Section 12.4 Invariants

Basics

- A Petri-net invariant is a property of a Petri-net, which holds for any marking, respectively transition word, of the net.
- We study place- and transition-invariants, which are based on a matrix representation of a net, respectively vector representation of markings and transitions.

Incidence Matrix

- Let $N = (P, T, F, V, m_0)$ a eS-Net, $T = \{t_1, \ldots, t_n\}$, $P = \{p_1, \ldots, p_m\}$, $n, m \geq 1$.
- A vector of dimension n (m) is called T- (P-)vector.
- For any $t \in T$, Δt can be represented as a column P-vector.
- The *incidence matrix* of N is given as a $m \times n$-matrix $C = (\Delta t_1, \ldots, \Delta t_n)$, respectively $C = (c_{i,j})_{1 \leq i \leq m, 1 \leq j \leq n}$, where $c_{ij} := \Delta t_j(s_i)$.

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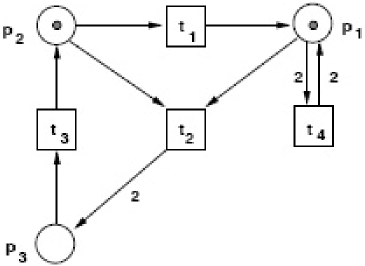
Example



$$C = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{pmatrix}$$

- Incidence matrices are independent of concrete markings,
- In case of loops, information concerning multiplicities is lost.

Parikh-Vektor

The transpose of a vector x, resp. matrix C is denoted by $x^\top$, bzw. $C^\top$.

The *Parikh-Vektor* $\bar{q}$ of some $q \in W(T)$ is a column T-vector, $n = \mid T \mid$, defined as follows:

$$\bar{q} : T \rightarrow NAT, \text{ where } \bar{q}(t) \text{ is the number of occurences of } t \text{ in } q.$$

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State Equation

Let $q \in W(T)$ and m, m' markings.

If $m[\, q \succ m'$, then $\sum_{t \in T} (\bar{q}(t) \cdot \Delta t) = C \cdot \bar{q} = \Delta q$.

Moreover, as $m[\, q \succ m'$, we have

- $m' = m + \Delta q^\top$.

  The equation:

  $$m' = m + (C \cdot \bar{q})^\top$$

  is called *state equation*.

- The system of linear equations given by

  $$C \cdot x = (m' - m)^\top$$

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If $C \cdot x = (m' - m)^\top$ has an integer nonnegative solution then

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I.e., the reachability problem cannot be solved, in general.

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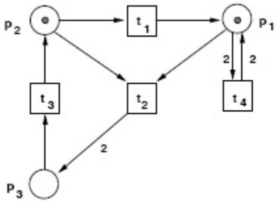

$$C = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{pmatrix}$$

Let $m = (1, 0, 0)$, $m' = (0, 0, 1)$.
$x = (0, 1, 1, 0)^\top$ is a solution for $C \cdot x = (m' - m)^\top$, however we cannot find a word which can be fired at m.

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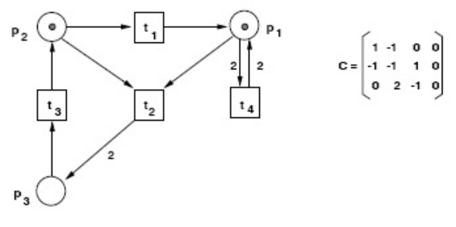

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$x = (0, 1, 1, 0)^\top$ is a solution for $C \cdot x = (m' - m)^\top$, however we cannot find a word which can be fired at m.

Theorem

Let N be a eS-Net and Δ a P-vector. There exists a marking m^* and a word $q \in W(T)$, such that $m^*[\, q \succ (m^* + \Delta)$, iff $C \cdot x = \Delta^\top$ has an integer nonnegative solution.

Proof:
"$\Rightarrow$": trivial.

"$\Leftarrow$": Let $m^* := \sum_{t \in T} x(t) \cdot t^-$.

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Corollary

Let $N = (P, T, F, V, m_0)$ be a eS-Net. There exists a marking m^* such that $N = (P, T, F, V, m^*)$ unbounded, iff $C \cdot x > 0$ has an integer nonnegative solution.

Useful application of the corollary:

If there does not exist an integer nonnegative solution for $C \cdot x > 0$, then for any initial marking, N is bounded.

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Transition-Invariants (T-Invariants)

Let $N = (P, T, F, V, m_0)$ be a eS-Net.

