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Freiburg, 12.11.2014
Discussion 19.11.2014

Exercises for the Lecture
Graph Theory
Winter 2014/15
Blatt 2 (10 points)

Task 1: 5 points

1. Given the following digraph as an adjacency matrix, decide whether the digraph is *acyclic* by applying the *topological sorting* algorithm from in the lecture! What is the topological sorting produced by this algorithm?

$$G_1 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

2. Present a directed graph G with n nodes, which possesses the maximal amount of different topological sorting. Compute the number of topological sortings and prove the maximality.

3. Find a directed graph G with n nodes, with minimal amount of different topological sorting. How many topological sortings exist in your graph?

Task 2: 5 points

1. Show that every path p in an undirected graph G has a corresponding path $\tilde{p}$ in $L(G)$ where the edges of p are nodes in $\tilde{p}$.

2. Show how the number of *connected components* in an undirected graph G relates to the *connected components* in $L(G)$ and the number of nodes v in G without neighbors.