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Freiburg, 10.12.2014
Discussion 16.12.2014

Exercises for the Lecture
Graph Theory
Winter 2014/15
Blatt 4 (10 points)

Task 1: 5 points

Let m be your immatriculation number. Show the equivalence of statement number s_1 and s_2 of Theorem 10, where $s_1 = 1 + (m \bmod 5)$ and statement number $s_2 = 1 + ((s_1 + m) \bmod 4)$.

1. G is a tree.
2. G contains no elementary cycle, but every proper super graph of G with the same vertex set contains an elementary cycle.
3. For every pair $u, v \in V$ there exists exactly one path with $\alpha(P) = u$, $\omega(P) = v$.
4. G is connected and $|E| = |V| - 1$.
5. G contains no elementary cycle and $|E| = |V| - 1$.

Task 2: 5 points

$$G = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

1. Apply the DFS-algorithm to G, where the nodes are chosen in the standard order $v_1, v_2, \ldots, v_9$.
2. Give for each node $v \in V(G)$ the values $d[v]$ and $f[v]$.
3. Use $f[v_1], f[v_2], \ldots, f[v_9]$ to compute a topological ordering.
4. What is the longest white path in the DFS-algorithm?
5. What happens, if we perform DFS on G^{-1} where the nodes are chosen according decreasing order of $f[v]$.