- Any nontrivial integer solution x of the homogenous linear equation system $C \cdot x = 0$ is called *transition-invariant (T-invariant)* of N.
- A T-invariant x is called *proper*, if $x \geq 0$.
- A T-invariant x is called *realizable* in N, if there exists a word $q \in W(T)$ with $\bar{q} = x$ and a reachable marking m such that $m[\, q \succ m$.
- N is called *covered with T-invariants*, if there exists a T-invariant x of N with all components positive, i.e. greater than 0.

Proper T-invariants denote *possible* cycles of the reachability graph - realizable T-invariants denote cycles which indeed may occur.

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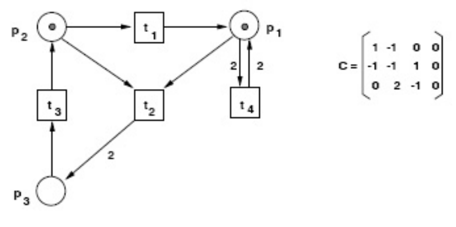

are as follows:

$$x = \lambda_1 \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

where λ_1, λ_2 integers.

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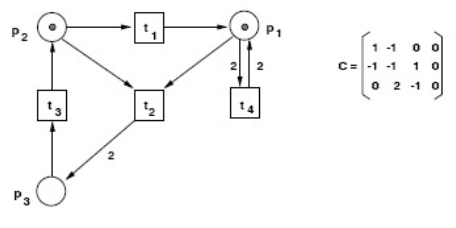

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Theorem

Let $N = (S, T, F, V, m_0)$ be a eS-Net. If there exists a marking m, such that N live and bounded at m, then N covered by T-invariants.

Proof: Let N live and bounded at some m.

As N is live at m, there exists a word $q_1 \in L_N(m)$, which contains all transitions in T and the marking $m + \Delta q_1$ is reachable from m.

Moreover, N is live at $m + \Delta q_1$ as well. Therefore, there exits a word $q_2 \in L_N(m)$, which contains all transitions in T and N is live at the marking $m + \Delta q_1 q_2$.

There exists an infinite sequence of markings (m_i), where $m_i := m + \Delta q_1 \ldots q_i$, such that:

$$m[\, q_1 \succ m_1[\, q_2 \succ m_2 \ldots m_i[\, q_{i+1} \succ m_{i+1} \ldots$$

As N is bounded at m, there is only a finite number of markings which are reachable. Therefore, there exist $i, j \in NAT : i < j$ such that $m_i = m_j$. Thus

$$m_i[\, q_{i+1} \ldots q_j \succ m_j = m_i$$

As all these q_i mention all transitions, we finally conclude

$$x = \bar{q}_{i+1} + \ldots + \bar{q}_j$$

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Useful application of the theorem:

Whenever N is not covered by T-invariants, then for every marking it holds N not live or not bounded.

Place-Invariants (P-Invariants)

Let $N = (P, T, F, V, m_0)$ be a eS-Net.

- Any nontrivial integer solution y of the homogeneous linear equation system $y \cdot C = 0$ is called *place-invariant (P-invariant)* of N.
- A P-invariant y is called *proper P-invariant*, if $y \geq 0$.
- N is called *covered with P-invariants*, if there exists a P-invariant y with all components positive, i.e. greater than 0.

If y is a P-invariant, then for any marking m the sum of the number of tokens on the places p is invariant with respect to the firing of the transitions weighted by $y(p)$.

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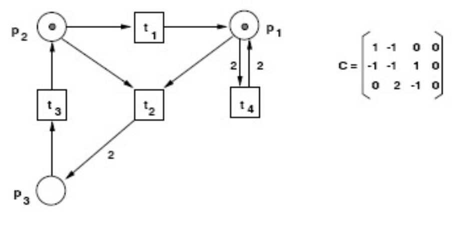

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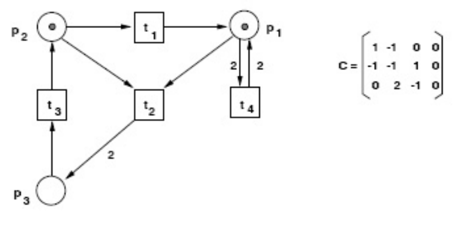

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Theorem

Let $N = (P, T, F, V, m_0)$ a eS-Net and let y a P-invariant of N. Then:

$$m \in R_N(m_0) \Rightarrow y \cdot m^\top = y \cdot m_0^\top .$$

Proof:

Assume $m_0[\, q \succ m$. Then $m = m_0 + (C \cdot \bar{q})^\top$ and also:

$$y \cdot m^\top = y \cdot m_0^\top + y \cdot (C \cdot \bar{q}) =$$

$$= y \cdot m_0^\top + (y \cdot C) \cdot \bar{q} = y \cdot m_0^\top + 0 \cdot \bar{q} = y \cdot m_0^\top .$$

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Corollary:

- Let y P-invariante of N, m marking.

