

Exercise for the lecture
Computational Complexity
Winter 2014/15
Sheet 3

EXERCISE 1:

Does the Boolean Hierarchy collapse if the following is true? And if so, to which level does it collapse?

1. $\mathcal{NL} = \mathcal{NP}$,
2. $\mathcal{NP} = \mathcal{BH}_2$,

Note that $\mathcal{NL} := \text{NSPACE}(O(\log n))$.

Hint: consider the relation to \mathcal{P} .

EXERCISE 2:

Prove that

$$\mathcal{PP} \subseteq \mathcal{PSPACE}.$$

EXERCISE 3:

For a language L and a polynomial p define

$$\exists^p L := \{ x \in \Sigma^* : \exists w \in \{0, 1\}^{p(|x|)} : (x, w) \in L \},$$

where (x, w) is the concatenation of x and w .

1. Prove that for $L \in \mathcal{P}$: $\exists^p L \in \mathcal{NP}$.
2. Prove that for all $L' \in \mathcal{NP}$ there exists $L \in \mathcal{P}$ and a polynomial p such that $\exists^p L = L'$.

Hint: Consider a canonic NTM and the directions of the computation path encoded by w .