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Exercises for the Lecture
Graph Theory
Winter 2014/15
Blatt 1 (10 points)

Task 1: 5 points

Consider the following adjacency matrices of two digraphs.

$$G_1 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \quad G_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix}$$

1. Draw graphs G_1 and G_2.
2. Which of the following features can be observed?
   (a) loop-free
   (b) parallel edges
   (c) anti-parallel edges
   (d) simple
3. Represent these graphs as
   (a) incidence matrix,
   (b) adjacency list,
   (c) on a tape of a Turing machine.

Task 2: 5 points

Line graphs and graph isomorphism. A d**-regular undirected graph** H consists only of nodes $u \in V$ with $d_H(u) = d$.
Prove or disprove:

1. For every 2-regular simple undirected graph H we have $H \cong L(H)$. [1]
2. For every 3-regular simple undirected graph H we have $H \cong L(H)$.

[1] $\cong$ means: *is isomorphic to*