8. Distributed Concurrency Control

General reference architecture.

Federated system
8.1: Preliminaries

Sites and subtransactions

- Let be given a fixed number of sites across which the data is distributed. The server at site $i$, $1 \leq i \leq n$ is responsible for a (finite) set $D_i$ of data items. The corresponding global database is given as $D = \bigcup_{i=1}^{n} D_i$.

- Data items are not replicated; thus $D_i \cap D_j = \emptyset$, $i \neq j$.

- Let $T = \{T_1, \ldots, T_m\}$ be a set of transactions, where $T_i = (OP_i, <_i)$, $1 \leq i \leq m$.

- Transaction $T_i$ is called global, if its actions are running at more than one server; otherwise it is called local.

- The part of a transaction $T_i$ being executed at a certain site $j$ is called subtransaction and is denoted by $T_{ij}$. 
Parallelism as prerequisite for distributed execution

A transaction $T$ is a partial order $<^1$ of actions in $OP$, $T = (OP, <)$, where $OP$ is a finite set of $T$'s actions $RX$ and $WX$, where $X$ is a data item.

Moreover, $< \subseteq OP \times OP$ is a partial order on $OP$ which fulfills the following properties:

- Each data item is read and written by $T$ at most once.
- If $p$ is a read action and $q$ is a write actions of $T$ and both access the same data item, then $p < q$.

Complete transaction

We call a transaction *complete*, if its first action is begin $b$ and its last action either is commit $c$ or abort $a$.

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$^1$A binary relation is a partial order , if it is reflexive, antisymmetric and transitive.
Histories and schedules

Let \( T = \{ T_1, \ldots, T_n \} \) be a (finite) set of complete transactions, where for each \( T_i \) we have \( T_i = (OP_i, <_i) \).

A history of \( T \) is a pair \( S = (OP_S, <_S) \), where

- \( OP_S = \bigcup_{i=1}^{n} OP_i \) and \( <_S \) a partial order on \( OP_S \) such that \( <_S \supseteq \bigcup_{i=1}^{n} <_i \).
- Let \( p, q \in OP_S \), where \( p \) and \( q \) belong to distinct transactions, however access the same data object. If \( p \) or \( q \) is a write action, then either \( p <_S q \) or \( q <_S p \); we say, \( p \) and \( q \) are in conflict; if \( p <_S q \) and \( p \) and \( q \) are in conflict, we write \( (p, q) \in \text{conf}(S) \).

A schedule of \( T \) is a prefix of a history.\(^2\)

Conflict graph

The conflict graph of a schedule \( S \) is given as \( G(S) = (V, E) \), where \( V \) is the set of transactions in \( S \) and the set of edges \( E \) is given by the conflicts in \( S \): \( T_i \rightarrow T_j \in E \), iff there are conflicting actions \( p \in OP_i, q \in OP_j \) and \( p <_S q \).

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\(^2\)A partial order \( L' = (A', <') \) is a prefix of a partial order \( L = (A, <) \), if \( A' \subseteq A \), \( <' \subseteq < \), for all \( a, b \in A' \): \( a <' b \) if \( a < b \), and for all \( p \in A, q \in A' \): \( p < q \Rightarrow p <' q \).
A parallel debit/credit transaction. \( b: \text{BEGIN}; \ c: \text{COMMIT} \).

When transactions are depicted as directed graphs, we omit transitive edges.

Two parallel debit/credit transactions, each prepared for parallel execution.

\[ \implies \text{Definition of a schedule? Definition of serializability?} \]
Two parallel debit/credit transactions, each prepared for parallel execution.

Locally observable schedules of the two transactions when executed in parallel by CPU PA and CPU PB.

(i) \( PA: R_1 A W_1 A R_2 A W_2 A \)
    \( PB: R_1 B W_1 B R_2 B W_2 B \)

(ii) \( PA: R_1 A W_1 A R_2 A W_2 A \)
    \( PB: R_2 B W_2 B R_1 B W_1 B \)

On each CPU in both cases the local schedules are serializable - however, globally, in the second case the transactions are not executed in a serializable manner!
A schedule/history of the two parallel debit/credit transactions.

The schedule is not serializable as its conflict graph is cyclic.

Serializability

- A schedule $S = (\text{OP}_S, <_S)$ is *serial*, if for any two transactions $T_1, T_2$ appearing in $S$, $<_S$ orders all actions of $T_1$ before all actions of $T_2$, or vice versa.
- A schedule is called (conflict-)serializable,\(^3\) if there exists a (conflict-)equivalent serial schedule over the same set of transactions.
- A schedule $S = (\text{OP}_S, <_S)$ is serializable, iff its conflict graph is acyclic.

