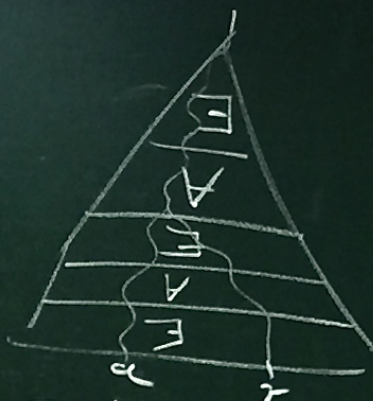


8 AP=PSPACE & PH

Alternating TM



Theorem

$$Q = \{q_{acc}, q_{rej}\}$$

$$\forall Q \cap Q \cap Q$$

$$\begin{aligned}
 & \text{ATIME(POL)} \\
 & = \text{PSPACE}
 \end{aligned}$$

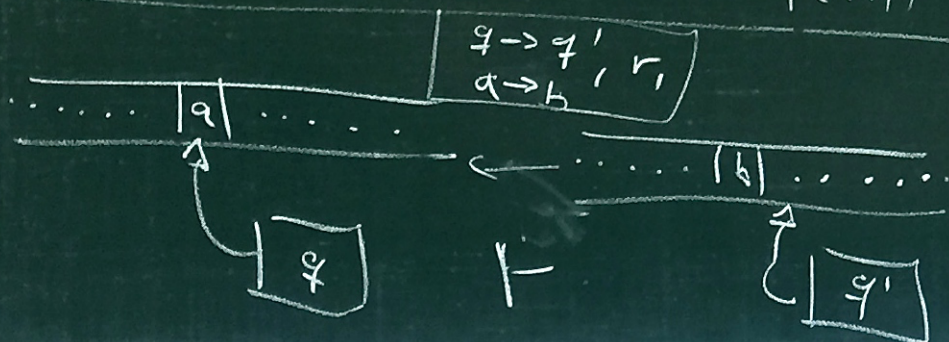
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Configuration of an ATM

tape inscription \times tape position $\times Q$

$$\in \text{space}_M(x) \leq \text{time}_M(x) \leq T(|x|)$$

Space needed for DFS $\in T(|x|)^2$



We can store all information to get from C' to C if $C \vdash C'$ in space $O(n)$.

\rightarrow space for DFS: $O(T)$

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Simulate (M, C, t)

