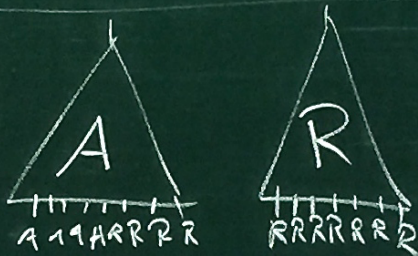


7 R, ZPP, BPP, Savitch and ATMs

R TM - accepts if
 50% of comp. paths
 are accepting
 - rejects if all leaves
 are rejecting.



$$R = RP = RTIME(n^{O(n)})$$

$$ZPP := R \cap co-R$$

$$PRIME := \{bin(m) \mid m \text{ is a prime number}\}$$

$$PRIME \in ZPP$$

$$P \subseteq ZPP, PRIME \in P!$$

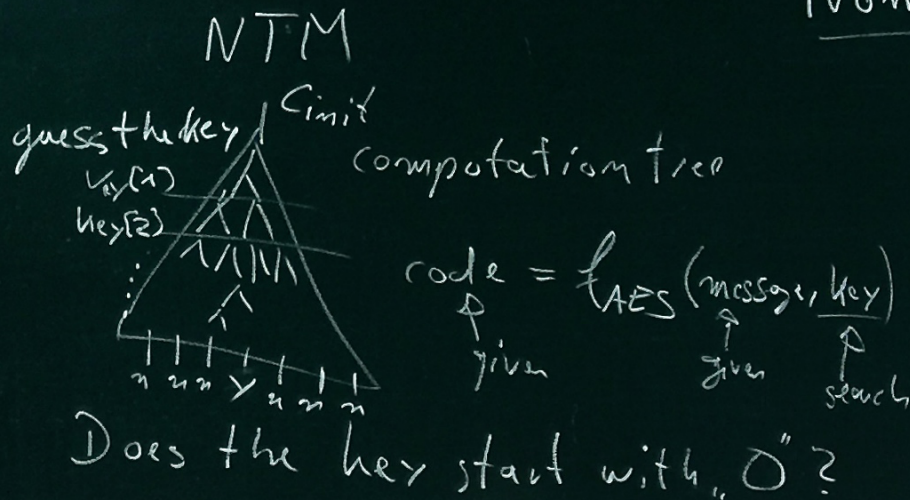
BPP: bounded pol. time
 accepts if more than $\frac{2}{3}$ accept.
 rejects if more than $\frac{2}{3}$ reject

$$R \cup co-R \in BPP \subseteq PP$$

if $NP \subseteq BPP \Rightarrow R = NP!$

7 R, ZPP, BPP, Savitch and ATMs

Non-Determinism



- Unambiguous TM (UTM)
there is at most one accepting leaf in the computation tree

UTM accepts, if it exists

$$UP := UTIME(n^{O(n)})$$

$$P \subseteq UP \subseteq NP$$

$$\begin{matrix} \text{in} \\ \oplus P \end{matrix}$$

- Few-TM
promise: the number of
accepting leafs is bounded
by a polynomial.

$$\text{FewTIME}(n^{O(n)}) =: \overline{\text{FewP}}$$

$$\text{UP} \subseteq \overline{\text{FewP}} \subseteq \text{NP}$$

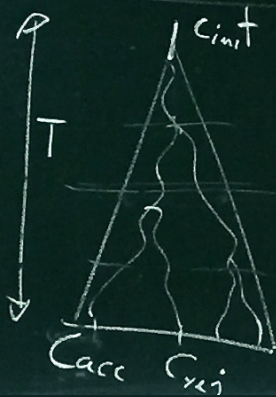
$$\overline{\text{FewP}} \subseteq \oplus \text{P} \quad !$$

7 R, ZPP, BPP, Savitch and ATMs

Theorem of Savitch

Theorem For any space-constructible function
 $s: \mathbb{N} \rightarrow \mathbb{N} : s(n) \geq n$
 $NSPACE(s(n)) \subseteq SPACE(s(n)^2)$

Proof:



computation tree
 unique accepting
 configuration c_{acc}
 time $T \in 2^{c \cdot s(n)}$
 $c > 0$ constant

Configuration
 $Q \times |\Sigma|^{k \cdot s(n)} \times \{\perp, \dots, \perp\}^k$
 $\# \text{config} \leq 2^{c \cdot s(n)}$

1. BAD idea: Backtrack in
in the comp. tree

Size of the stack:
 - one entry config Space: $s(n)$
 - T entries
 \Rightarrow Size $s(n) \cdot 2^{s(n)}$