\(^3\)We consider only conflict-serializability and therefore talk about serializability in the sequel, for short.
Local and global schedules

We are interested in deciding whether or not the execution of a set of transactions is serializable, or not.

- At the local sites we can observe an evolving sequence of the respective transactions’ actions.
- We would like to decide whether or not all these locally observable sequences imply a (globally) serializable schedule.
- However, on the global level we cannot observe an evolving sequence, as there does not exist a notion of global physical time.
Example

Schedule:

Observed local schedules:

Site 1 (PA): \( R_1A \ W_1A \ R_2A \ W_2A \)
Site 2 (PB): \( R_2B \ W_2B \ R_1B \ W_1B \)

Can schedules be represented as action sequences, as well?
... yes, we call them *global schedules.*
From now on local and global schedules are sequences of actions!

Let $\mathcal{T} = \{T_1, \ldots, T_m\}$ be a set of transactions being executed at $n$ sites. Let $S_1, \ldots, S_n$ be the corresponding local schedules.

A *global schedule* of $\mathcal{T}$ with respect to $S_1, \ldots, S_n$ is any sequence $S$ of the actions of the transactions in $\mathcal{T}$, such that its projection onto the local sites equals the corresponding local schedules $S_1, \ldots, S_n$.

**Example**

Consider local schedules $S_1 = R_1 A W_2 A$ and $S_2 = W_1 B R_2 B$.

Global schedules:

$S : R_1 A W_1 B W_2 A R_2 B$

$S' : R_1 A W_1 B R_2 B W_2 A$

Not a global schedule: $S'' : R_1 A R_2 B W_1 B W_2 A$
Examples where there does not exist a serializable global schedule

- \( T_1 = R_1 A \ W_1 B, \ T_2 = R_2 C \ W_2 A \) are global transactions and \( T_3 = R_3 B \ W_3 C \) is a local transaction.

  \[ S_1 : \quad R_1 A \quad W_2 A \]
  \[ S_2 : \quad R_3 B \quad W_1 B \quad R_2 C \quad W_3 C \]

  Note, in \( S_2 \) subtransactions \( T_{12} \) and \( T_{22} \) have no conflicting actions!

- \( T_1 = RA \ RD \) und \( T_2 = RB \ RC \) are global transactions, while \( T_3 = RA \ RB \ WA \ WB \) and \( T_4 = RD \ WD \ RC \ WC \) are local transactions.

  \[ S_1 : \quad R_1 A \quad R_3 A \quad R_3 B \quad W_3 A \quad W_3 B \quad R_2 B \]
  \[ S_2 : \quad R_4 D \quad W_4 D \quad R_1 D \quad R_2 C \quad R_4 C \quad W_4 C \]

  Note, both global transactions are only reading and, in particular, disjoint data sets!

In both examples the local schedules are serializable, however no serializable global schedule exists.
Serializability of global schedules

- As we do not have replication of data items, whenever there is a conflict in a global schedule, the same conflict must be part of exactly one local schedule.

- Consequently, the conflict graph of a global schedule is given as the union of the conflict graphs of the respective local schedules.

- In particular, given a set of local schedules, either all or none corresponding global schedule is serializable.
### Examples

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Types of federation

- **homogeneous** federation:
  Same services and protocols at all servers. Characterized by *distribution transparency*: the federation is perceived by the outside world as if it were not distributed at all.

- **heterogenous** federation:
  Servers are autonomous and independent of each other; no uniformity of services and protocols across the federation.

Interface to recovery

Every global transactions runs the 2-phase-commit protocol. By that protocol the subtransactions of a global transaction synchronize such that either all subtransactions commit, or none of them, i.e. all abort. Details are given in Chapter 10.
8.2: Homogeneous Concurrency Control

Serializability by distributed 2-Phase Locking (2PL)

A transactions entry into the unlock-phase has to be synchronized among all sites the transaction is being executed.

**Primary Site 2PL:**
- One site is selected at which lock maintenance is performed exclusively.
- This site thus has global knowledge and enforcing the 2PL rule for global and local transactions is possible.
- The lock manager simply has to refuse any further locking of a subtransaction $T_{ij}$ whenever a subtransaction $T_{ik}$ has started unlocking already.
- Much communication is resulting which may create a bottleneck at the primary site.

Example

$S_1 : \quad R_1 A \quad W_1 A \quad R_2 A \quad W_2 A$

$S_2 : \quad R_2 B \quad W_2 B \quad R_1 B \quad W_1 B$
Distributed 2PL:

- When a server wants to start unlocking data items on behalf of a transaction, it communicates with all other servers regarding the lock point of the other respective subtransaction.
- The server has to receive a *locking completed*-message from each of these servers.
- This implies extra communication between servers.

