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Distributed Systems

Chapter 4 Coordination and Agreement

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19. May 2014

Implementing Causal Ordering

- Uses vector clocks to keep causal ordering (piggybacked to messages)
- Vector clock $V_i^g[i]$ counts all multicast messages of process i in group g
- hold-back queue reflects vector clocks

Algorithm for group member p_i $(i = 1, 2\ldots, N)$

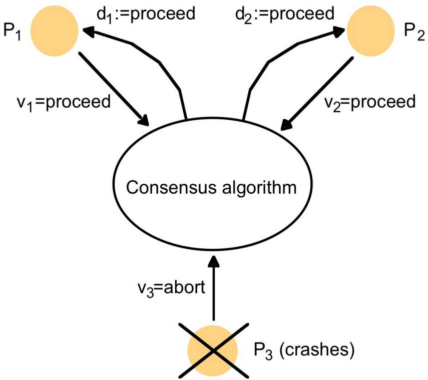
On initialization
$V_i^g[j] := 0$ $(j = 1, 2\ldots, N)$;

To CO-multicast message m to group g
$V_i^g[i] := V_i^g[i] + 1$;
B-multicast$(g, <V_i^g, m>)$;

On B-deliver$(<V_j^g, m>)$ *from* p_j, *with* $g = group(m)$
place $<V_j^g, m>$ in hold-back queue;
wait until $V_j^g[j] = V_i^g[j] + 1$ and $V_j^g[k] \le V_i^g[k]$ $(k \neq j)$;
CO-deliver m; // after removing it from the hold-back queue
$V_i^g[j] := V_i^g[j] + 1$;

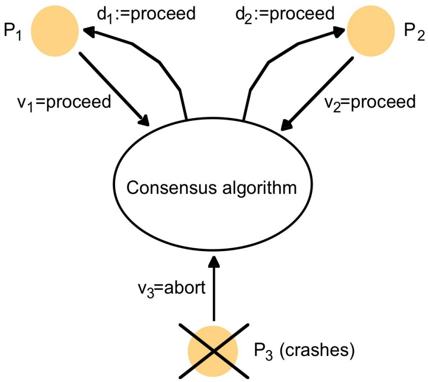
4.5: Consensus

- n processes $p_1, \ldots, p_n$
- at most f processes have arbitrary (Byzantine) failures
- Every process starts in the *undecided* state and *proposes* a value v_i
- Eventually all correct processes p_i
  - choose the *decided* state
  - and choose the same value $d_i \in \{v_1, \ldots, v_n\}$
  - and stay in this state



Consensus Problem

- *Termination:* Eventually each correct process p_i is *decided* by setting variable d_i
- *Agreement:* The decision value d_i of all correct processes is the same
- *Integrity:* If all correct process proposed the same value v, then $d_i = v$ for all correct p_i



- Possible decision functions: *majority, minimum, maximum, . . .*
- Byzantine failures can cause irritating and adversarial messages
- System crashes may not be detected

Byzantine Generals Problem

- n generals have to agree on attack or retreat
- one of them is the commander and issues the order
- at most f generals are traitors (possibly also the commander) and have adversarial behavior
- all correct generals have eventually to agree on the commander decision if he acts correctly

Consensus Problem

- *Termination:* Eventually each correct process p_i is *decided* by setting variable d_i
- *Agreement:* The decision value d_i of all correct processes is the same
- *Integrity:* If the commander is correct then all correct processes choose the commander's proposal

Interactive Consistency

- n processes need to agree on a *vector* of values
- Each process proposes a value v_i
- A correct processes eventually decide on a vector $d_i = \{d_{i,1}, \ldots, d_{i,n}\}$ where

$$d_{i,j} = v_j \quad \text{if } p_j \text{ is correct}$$

Interactive Consistency

- *Termination:* Eventually each correct process p_i is *decided* by setting variable d_i
- *Agreement:* The decision value d_i of all correct processes is the same
- *Integrity:* If the p_j is correct then all correct processes p_i set $d_{i,j} = v_j$

The Relationship between Consensus Problems

Assume solutions to Consensus (C), Byzantine generals (BG), interactive consistency (IC)

$$
\begin{aligned}
C_i(v_1, \ldots, v_n) &= \text{consensus decision value of } p_i \text{ for proposals } v_i \\
BG_i(j, v) &= \text{BG decision value of } p_i \text{ for commander } p_j \text{ proposal } v_j \\
IC_i(v_1, \ldots, v_n)[j] &= j\text{-th position of interactive consistency decision vector of } p_i \text{ for proposals } v_i
\end{aligned}
$$

