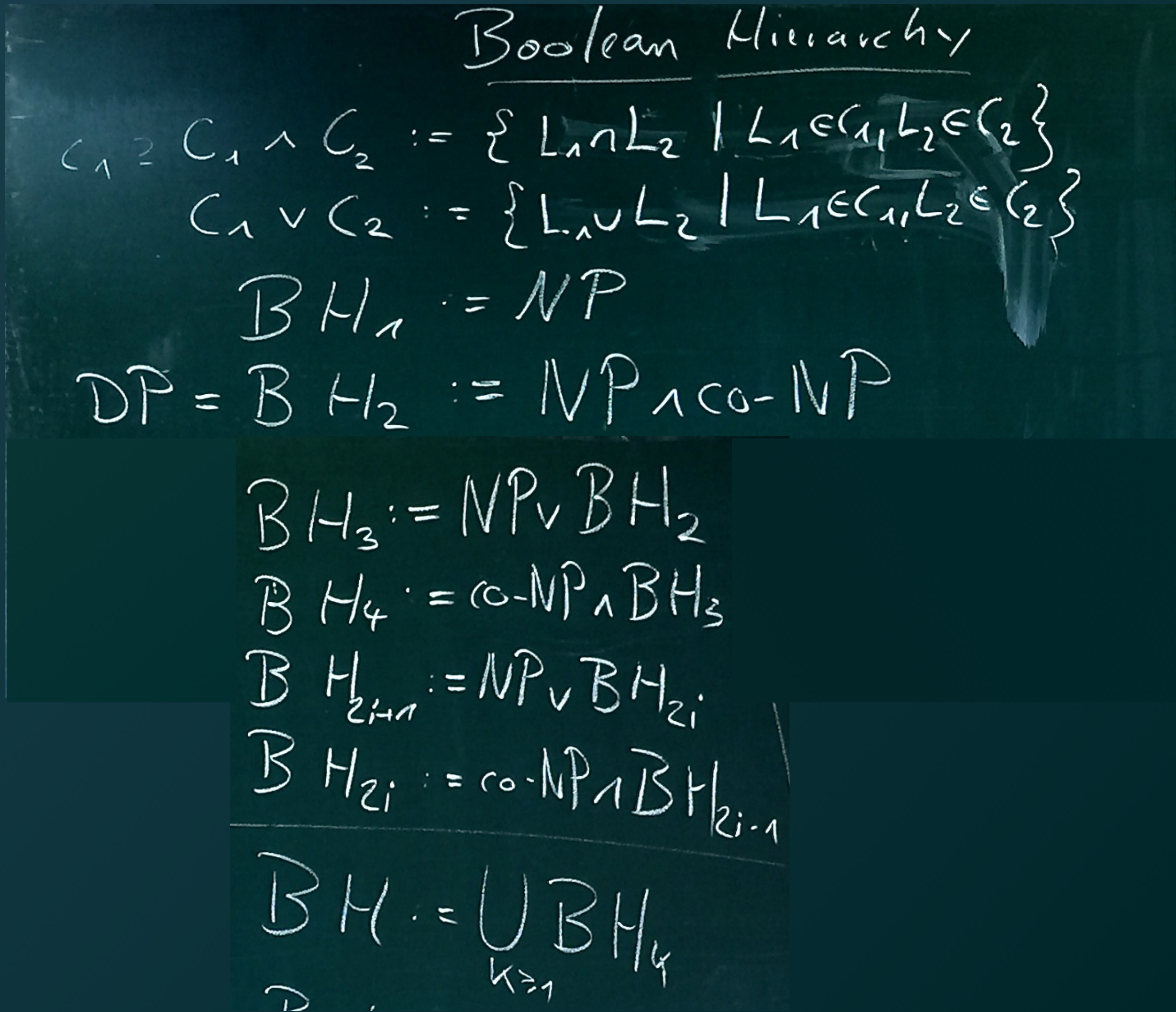
6 BH and PP


Boolean Hierarchy
$C_1 \wedge C_2 := \{L_1 \cap L_2 \mid L_1 \in C_1, L_2 \in C_2\}$
$C_1 \vee C_2 := \{L_1 \cup L_2 \mid L_1 \in C_1, L_2 \in C_2\}$
$BH_1 := NP$
$DP = BH_2 := NP \wedge co\text{-}NP$
$BH_3 := NP \vee BH_2$
$BH_4 := co\text{-}NP \wedge BH_3$
$BH_{2i+1} := NP \vee BH_{2i}$
$BH_{2i} := co\text{-}NP \wedge BH_{2i-1}$
$BH := \bigcup_{k \geq 1} BH_k$

6 BH and PP

$DP = BH_2 := NP \wedge \text{co-}NP$

differential pol. time

$DP \ni \text{SAT-UNSAT} := \left\{ (f_1, f_2) \;\middle|\; \begin{array}{l} f_1 \text{ is satisfiable} \\ f_2 \text{ is not satisfiable} \end{array} \right\}$

$\text{SAT-x} := \left\{ (f_1, f_2) \;\middle|\; \begin{array}{l} \text{Boolean formula} \\ f_1 \text{ is satisfiable} \end{array} \right\}$