Example

\[
S_1 : \begin{array}{cccc}
R_1 A & W_1 A & R_2 A & W_2 A
\end{array}
\]

\[
S_2 : \begin{array}{cccc}
R_2 B & W_2 B & R_1 B & W_1 B
\end{array}
\]
**Distributed Strong 2PL:**

- Every subtransaction of a global transaction and every local transaction holds locks until commit.
- Then by the 2-phase-commit protocol the 2PL-rule is enforced as a side-effect.

Applying strong 2PL the global 2PL-property is self-guaranteed without any explicit measures!
Locking protocols are prone to deadlocks!

**Global deadlock**
Global deadlock detection is difficult. Detection strategies:

- **Centralized detection**: Each site maintains its local wait-for graph. One distinguished site is selected to which all local wait-for graphs are send periodically. The selected site computes the union of all local wait-for graphs and checks for deadlocks.

- **Time-out based detection**: Whenever during a wait a time-out occurs, the respective transaction decides for a deadlock and aborts itself.

- **Edge chasing**: Whenever a transaction $T$ waits for a transaction $T'$, it sends its identification to $T'$. Whenever a transaction $T'$ receives such a message, it sends the identification of such $T$ to all transactions it is waiting for. If a transaction receives its own identification, it decides for a deadlock and it aborts itself.

- **Path pushing**:
  
  (i) Each server that has a waits-for path from transaction $t_i$ to transaction $t_j$ such that $T_i$ has an incoming waits-for-message edge and $T_j$ has an outgoing waits-for-message edge sends that path to the server along the outgoing edge.

  (ii) Upon receiving a path the server concatenates this with the local paths that already exist, and forwards the result along its outgoing edges again. If there exists a cycle among $k$ servers, at least one of them will detect the cycle in at most $k$ rounds.
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  (ii) Upon receiving a path the server concatenates this with the local paths that already exist, and forwards the result along its outgoing edges again. If there exists a cycle among $k$ servers, at least one of them will detect the cycle in at most $k$ rounds.
Serializability by assigning timestamps to transactions

- Global and local transactions are timestamped; all subtransactions of a transaction obtain the same timestamp.
- Timestamps must be system-wide unique and based on synchronized clocks.
- To be system-wide unique, timestamps are values of local clocks concatenated with the site ID.

Time Stamp Protocol TS

- To each transaction $T$ it is assigned a unique timestamp $Z(T)$ when it is started.
- A transaction $T$ must not write an object which has been read by any $T'$ where $Z(T') > Z(T)$.
- A transaction $T$ must not write an object which has been written by any $T'$ where $Z(T') > Z(T)$.
- A transaction $T$ must not read an object which has been written by any $T'$ where $Z(T') > Z(T)$. 
The TS-protocol guarantees serializability of schedules.

Let $S$ be a global schedule of a set of transactions $T = \{T_1, \ldots, T_n\}$, which all apply TS.

Assume, $S$ is not serializable, i.e. the conflict graph $G(S)$ is cyclic, where w.l.o.g. $T_1 \rightarrow T_2 \rightarrow \cdots \rightarrow T_k \rightarrow T_1$.

- Each edge $T \rightarrow T'$ implies $T$ and $T'$ have conflicting actions, where the action of $T$ preceds the one of $T'$.
- Because of TS we know $Z(T) < Z(T')$. This implies the following:

$$Z(T_1) < Z(T_2) < \ldots < Z(T_n) < Z(T_1),$$

a contradiction. Therefore $S$ is serializable.
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a contradiction. Therefore $S$ is serializable.
8.3: Heterogeneous Concurrency Control

Local and global transaction managers

- Each server runs its own *local* transaction manager which guarantees local serializability, i.e. the serializable execution of its local transactions and subtransactions.

- To guarantee global serializability a *global* transaction manager controls the execution of the global transactions. This could either be based on ordering the commit of the transaction, or by introducing artificial data objects called *tickets* which have to be accessed by the subtransactions.
Global serializability through local guarantees: rigorous local schedules

Rigorous schedules

A local schedule $S = (OP_S, <_S)$ of a set of complete transactions is rigorous if for all involved transactions (local and subtransactions) $T_i, T_j$ there holds:

Let $p_j \in OP_j, q_i \in OP_i, i \neq j$ such that $(p_j, q_i) \in conf(S)$. Then either $a_j <_S q_i$ or $c_j <_S q_i$.

Commit-deferred transaction

A global transaction $T$ is commit-deferred if its commit action is sent by the global transaction manager to the local sites of $T$ only after the local executions of all subtransactions of $T$ at those sites have been acknowledged.