7 R, ZPP, BPP, Savitch and ATMs

$$\text{CanYield}(C_1, C_2, 0) = (C_1 = C_2) \quad \checkmark$$

$$\text{CanYield}(C_1, C_2, 1) = \begin{cases} 1, & C_1 \neq C_2 \\ 0, & \text{else} \end{cases} \quad \checkmark$$

$$\text{CanYield}(C_1, C_2, t) =$$

$\exists \text{ conf. } C \text{ s.t.}$

$$\text{CanYield}(C_1, C_1 \upharpoonright_{\lfloor \frac{t}{2} \rfloor}) \wedge \text{CanYield}(C_2, C_2 \upharpoonright_{\lfloor \frac{t}{2} \rfloor})$$

$$t = \lfloor \frac{t}{2} \rfloor + \lfloor \frac{t}{2} \rfloor$$

$$5 = \lfloor 2.5 \rfloor + \lfloor 2.5 \rfloor = 2 + 3 = 5$$

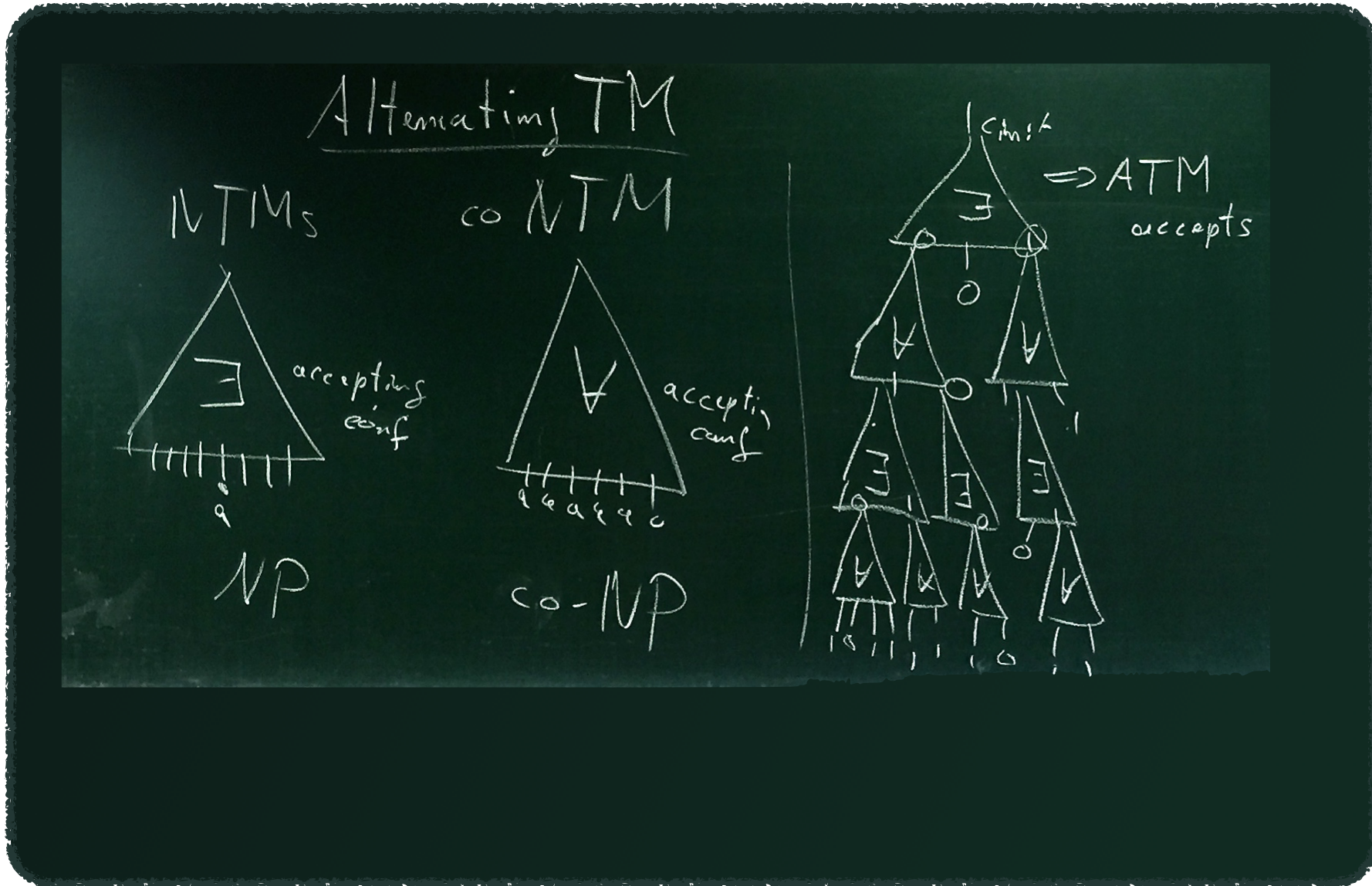
7 R, ZPP, BPP, Savitch and ATMs

Alg CanYield(C_1, C_2, ϵ)
 if $t=0$ then if $C_1 = C_2$ then
 RETURN true
 else RETURN false
 else if $t=1$ then if $C_1 \neq C_2$ then
 else true
 else false
 else forall $c \in C_1$ of size $s(n)$ do
 if CanYield($C_1, C_2, \frac{\epsilon}{2}$)
 and CanYield($C_1, C_2, \frac{\epsilon}{2}$)
 then RETURN TRUE
 od
 RETURN FALSE