Solving *IC* from *BG*

- In parallel n Byzantine generals problems are solved
- each process p_j acts as commander once

$$IC_i(v_1, \ldots, v_n)[j] = BG_i(j, v)$$

The Relationship between Consensus Problems

Solving C from IC

- *majority* returns the most often parameter or $\perp$ if no such value exists
- for all $i = 1, \ldots, n$

$$C_i(v_1, \ldots, v_n) = majority(IC_i(v_1, \ldots, v_n)[1], \ldots, IC_i(v_1, \ldots, v_n)[n])$$

Solving BG from C

- The commander p_j sends its proposed value to itself and each other process
- All processes run consenus with the values $v_1, \ldots, v_n$ received from the commander
- for all $i = 1, \ldots, n$

$$BG_i(j, v) = C_i(v_1, \ldots, v_n)$$

Consensus in a Synchronous System

- Assume that there are no arbitrary (Byzantine) errors
- Given a synchronous distributed systems (fail-stop model)
- Use basic multicast for $f+1$ rounds
- Multicast all known values of all participants
- $Values_i^r$ denotes the set of proposed variables at the beginning of round r
- Reduce communication overhead by multicasting only freshly arrived variables $Values_i^r - Values_i^{r-1}$
- Choose the minimum of all known values as final value

Consensus in a Synchronous System

Algorithm for process $p_i \in g$; algorithm proceeds in $f + 1$ rounds

On initialization

$Values_i^1 := \{v_i\}$; $Values_i^0 = \{\}$;

In round r $(1 \le r \le f + 1)$

B-multicast$(g, Values_i^r - Values_i^{r-1})$; // Send only values that have not been sent

$Values_i^{r+1} := Values_i^r$;

while (in round r)

{

On B-deliver(V_j) *from some* p_j

$Values_i^{r+1} := Values_i^{r+1} \cup V_j$;

}

After $(f + 1)$ *rounds*

Assign $d_i = minimum(Values_i^{f+1})$;

Consensus in a Synchronous System

- There are no arbitrary errors only processes that crash and are correctly detected
- Given a synchronous distributed systems (fail-stop model)
- Correctness
  - Assume that two processes p_i and p_j have different values at round r
  - Then, in round $r - 1$ at least one process p_k has sent different values to p_i and p_j
  - Then, p_k has crashed in this round
  - Since the number of crashes is limited to f there are not enough crashes to cover each of the $f + 1$ rounds

Byzantine Generals Problem in a Synchronous System

- Assume that there are **Byzantine** errors
- Given a synchronous distributed system
  - crashes are detected
  - other wrong behavior cannot be detected, e.g. strange messages
- messages are not (digitally) signed
- at most f faulty processes

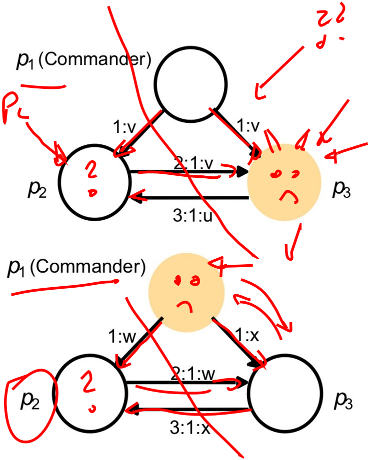
Impossibility of a solution of the Byzantine generals problem [Lamport, Shostak, Pease 1982]

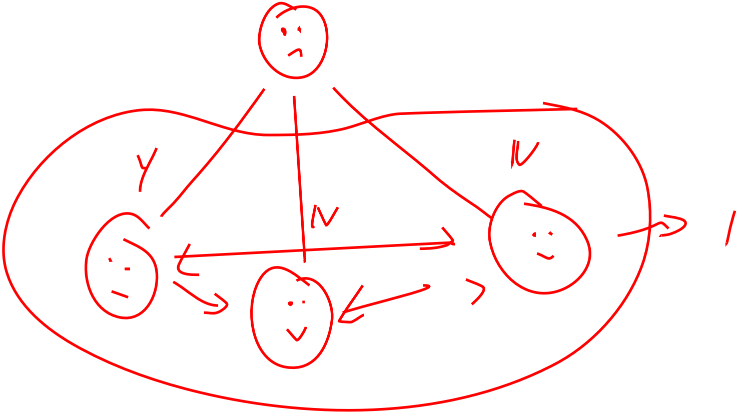
- The byzantine generals problem cannot be solved for $n = 3$ and $f = 1$.
- The byzantine generals problem cannot be solved for $n \leq 3f$.

