

## Exercises for the Lecture

**Graph Theory**

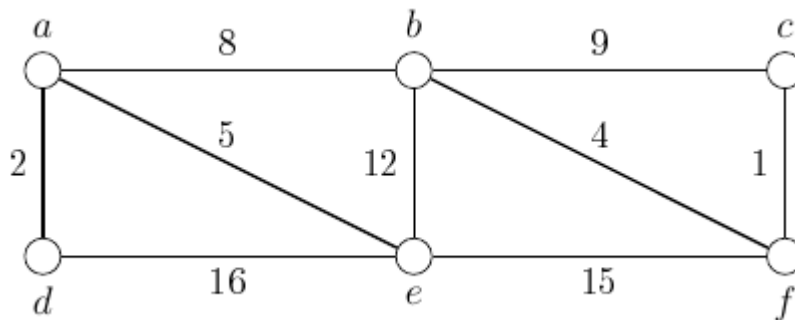
Winter 2014/15

Blatt 5 (10 points)

**Task 1:**

10 points

1. Look at the graph. Apply the algorithm of Kruskal and mark the edges which are added to the *Minimum Spanning Tree*. Also give the order in which the edges are chosen.

**Algorithm 1** Algorithm of Prim**Input:** A non-empty connected weighted graph with vertices  $V$  and edges  $E$ **Output:**  $U$  and  $T$  describe a minimal spanning tree

- 1:  $T \leftarrow \phi$ ;
- 2:  $U \leftarrow v_1$ ;
- 3: **while**  $U \neq V$  **do**
- 4:   Let  $(u, v)$  be the edge with lowest cost such that  $u \in U$  and  $v \in V - U$ ;
- 5:    $T \leftarrow T \cup (u, v)$ ;
- 6:    $U \leftarrow U \cup v$ ;
- 7: **end while**

2. Apply the algorithm of Prim to the given graph and discuss the different output.
3. Assume negative edge costs are allowed and define a minimum spanning graph as the subgraph connecting all nodes of each connecting component with minimum edge weight. Is every minimum spanning graph still a forest? Prove your answer!
4. Give an algorithm that produces the minimum spanning graph of a graph with negative edge costs allowed! Hint: Modify Kruskal's or Prim's algorithm.