  $y \cdot m^\top \neq y \cdot m_0^\top \Rightarrow m \notin R_N(m_0)$.
- Let y proper P-invariant of N. Let $p \in P$ such that $y(p) > 0$.

  Then, for any initial marking, p is bounded.

  Proof: $y \cdot m_0^\top = y \cdot m^\top \geq y(p) \cdot m(p) \geq m(p)$.
- Let N be covered by P-invariants. N is bounded for any initial marking.

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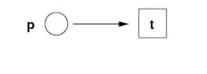
- Let N be covered by P-invariants. N is bounded for any initial marking.

Note, the following net is bounded for any initial marking, however does not have a P-invariant:



P-invariants allow sufficient tests for non-reachability and boundedeness.

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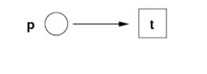
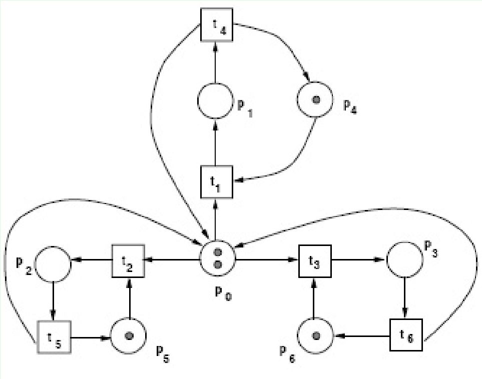


P-invariants allow sufficient tests for non-reachability and boundedeness.

Example: Prove freedom from deadlocks.



$$C = \begin{bmatrix} -1 & -1 & -1 & 1 & 1 & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

P-invariants:

$$Y_1 = (0, 1, 0, 0, 1, 0, 0)$$
$$Y_2 = (0, 0, 1, 0, 0, 1, 0)$$
$$Y_3 = (0, 0, 0, 1, 0, 0, 1)$$
$$Y_4 = (1, 1, 1, 1, 0, 0, 0)$$

Initial marking is given by $m_0 = (2, 0, 0, 0, 1, 1, 1)$. Assume there exist a dead marking m, $m_0[\, q \succ m$. Then it must hold $m(p_1) = m(p_2) = m(p_3) = 0$. Because of Y_4 it follows $m(p_0) = 2$. As m dead it follows $m(p_4) = m(p_5) = m(p_6) = 0$. However this contradicts $Y_1 m_0 = Y_1 m$.

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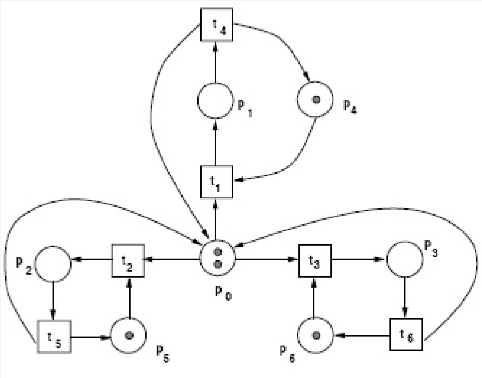

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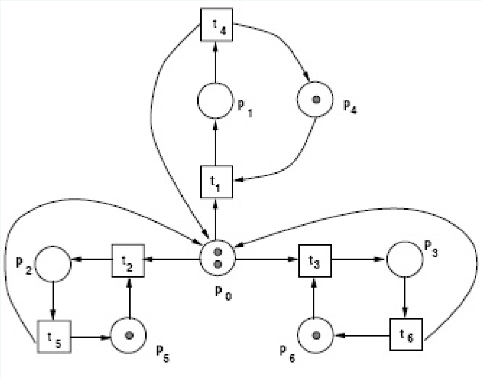

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Section 12.5 Place Capacities

Sometimes when modelling we would like to fix an upper bound for the number of tokens in a place.

- Let $N = (P, T, F, V, m_0)$ be a eS-Net, c a ω-marking of P and let $m_0 \leq c$. (N, c) is called *eS-Net with capacities*. $c(p), p \in P$ is called *capacity* of p.
- For eS-nets with capacities the notion of being enabled is adapted:

  a transition $t \in T$ *is* enabled at marking m, *if* $t^- \leq m$ *and* $m + \Delta t \leq c$.

- Capacities graphically are labels of places - no label means capacity ω.

