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Due until 8 June 2007

Exercises of lecture
Mobile Ad Hoc Networks
Summer 2007
Sheet 6

SECTION 1:
Spanner, Weak Spanner, Power Spanner

1. Show that every c-Spanner is:
   - a weak c-Spanner.
   - a (c^d,d)-Power Spanner.
2. Draw a finite connected graph that is **not** a c-Spanner for any c.
3. Draw a finite connected graph that is also **not** even a weak c-Spanner for any c.
4. Draw a connected graph consisting of five nodes, which is a 1-Spanner.

Solution:

1. According to the definition of weak c-Spanner, for every node pair u,v, a path P exists inside the disk $C(u, c \cdot \|u, v\|)$. Based on the definition of c-Spanner, the length of the path P connecting any two vertices u and v in a c-Spanner, $\|P\|$ is bounded by $c \cdot \|u, v\|$. Every path P is within the circle of u with radius $c \cdot \|u, v\|$.
2. Let G be a c-Spanner and $P_{opt} = (u_1, u_2, ..., u_n)$ be its energy-optimal path, which has the energy cost of $E(P_{opt}) := \sum_{i=1}^{n} \|u_i - u_{i+1}\|^d$.

   Replace all the link of optimal path to include path of c-spanner, P_i so that we have the path, $P' = (P_1, P_2, ..., P_m)$. Every path P_i follows the spanner property of $\|P_i\| \leq (c \cdot \parallel u_i - u_{i+1} \parallel)$. The energy of each path P_i follows $Energy(P_i) < (c \cdot \parallel u_i - u_{i+1} \parallel)^d$.

   Get the total energy cost for these path of c-spanner.
   $Energy(P') = \Sigma_{i=1}^{n-1} Energy(P_i) \leq \Sigma_{i=1}^{n-1} (c \cdot \parallel u_i - u_{i+1} \parallel)^d = c^d \Sigma_{i=1}^{n-1} (\parallel u_i - u_{i+1} \parallel)^d$

   Thus, G is a (c^d, d)-power spanner.
3. Every finite connected graph is a c-Spanner for some value of c. Hence, such a graph does not exist.
4. Every finite connected graph is a weak c-Spanner for some value of c. Hence, such a graph does not exist.
5. Connect all the five nodes in a straight line with the distances between nodes be either equal or not equal.