Mobile Ad Hoc Networks
Theory of Interferences, Trade-Offs between Energy, Congestion and Delay

5th Week

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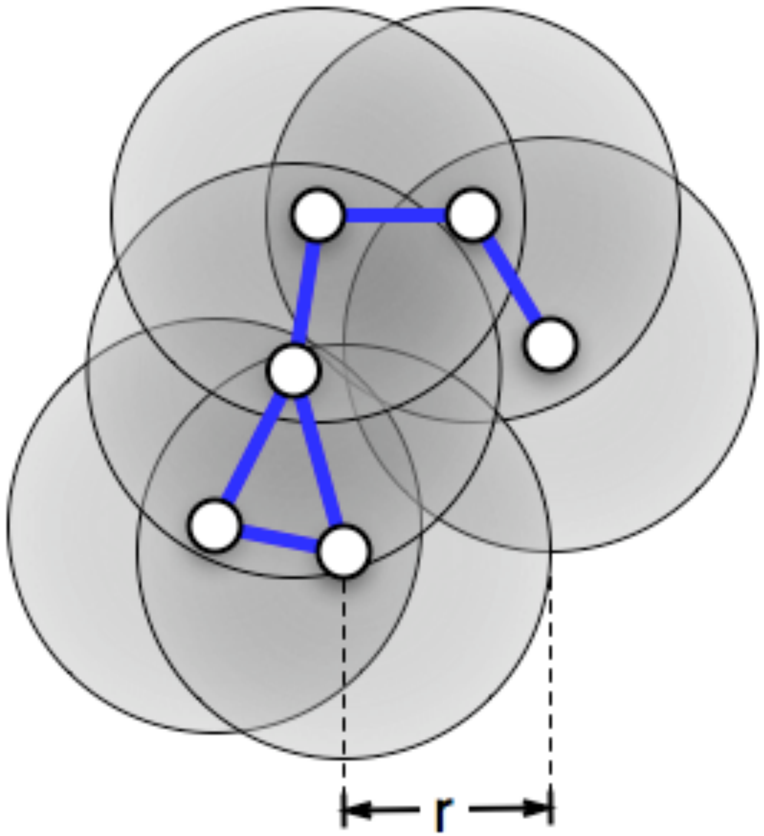
Unit Disk Graphs

- **Motivation:**
  - Received Signal Strength decreases proportionally to $d^{-\gamma}$,
    - where γ is the path loss exponent
  - Connections only exists if the signal/noise ratio is beyond a threshold
- **Definition**
  - Given a finite point set V in $\mathbf{R}^2$ or $\mathbf{R}^3$,
  - then a Unit Disk Graph with radius r G=(V,E) of the point set is defined by the undirected edge set:

$$E = \{\{u,v\} \mid ||u,v||_2 \leq r\}$$

  - where $||u,v||_2$ is the Euclidean distance:

