

Exercises for the Lecture

Graph Theory

Winter 2014/15

Blatt 7 (10 points)

Task 1:

3 points

Show that a graph is bipartite if and only if it can be properly colored with at most two colors.

Task 2:

4 points

- a) Given two graphs $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$. The union of G_1 and G_2 is defined as follows: $G_1 \cup G_2 := (V, E_1 \cup E_2)$. Prove the following statement: $\chi(G_1 \cup G_2) \leq \chi(G_1) \cdot \chi(G_2)$
- b) Show that $\chi(G - v)$ is either $\chi(G)$ or $\chi(G) - 1$.

Task 3:

3 points

Consider the following properties of undirected simple graphs.

1. perfect
2. chordal
3. connected
4. Eulerian
5. Hamiltonian
6. regular

Prove if there is a non empty graph satisfying...

- a) ... all properties!
- b) ... no property!
- c) ... exactly three properties!