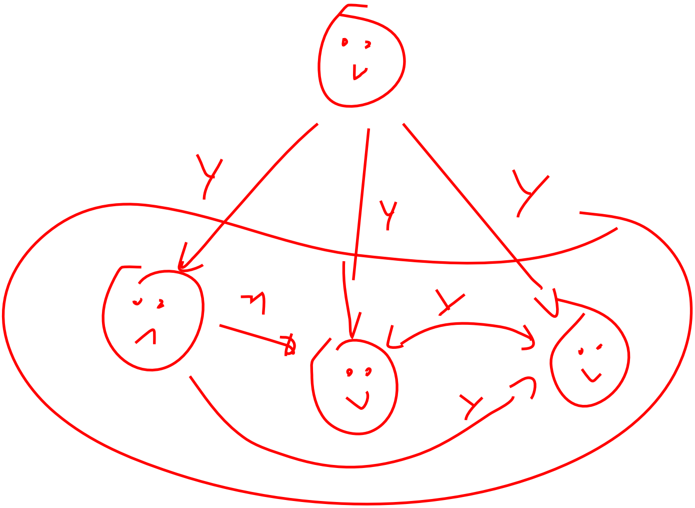
Byzantine Generals Problem in a Synchronous System

Impossibility of a solution of the Byzantine generals problem for $n = 3$

- The byzantine generals problem with arbitrary failures cannot be solved for $n = 3$ and $f = 1$ in a synchronous system.
  - a faulty commander sending different values to his generals
  - cannot be distinguished from a faulty general forwarding wrong values







Solution of the Byzantine Generals Problem

- Assume that there are **Byzantine** errors
- Given a synchronous distributed system
- messages are not (digitally) signed
- at most f faulty processes

Solution of the Byzantine generals problem [Pease, Shostak, Lamport 1980]

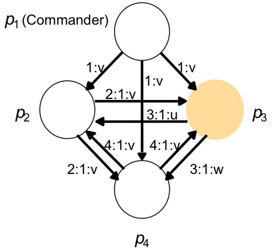
- The byzantine generals problem can be solved for $n = 4$ and $f = 1$.
- The byzantine generals problem can be solved for $n \geq 3f + 1$.

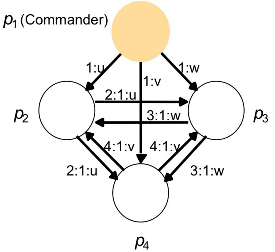
Solution for Four Generals and One Faulty Process

- The byzantine generals problem can be solved for $n \geq 4$ and $f = 1$.

Algorithm of Pease et al.

1. The commander sends a value to all other generals (lieutenants)
2. All lieutenants send the received value to all other lieutenants
3. The commander chooses its value; the lieutenants compute the majority of all received values

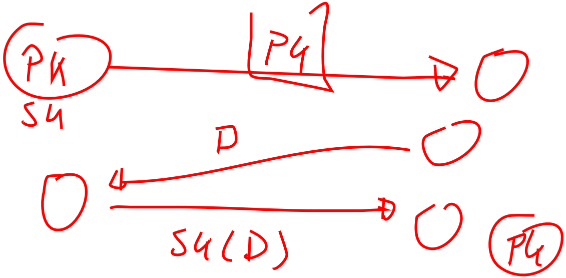




- Since $n \geq 4$ the majority function always can be computed if at most one process is faulty
- If the commander crashes very early then all lieutenants agree on $\perp$

More About the Byzantine Generals Problems

- For $f > 1$ the algorithm can be used recursively
  - Complexity: $f+1$ rounds and $O(n^{f+1})$ messages
  - The time complexity of $f+1$ rounds is optimal
- With the help of signed messages
  - any number of faulty generals $f < n$ can be dealt with
  - with signed messages the Byzantine Generals problem can be solved in $f+1$ rounds with $O(n^2)$ messages [Dolev & Strong 1983]
- For asynchronous systems with crash failures
  - No algorithm can reach consensus even if only **one processor** is faulty [Fischer, Lynch, Paterson 1985]
  - Each algorithm that tries to reach consensus can be confronted with a faulty process which influences the result if it continues (instead of crashing)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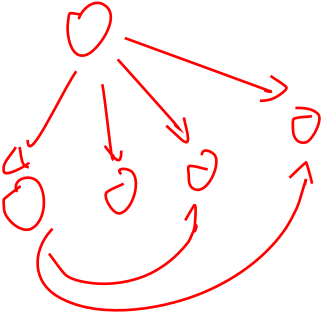

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