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Exercise for the lecture
Computational Complexity
Winter 2014/15
Sheet 4

EXERCISE 1:
Consider the following functions $\text{MX}_n : \{0,1\}^n \times \{0,1\}^k \to \{0,1\}$ for $n = 2^k$ with

$$\text{MX}(x_{0\ldots00}, x_{0\ldots01}, \ldots, x_{1\ldots1}, i_0, \ldots, i_{k-1}) := x_{i_0,\ldots,i_{k-1}}$$

and the function $\text{STORE}_n : \{0,1\}^n \times \{0,1\}^k \times \{0,1\} \to \{0,1\}^n$ for $n = 2^k$ with

$$\text{STORE}(x_0, x_1, \ldots, x_{\text{bin}(n)}, i_0, \ldots, i_{k-1}, y) := (x_0, x_1, \ldots, x_{\text{bin}(i-1)}, y, x_{\text{bin}(i+1)}, \ldots, x_{1\ldots1}) \ .$$

Show that

1. Show that a Boolean circuit for MX_n can be constructed by a DTM with space $O(\log n)$.
2. Show the same for STORE_n.

EXERCISE 2:
Consider the monotone circuit value problem MCV:

$$\text{MCV} := \{(C, x) \mid C \text{ is a } \{0,1,\wedge,\vee\}\text{-circuit and } \text{val}_C(x) = 1\} \ .$$

Prove that MCV is $\mathcal{P}$-complete.
Hint: *Consider the railway code, where a bit b is encoded by two bits $(b, \overline{b})$. Show that the negation can be presented by switching the positions and Or can be represented by a combination of Or and And.*

EXERCISE 3:
A quantified Boolean formula is given by

$$Y_1 x_1 Y_2 x_2 Y_3 x_3 \ldots Y_n x_n \ \ f(x_1, x_2, \ldots, x_n)$$

for $\forall i \in \{1, \ldots, n\} : Y_i \in \{\exists, \forall\}$, $x_i \in \{0,1\}$ and a Boolean formula $f : \{0,1\}^n \to \{0,1\}$. Now define

$$\text{QBF} := \{f \mid \text{quantified Boolean formula } f \text{ with } f = 1\} \ .$$

Show that QBF is $\mathcal{PSPACE}$-complete.