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Any eS-net with capacities can be simulated by a eS-Net without capacities.

Construction

- Let p a palce with capacity $k = c(p), k \geq 1$. Let p^{co} be the complementary place of p which is assigned the initial marking $k - m_0(p)$.
- Whenever for a transition t we have $\Delta t(p) > 0$, we introduce an arc from p^{co} to t with multiplicity $\Delta t(p)$;
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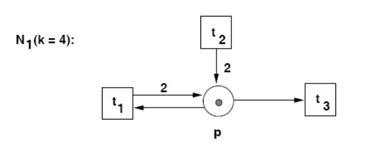
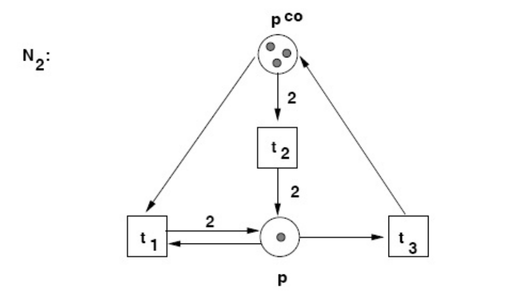
A eS-Net with capacities and its simulation by a bounded eS-Net.

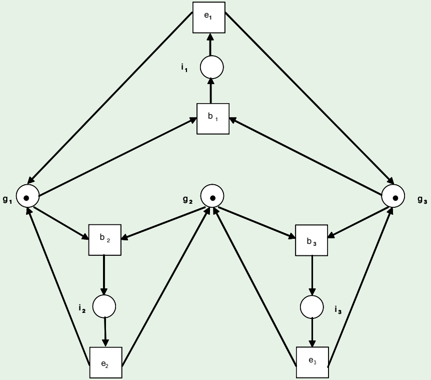




Section 12.6 S-Nets with Colors

- eS-Nets in practice may become huge and difficult to understand.
- Sometimes such nets exhibit certain regularities which give rise to questions how to reduce the size of the net without losing modeling properties.

What about a n-philosopher problem with $n >> 3$?



Why not introduce tokens with individual information?

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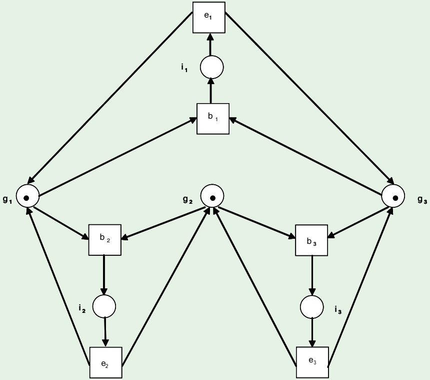
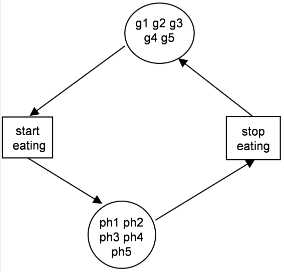


Why not introduce tokens with individual information?

Abstraction 5-philosopher problem

Note: the intention of the marking shown only is to demonstrate „individual“ tokens.



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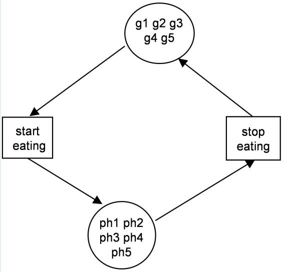

What about being enabled and firing?

Colored System-Nets

A colored System-Net distinguishes different kinds of sorts for markings - the so called *colors* - and functions over these sorts which are used to label the edges of the net.

Generalizing eS-Nets, in a colored net a transition will be called enabled, if certain conditions are true, which are based on the functions which are assigned to the edges of the transitions surrounding.

Thus, we have colors, to characterize markings (*place colors*), and colors, to characterize the firing of transitions (*transition colors*).

As a marking of a place now can be built out of different kind of tokens, we introduce multisets.

- Let A be a set. A *multiset m* over A is given by a maping $m : A \rightarrow NAT$.
- Let $a \in A$. If $m[a] = k$ then there exist k occurences of a in m.
- A multiset oftenly is written as a (formal) sum, e.g. $[Apple, Apple, Pear]$ is written as $2 \cdot Apple + 1 \cdot Pear$.