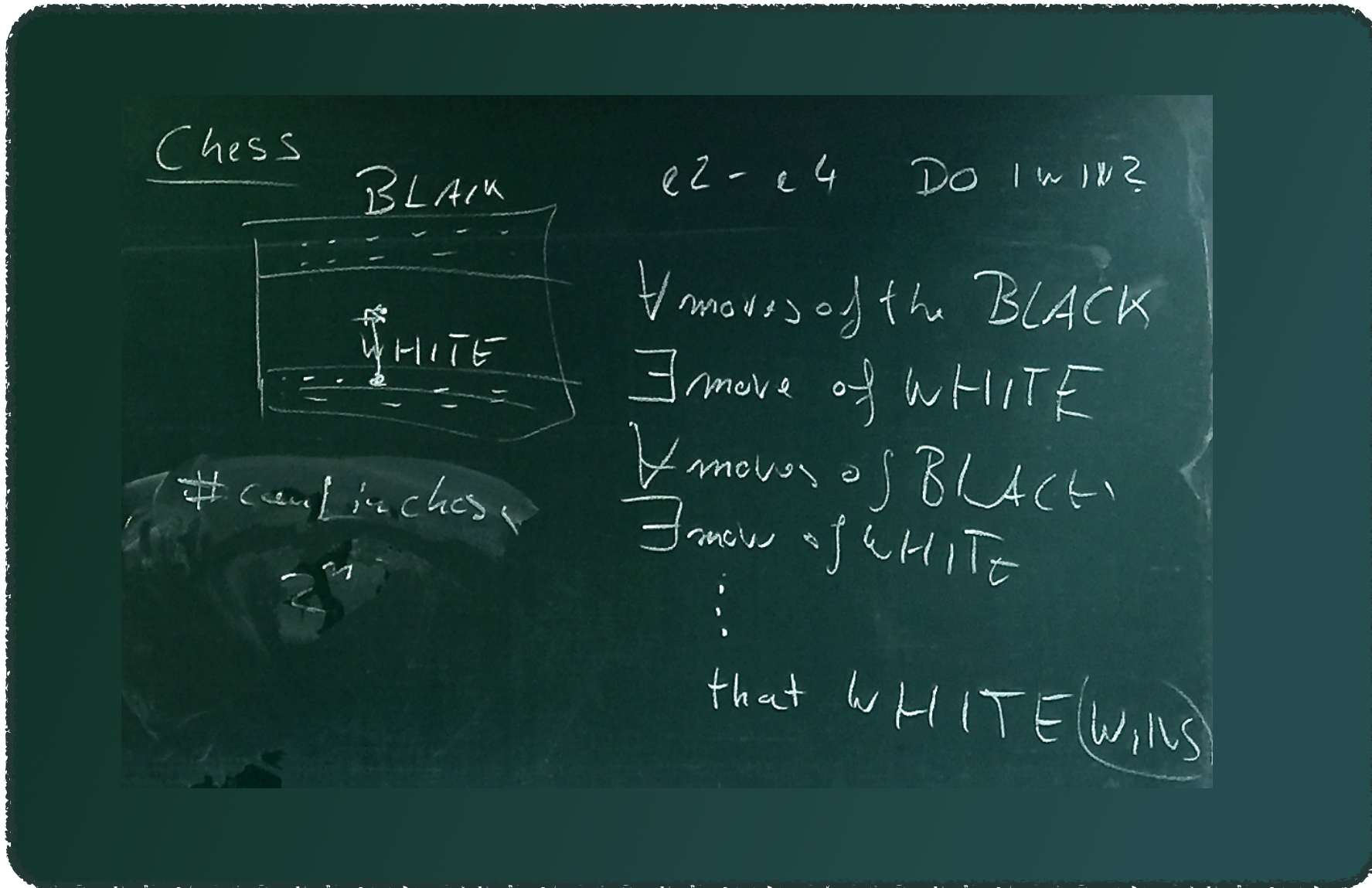
space needed per
 recursion : $O(s(n))$
 stack size $\log T + 1$
 $T \rightarrow T/2 \rightarrow T/4 \rightarrow \dots \rightarrow 1$
 $\log T = O(s(n))$
 overall space needed
 is $O(s(n)^2)$
 Now compute
 CanYield(C_1, C_2, ϵ)

\Rightarrow
 PSPACE = NPSPACE

7 R, ZPP, BPP, Savitch and ATMs



7 R, ZPP, BPP, Savitch and ATMs



7 R, ZPP, BPP, Savitch and ATMs

ATM has states Q

$$Q = \{q_{acc}\} \cup \{q_{rej}\} \cup Q_{\exists} \cup Q_{\forall}$$

ATM M accepts in conf C .

1. C is in q_{acc}
2. C is in Q_{\exists} and $\exists C' : C \vdash C'$ and C' accepts
3. C is in Q_{\forall} and $\forall C' : C \vdash C' \Rightarrow C'$ accepts

$$AP = ATIME(n^{O(n)})$$

$$AP = PSPACE$$

(next time...)