 if $t = 0$ and C non-accepting \rightarrow Return 0

 else if C accepts then 1

 else if C rejects then 0

 else if $C \in \Sigma^*$ then

 $v = 0$

 forall $C' : C \vdash C'$ do

 or $v = v$ or Simulate $(M, C', t-1)$

 else $(C \in \Sigma^k)$ then $v = 1$

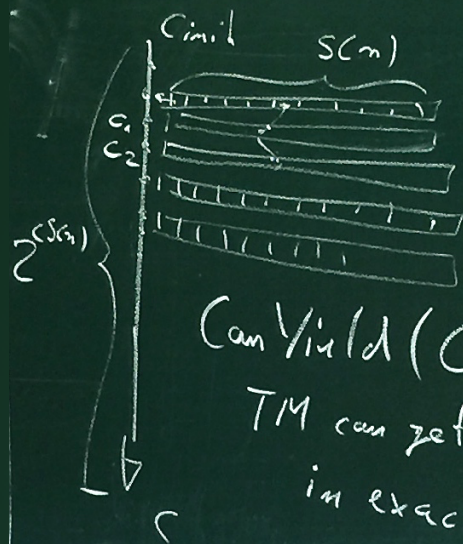
 forall $C' : C \vdash C'$ do

 if or $v_i = v$ and Simulate $(M, C', t-1)$

 Return v

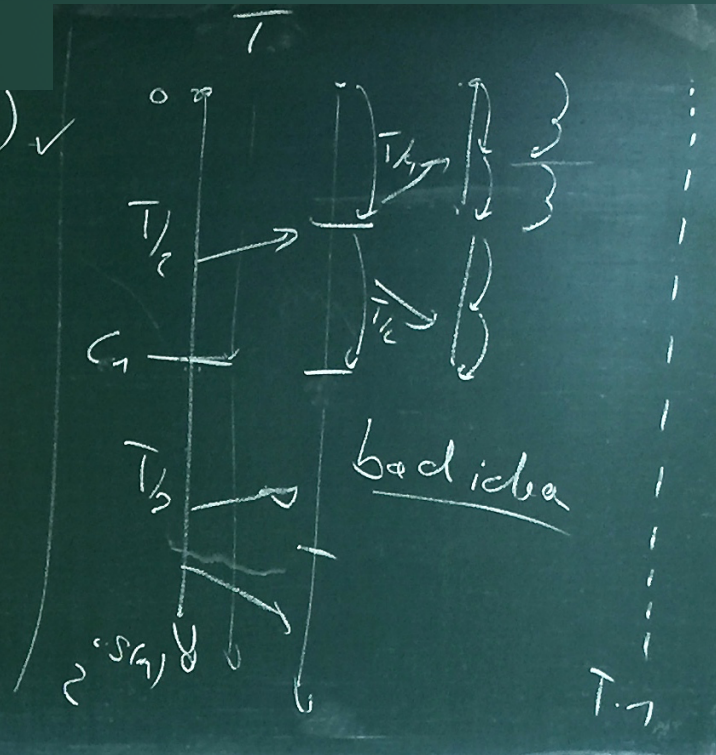
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2.) $SPACE(S) \subseteq ATIME(S^2)$ ✓



$c_1 \vdash c_2$
 can be decided
 in time $O(S(n))$
 by a DTM!

Can Yield $(C_1, C_2, t) :=$
 TM can get from c_1 to c_2
 in exactly t steps



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Assume that T is a power of 2
 $t \geq 2$.

$$\text{CanYield}(C_1, C_2, t) =$$

$$\exists C : (\text{CanYield}(C_1, C_1, t/2) \wedge \text{CanYield}(C_1, C_2, t/2))$$

$$\exists C \forall C'_1, C'_2 : ((C_1, C_2) = (C'_1, C'_2) \vee (C_1, C_2) = (C'_2, C'_1))$$

$$\Rightarrow \text{CanYield}(C'_1, C'_2, t/2)$$

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ATM $M'(c_1, c_2, t)$

if $t=1$ then Return (c_1, c_2) ?

\exists guess configuration C

\forall guess configuration C_1, C_2

$CY = \text{ANYield}(C_1, C_2)$

RETURN $(C_1 = C_1)$ and $(C_2 = C)$
 or $(C_1 = C)$ and $(C_2 = C_2)$
 $\Rightarrow CY$

for $t=1$: time: $s(n)$

$$\begin{aligned}
 f(t) &= f(t/2) + c \cdot s(n) \\
 &= c \cdot s(n) \cdot \log_2 t = O(s(n)^2)
 \end{aligned}$$

$\underbrace{\hspace{1cm}}_{c \cdot s(n)}$

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Def Σ_k -TM is an ATM

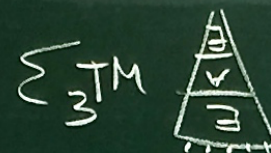
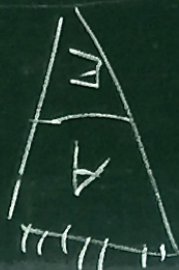
 which starts at an existential

 state and alternates at

 most k -times

$$\Sigma_1\text{-TM} = \text{NTM}$$

Σ_2 -TM



$$\Sigma_k^P = \Sigma_k \text{TIME}(n^{O(k)})$$

$$\Sigma_1^P = \text{NP}$$

$$\text{BPP} \subseteq \Sigma_2^P \text{ [Sipser] } \circ$$

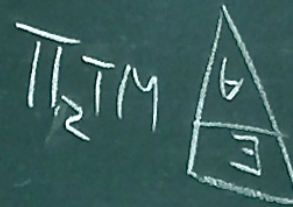
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Def Π_k -TM is an ATM starting with a universal state and alternating k times

$$\Pi_1\text{-TM} = \text{co-NP}$$

$$\Pi_k^P = \Pi_k \text{ TIME}(n^{\text{O}(n)})$$

$$\Pi_1^P = \text{co-NP}$$



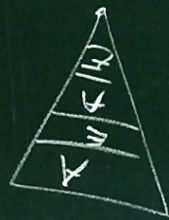
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$$APSPACE = \bigcup_{k \in \mathbb{N}} ASPACE(n^k)$$

$$\begin{aligned}
 APSPACE &= EXP \\
 &= \bigcup_{c \in \mathbb{N}} DTIME(2^{n^c})
 \end{aligned}$$

$$\begin{aligned}
 BPP &\subseteq \Sigma_2^P \text{ [Sipser]} \\
 &\subseteq \Pi_2^P
 \end{aligned}$$

$$co-\Sigma_k^P = \Pi_k^P$$



$$\neg \exists x : P(x) \Leftrightarrow \forall x : \neg P(x)$$

$$\neg \exists x \forall y : P(x) \Leftrightarrow \forall x \exists y : \neg P(x)$$

8 AP=PSPACE & PH