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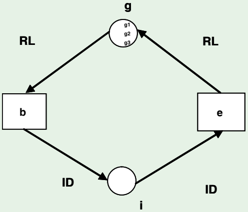
A colored version of the 3-Philosopher-Problem



Colors

$$C(g) = \{g_1, g_2, g_3\},\ C(i) = \{ph_1, ph_2, ph_3\} \quad \text{place colors}$$
$$C(b) = \{ph_1, ph_2, ph_3\},\ C(e) = \{ph_1, ph_2, ph_3\} \quad \text{transition colors}$$

Functions

$$ID(ph_j) := 1 \cdot ph_j, 1 \leq j \leq 3$$
$$RL(ph_j) := \begin{cases} 1 \cdot g_1 + 1 \cdot g_3 & \text{if } j = 1, \\ 1 \cdot g_{j-1} + 1 \cdot g_j & \text{if } j \in \{2, 3\}. \end{cases}$$

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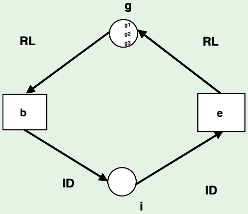

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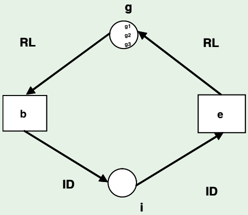

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Multiplicities

A *multiplicity* assigned to an edge between a place p and a transition t is a mapping from the set of transition colors of t into the set of multisets over the colors of p.

In the example:

$$V(b,i) = V(i,e) = ID, \ \ V(g,b) = V(e,g) = RL,$$

where:

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ID denotes the identity mapping.

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Markings are multisets over the respective place colors.

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$$m_0(p) := \begin{cases} 1 \cdot g_1 + 1 \cdot g_2 + 1 \cdot g_3 & \text{if } p = g, \\ 0 & \text{otherwise.} \end{cases}$$

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A *colored Net* $CN = (P, T, F, C, V, m_0)$ is given by:

- A net (P, T, F).
- A mapping C which assignes to each $x \in P \cup T$ a finite nonempty set $C(x)$ of *colors*.
- Mapping V assignes to each edge $f \in F$ a mapping $V(f)$.

  Let f be an edge connecting palce p and transition t.
  $V(f)$ is a mapping from $C(t)$ into the set of multisets over $C(p)$.
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- A transition t *is enabled in color* $d \in C(t)$ at m, if for all pre-places $p \in Ft$ there holds:

$$V(p,t)(d) \leq m(p).$$

- Assume t is enabled in color d at marking m. Firing of t in color d transforms m to a marking m':

$$m'(p) := \begin{cases} m(p) - V(p,t)(d) + V(t,p)(d) & \text{if } p \in Ft, \\ & p \in tF, \\ m(p) - V(p,t)(d) & \text{if } p \in Ft,, \\ & p \notin tF, \\ m(p) + V(t,p)(d) & \text{if } p \notin Ft,, \\ & p \in tF, \\ m(p) & \text{otherwise.} \end{cases}$$

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- Assume t is enabled in color d at marking m. Firing of t in color d transforms m to a marking m':

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Let $CN = (P, T, F, C, V, m_0)$ be a colored System-Net.

- A marking m of P is mapping which assignes to each place p a multiset $m(p)$ over $C(p)$.
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Fold and Unfold of a Colored System-Net

Folding

By folding of a eS-Net we can reduce the number of places and transitions; places and transitions are represented by appropriate place and transition colors, on which certain functions defining the multiplicities are defined.

Let $N = (P, T, F, V, m_0)$ a eS-Net. A folding is defined by π and τ:

- $\pi = \{q_1, \ldots, q_k\}$ a (disjoint) partition of P,
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Two special cases

Call $GN(\pi, \tau) := (P', T', F', C', V', m'_0)$ the result of folding.

- All elements of π, τ are one-elementary:

$$\Rightarrow N \text{ and } GN(\pi, \tau) \text{ are isomorph,}$$

- π, τ contain only one element:

$$\Rightarrow |P'| = |T'| = 1, \text{"the model is represented by the labellings"}.$$

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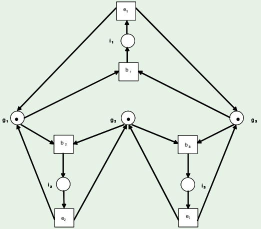
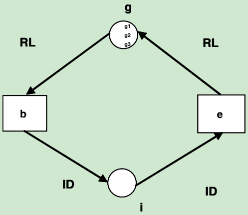


Folding $\pi = \{\{g_1, g_2, g_3\}, \{i_1, i_2, i_3\}\}$, $\tau = \{\{b_1, b_2, b_3\}, \{e_1, e_2, e_3\}\}$.