Boolean function $f(x_1, x_2, \ldots, x_n)$ is satisfiable

if $\exists x_1 \ldots x_n \in \{0,1\}: f(x_1, \ldots, x_n) = 1$

f is unsatisfiable if $\forall x_1 \ldots x_n: f(x_1, \ldots, x_n) = 0$

$\text{SAT-x} \in NP$

$\text{x-UNSAT} := \left\{ (f_1, f_2) \;\middle|\; \begin{array}{l} \text{Boolean function} \\ f_2 \text{ is unsatisfiable} \end{array} \right\}$

$\in \text{co-}NP$

6 BH and PP

$\mathrm{SAT}\text{-}x \wedge x\text{-}\mathrm{UNSAT} = \mathrm{SAT}\text{-}\mathrm{UNSAT} \in BH_2 = DP$

$\mathrm{EXACT}\text{-}\mathrm{TSP} := \{(G,k) \mid \exists$ TSP of length k (exactly)$\}$

$NP \ni \mathrm{TSP} := \{(G,k) \mid \exists$ TSP of length $\leq k\}$

$\text{co-}NP \ni \mathrm{MORE}\text{-}\mathrm{TSP} : \{(G,k) \mid \forall$ TSPs have length $\geq k\}$

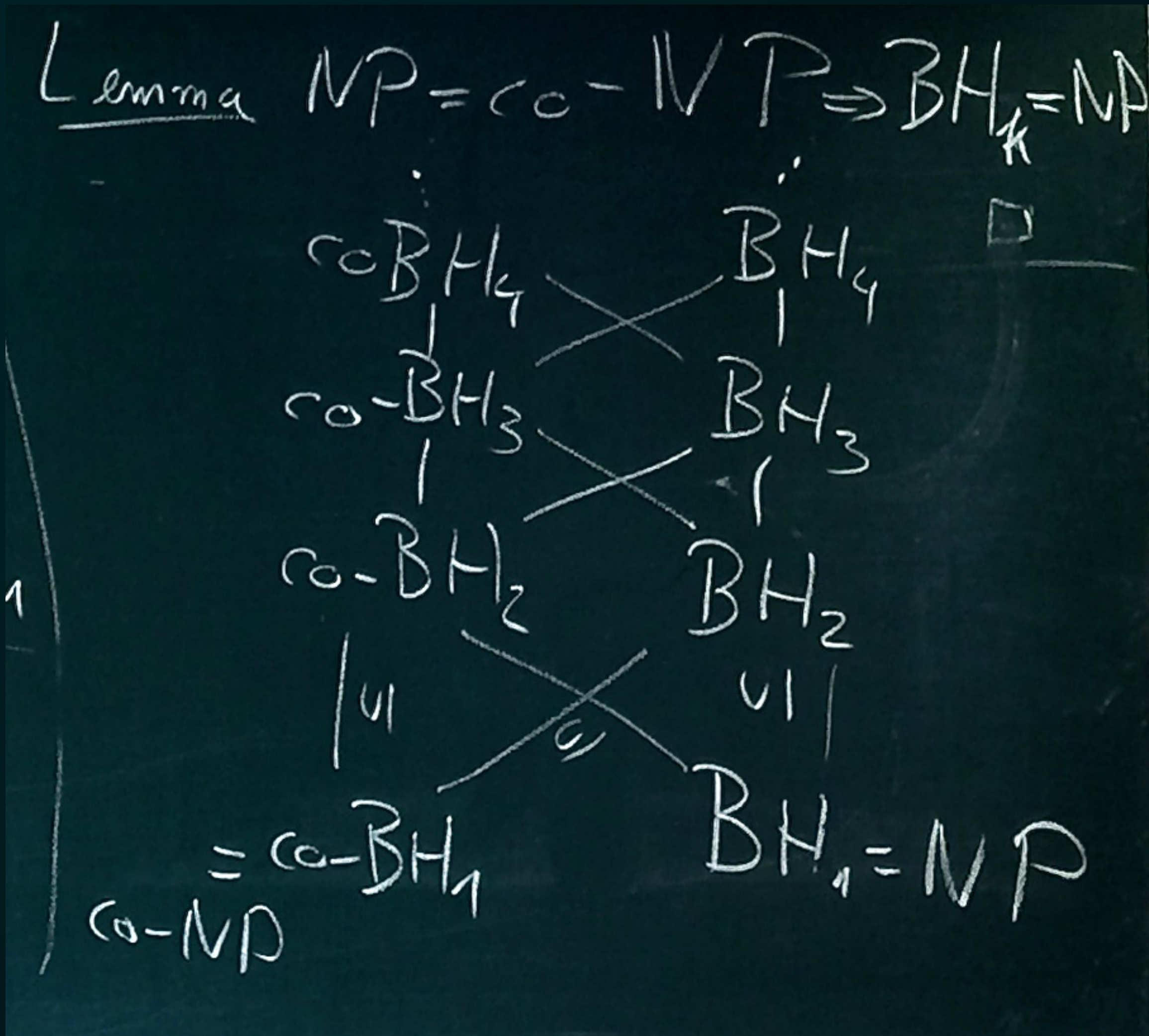
$\{\emptyset, \Sigma^*\} \subseteq \text{co-}NP$?