Commit-deferment is achieved as a side-effect of the 2-phase-commit protocol.
Examples

Consider two servers where \( D_1 = \{A, B\} \) and \( D_2 = \{C, D\} \). We have the following transactions:

- global: \( T_1 = WA \ W D \)
- local: \( T_3 = RA \ R B \)
- \( T_2 = WC \ W B \)
- \( T_4 = RC \ R D \)

We have the following local schedules:

- \( S_1 : \) \( W_1 A \ c_1 \ R_3 A \ R_3 B \ c_3 \ W_2 B \ c_2 \)
- \( S_2 : \) \( W_2 C \ c_2 \ R_4 C \ R_4 D \ c_4 \ W_1 D \ c_1 \)

Even though the local schedules are serializable, the two global transactions are not executed in a serializable manner. The local schedules are rigorous, however not commit-deferred.
Lemma

A schedule is serializable, whenever it is rigorous.

Sketch of proof: Assume the contrary. Then there exists a history which has a cyclic conflict graph, though rigorousness holds. As a commit is the final action of a transaction, rigorousness makes such a cycle impossible.

Theorem

Let $S$ be a global history for local histories $S_1, \ldots, S_n$. If $S_i$ rigorous, $1 \leq i \leq n$ and all global transactions are commit-deferred, then $S$ is globally serializable.

Sketch of proof: Assume the contrary. Then there exists a history which has a cyclic conflict graph, though rigorousness and commit-deferment hold. As rigorousness guarantees local serializability, such a cycle must involve at least two sites. As a commit is the final action of a transaction, commit-deferment makes such a cycle impossible.

Because of the 2-phase-commit protocol, under rigorousness global serializability practically comes for free!
Global serializability through explicit measures: tickets

**Ticket-based concurrency control**

- Each server guarantees serializable local schedules in a way unknown for the global transactions.
- Each server maintains a special counter as database object, which is called *ticket*. Each subtransaction of a global transaction being executed at that server increments (reads and writes) the ticket (*take-a-ticket*-Operation). Doing so we introduce explicit conflicts between global transactions running at the same server.
- The global transaction manager guarantees that the order in which the tickets are accessed by the subtransactions will imply a linear order on the global transactions.
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Applying ticketing by examples

By $l_j$ we denote the ticket at server $j$.

- Let $T_1 = R_1 A R_1 D$ and $T_2 = R_2 B R_2 C$ be global transactions and let $T_3 = R_3 A R_3 B W_3 A W_3 B$ and $T_4 = R_4 D W_4 D R_4 C W_4 C$ be local transactions.
  
  $S_1 : R_1 (l_1) W_1 (l_1) R_1 A R_3 A R_3 B W_3 A W_3 B R_2 (l_1) W_2 (l_1) R_2 B$
  $S_2 : R_4 D W_4 D R_1 (l_2) W_1 (l_2) R_1 D R_2 (l_2) W_2 (l_2) R_2 C R_4 C W_4 C$

Not serializable - could be detected at server 2.

- Let $T_1 = R_1 A W_1 B$ and $T_2 = R_2 B W_2 A$ be global transactions.
  
  $S_1 : R_1 (l_1) W_1 (l_1) R_1 A R_2 (l_1) W_2 (l_1) W_2 A$
  $S_2 : R_2 (l_2) W_2 (l_2) R_2 B R_1 (l_2) W_1 (l_2) W_1 B$

Not serializable, could not be detected neither at server 1 nor at server 2, however the order of take-a-ticket operations does not imply a linear order on the global transactions.
Applying ticketing by examples

By $I_j$ we denote the ticket at server $j$.

- Let $T_1 = R_1A R_1D$ and $T_2 = R_2B R_2C$ be global transactions and let $T_3 = R_3A R_3B W_3A W_3B$ and $T_4 = R_4D W_4D R_4C W_4C$ be local transactions.
  
  $S_1$ : $R_1(I_1) W_1(I_1) R_1A R_3A R_3B W_3A W_3B R_2(I_1) W_2(I_1) R_2B$
  
  $S_2$ : $R_4D W_4D R_1(I_2) W_1(I_2) R_1D R_2(I_2) W_2(I_2) R_2C R_4C W_4C$

Not serializable - could be detected at server 2.

- Let $T_1 = R_1A W_1B$ and $T_2 = R_2B W_2A$ be global transactions.
  
  $S_1$ : $R_1(I_1) W_1(I_1) R_1A R_2(I_1) W_2(I_1) W_2A$
  
  $S_2$ : $R_2(I_2) W_2(I_2) R_2B R_1(I_2) W_1(I_2) W_1B$

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