Colors from folding:
$C(g) = \{g_1, g_2, g_3\}, C(i) = \{i_1, i_2, i_3\}, C(b) = \{b_1, b_2, b_3\}, C(e) = \{e_1, e_2, e_3\}$

Multiplicities: ID, RL analogously to previous version.



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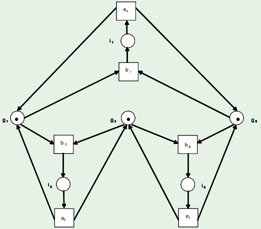

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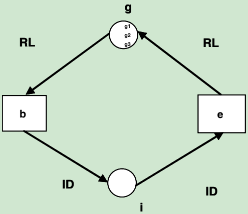

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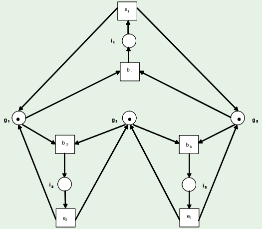

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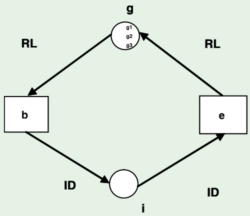
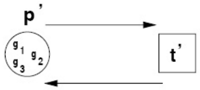


3-Philosopher-Problem?

$\pi = \{P\},\ \tau = \{T\}$:
$S' = \{s'\},\ T' = \{t'\},$
$C(s') = \{g_1, g_2, g_3, i_1, i_2, i_3\},$
$C(t') = \{b_1, b_2, b_3, e_1, e_2, e_3\},$
$m'_0(s') = g_1 + g_2 + g_3,$



$$V'(s', t')(t) = \begin{cases} g_1 + g_3 & \textit{falls } t = b_1, \\ g_1 + g_2 & \textit{falls } t = b_2, \\ g_2 + g_3 & \textit{falls } t = b_3, \\ i_1 & \textit{falls } t = e_1, \\ i_2 & \textit{falls } t = e_2, \\ i_3 & \textit{falls } t = e_3, \end{cases} \qquad V'(t', s')(t) = \begin{cases} g_1 + g_3 & \textit{falls } t = e_1, \\ g_1 + g_2 & \textit{falls } t = e_2, \\ g_2 + g_3 & \textit{falls } t = e_3, \\ i_1 & \textit{falls } t = b_1, \\ i_2 & \textit{falls } t = b_2, \\ i_3 & \textit{falls } t = b_3. \end{cases}$$

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- $P' := \{p_1', \ldots, p_k'\}$; $T' := \{t_1', \ldots, t_n'\}$,
- $C'(p_i') = q_i$ für $i = 1, \ldots, k$; $C'(t_j') = u_j$ für $j = 1, \ldots, n$,
- $$F' := \begin{array}{l} \{(p', t') \mid C'(p') \times C'(t') \cap F \neq \emptyset\} \cup \\ \{(t', p') \mid C'(t') \times C'(p') \cap F \neq \emptyset\}, \end{array}$$
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Let $GN = (P, T, F, C, V, m_0)$ a CN-Net.

The *Unfolding* of GN is a eS-Net $GN^* := (P^*, T^*, F^*, V^*, m_0^*)$ given as follows:

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Definition

Let E be a certain property of a net, e.g. boundedness, liveness, or reachability.

A CS-Net GN has property E, whenever its unfolding GN^* has property E.

Analysis of colored System Nets

- Analyse unfolding:

  Advantage: Methods exist,
  Pitfall: Unfoldings may be huge eS-Nets.

- Analyse colored net:
  - Reachability graph and coverability graph can be defined in analogous way to eS-Nets.
  - There exists a theory for invariants, as well.
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Section 12.7 Workflow-Nets

Literature:

van der Aalst, Hofstede: http://is.tm.tue.nl/staff/wvdaalst/publications/p174.pdf

Workflow (WF)-Net

A eS-Net $N = (P, T, F)$ is a WF-Net, if

- There exists an *input-place* $i \in P$ where $Fi = \emptyset$.
- There exists an *output-place* $o \in P$ where $oF = \emptyset$.
- In N, every $x \in P \cup T$ is contained in a path from i to o.