$\chi_{\Sigma^*}(x) = \underline{1}$

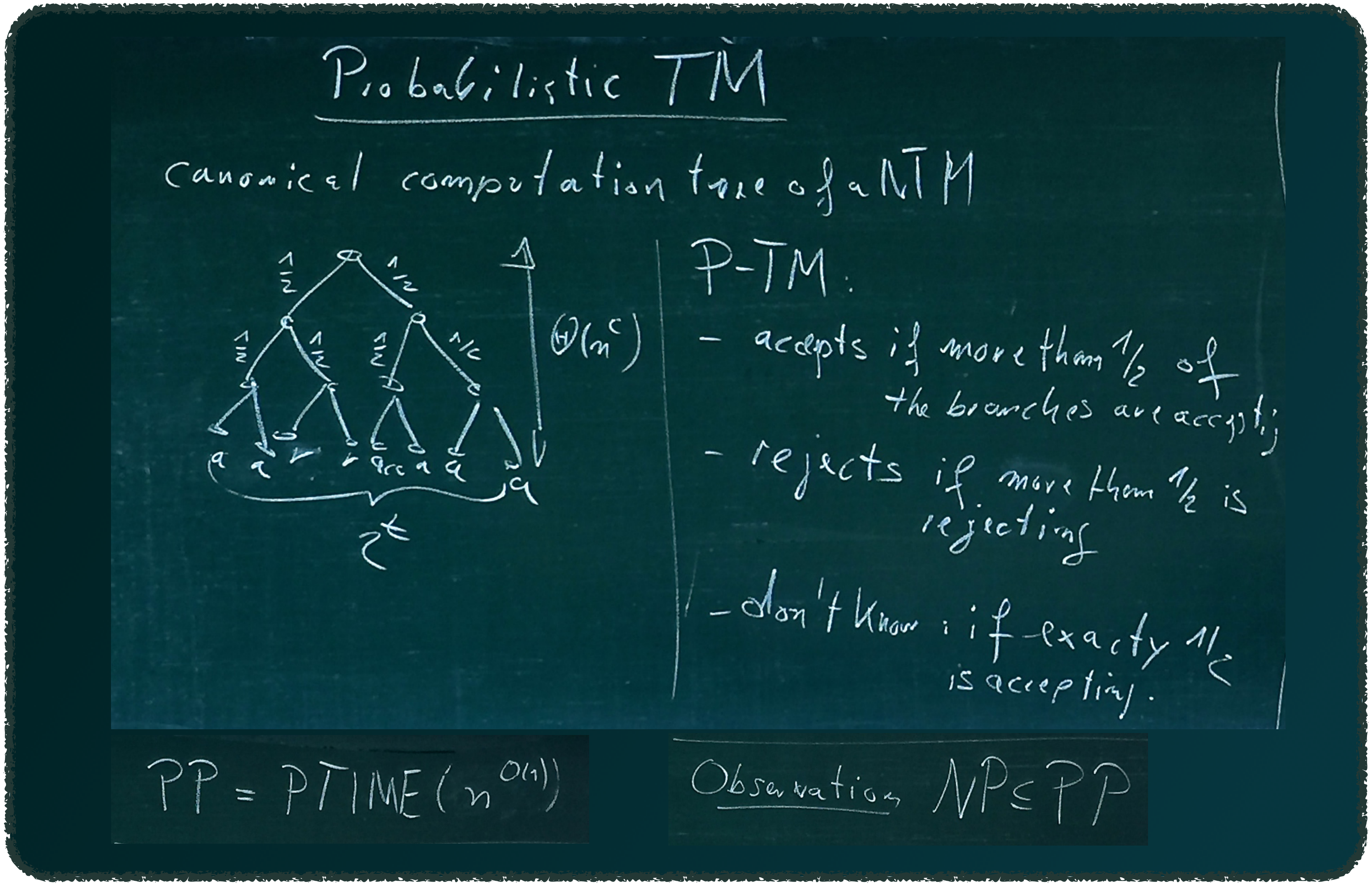
if $\Sigma^* \in C_2 \Rightarrow C_1 \wedge C_2 \supseteq C_1$

$\emptyset \in C_2 \Rightarrow C_1 \vee C_2 \supseteq C_1$

6 BH and PP


Lemma NP = co-NP ⇒ BH_k = NP
co-BH_4
BH_4
co-BH_3
BH_3
co-BH_2
BH_2
= co-BH_1
BH_1 = NP
co-NP

6 BH and PP


Probabilistic TM
canonical computation tree of a NTM
1/2
Θ(n^c)
2^t
P-TM:
- accepts if more than 1/2 of the branches are accepting
- rejects if more than 1/2 is rejecting
- don't know: if exactly 1/2 is accepting.
PP = PTIME(n^O(1))
Observation NP ⊆ PP