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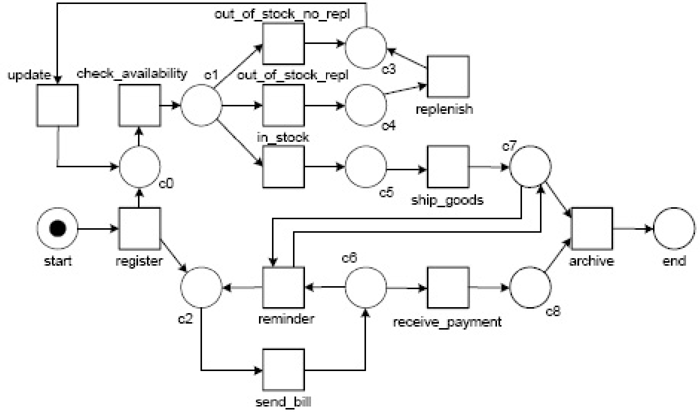
Example: WF-net order handling



Properties of a WF-Net

Let $N = (P, T, F)$ a WF-Net with input-place $i$and output-place o.

- For $p \in P$ there holds $Fp \neq \emptyset$ or $p = i$.
- For $p \in P$ there holds $pF \neq \emptyset$ or $p = o$.
- Let $\overline{N} = (\overline{P}, \overline{T}, \overline{F})$, where $\overline{P} = P$, $\overline{T} = T \cup \{t^*\}$ and $\overline{F} = F \cup \{(o, t^*), (t^*, i)\}$.

  $\overline{N}$ is called the *shortcut* net of N.

  $\overline{N}$ is strongly connected.

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- For $p \in P$ there holds $pF \neq \emptyset$ or $p = o$.
- Let $\overline{N} = (\overline{P}, \overline{T}, \overline{F})$, where $\overline{P} = P$, $\overline{T} = T \cup \{t^*\}$ and $\overline{F} = F \cup \{(o, t^*), (t^*, i)$.

$\overline{N}$ is called the *shortcut* net of N.

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Properties of a WF-Net

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Sound WF-Nets

A WF-Net is called *sound*, if the following holds.

Let m_i be a initial marking, such that only the input place i is marked.
Let m_o be a output marking, such that only the out-put place o is marked.

- From every marking m, which is reachable from m_i, marking m_o is reachable.
- m_o is the only marking reachable from m_i for which o is marked.
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A WF-Net N is sound iff $(\overline{N}, m_i)$ is life and bounded.

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Proof

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Consider an arbitrary such marking m' which is reachable from m_i, i.e. $m' = m'' + m_o$. t^* is enabled in m'. Thus marking $m'' + m_i$ is reachable from m_i. As $(\overline{N}, m_i)$ is bounded we have $m'' = 0$.

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Whenever a WF-Net N is sound, then $(\overline{N}, m_i)$ is bounded.

Proof

We show (N, m_i) bounded.

Assume (N, m_i) is not bounded. Then there exist markings m_1, m_2, such that $m_i[*\succ m_1$, $m_1[*\succ m_2$ and $m_2 > m_1$.

As N sound we have $m_1[q\succ m_o$. Moreover, because of $m_2 > m_1$, there exists a marking m with $m_2[q\succ m$ and $m > m_o$. This is a contradiction to N sound.

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Therefore, from any m', which is reachable in $(\overline{N}, m_i)$, we can reach m_i. As N does not have any dead transitions w.r.t. m_i, it follows $(\overline{N}, m_i)$ is live.

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Excursus: Net-Classes

Let $N = (P, T, F, V, m_0)$.

- N is called *Synchronization-Graph*, if for each place p it holds $\mid Fp \mid = \mid pF \mid = 1$.
- N is called *Statemachine*, if for each transition t it holds $\mid Ft \mid = \mid tF \mid = 1$.
- N is called *Free-Choice-Net (FC-Net)*, if $t, t' \in pF \Rightarrow Ft = \{s\} = Ft'$.
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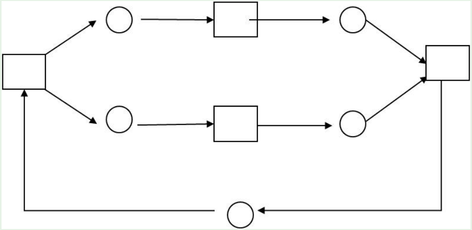
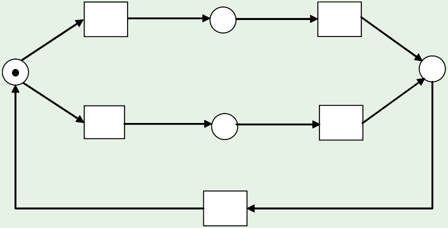
- A synchronization-graph is also a FC-Net.
- A statemachine is also a FC-Net.
- A FC-Net is also a EFC-Net.

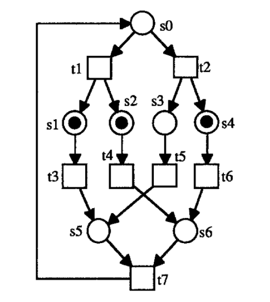
Synchronization-Graph



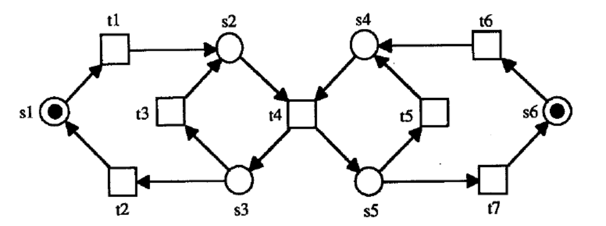
Statemachine



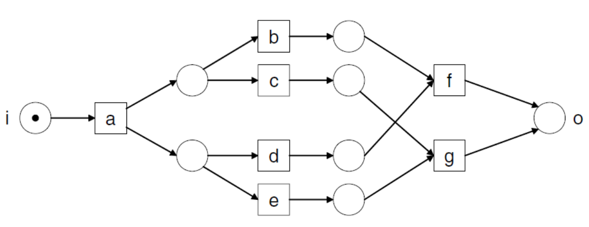
FC-Net


s0
t1
t2
s1
s2
s3
s4
t3
t4
t5
t6
s5
s6
t7

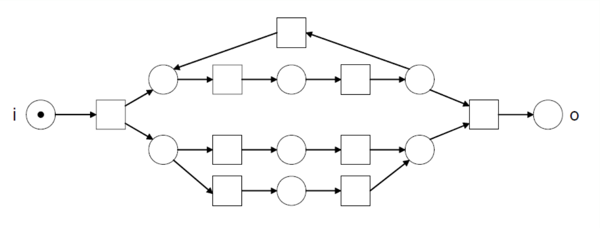
FC-Net


t1
s2
s4
t6
s1
t3
t4
t5
s6
t2
s3
s5
t7

A not sound WF-Net; the WF-Net is free-choice



A WF-Net which is sound, however not free-choice



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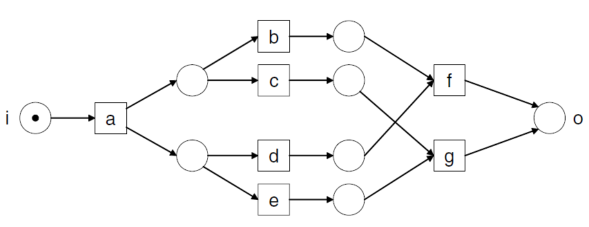

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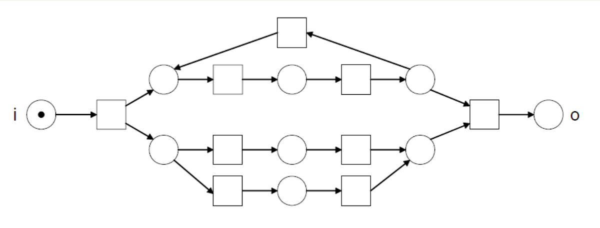

Soundness of a WF-Net

A WF-Net, which is a FC-Net, can be checked for soundness in polynomial time.

... from practical experiences:

For modeling in practical applications FC-Nets are sufficient.

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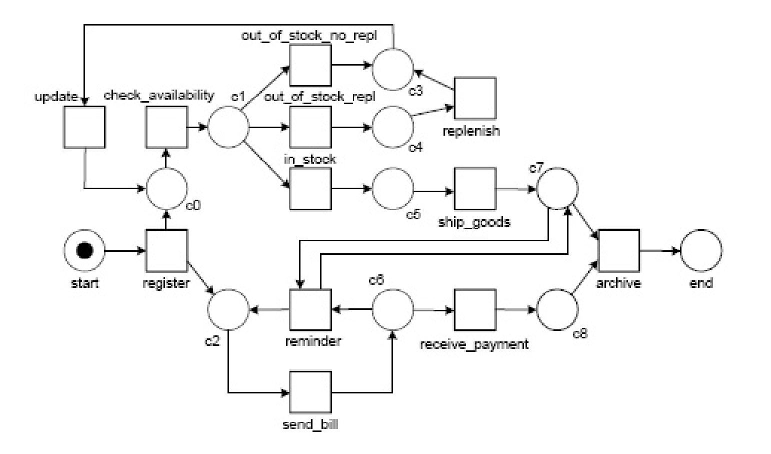
Example: WF-Net order handling - make it free-choice!



Split `send_bill` to `send_bill_reminder` and `send_bill_receive_payment`; now `reminder` and `receive_payment` do not share a common input-place.