$$||u,v||_2 = \sqrt{(u_x - v_x)^2 + (u_y - v_y)^2}$$



Topology Control

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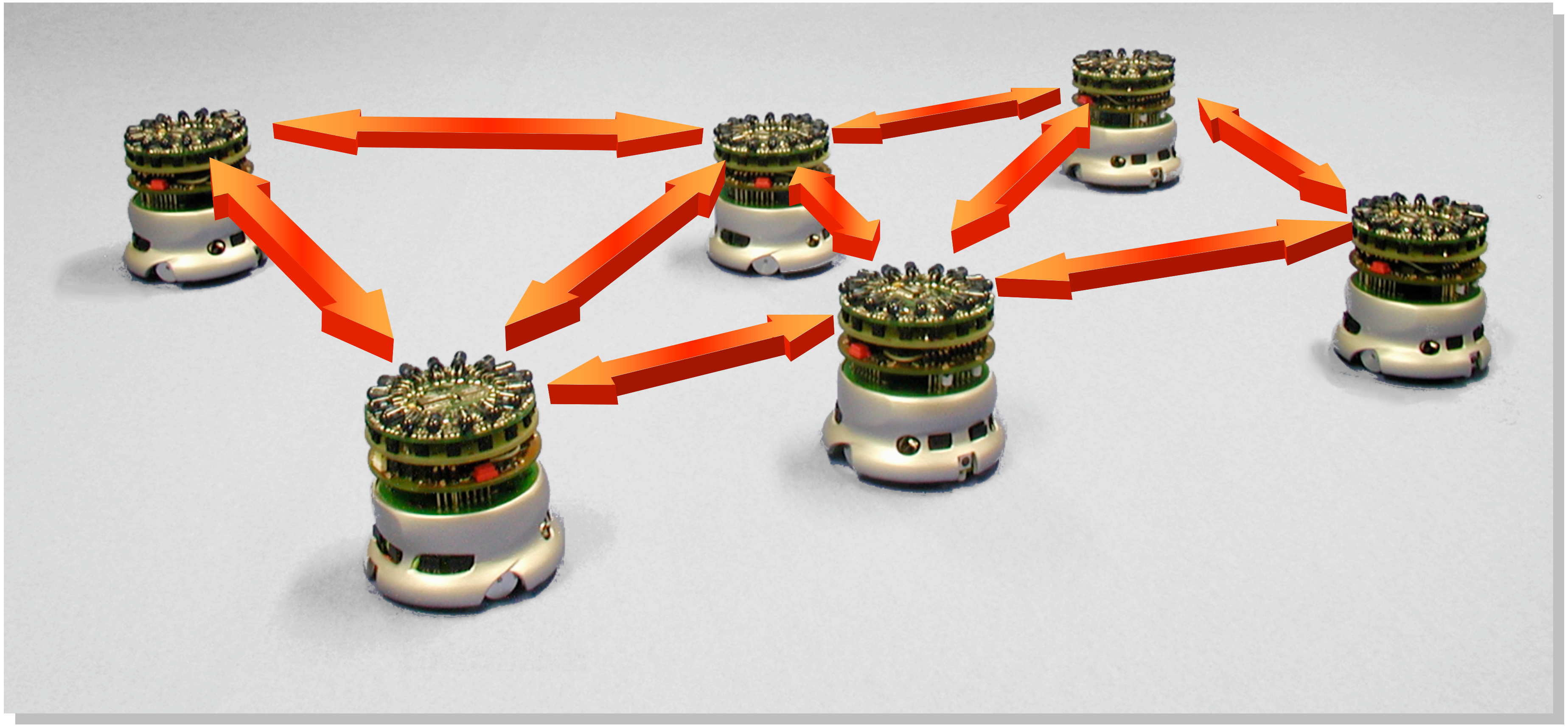

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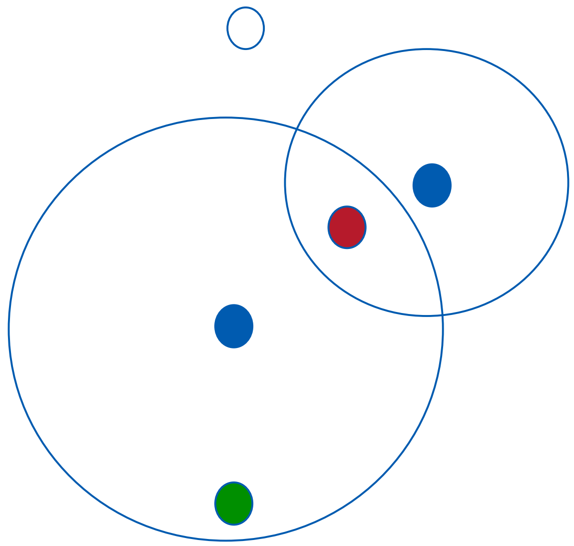
A Simple Physical Network Model

- **Homogenous Network of**
  - n radio stations $s_1,..,s_n$ on the plane
- **Radio transmission**
  - One frequency
  - Adjustable transmission range
    - Maximum range > maximum distance of radio stations
    - Inside the transmission area of sender: clear signal or radio interference
    - Outside: no signal
  - Packets of unit length



The Routing Problem

- **Given:**
  - n points in the plane, $V=(v_1,..,v_n)$
    - representing mobile nodes of a mobile ad hoc network
  - the complete undirected graph G = (V,E) as possible communication network
    - representing a MANET where every connection can be established
- **Routing problem (multi-commodity flow problem):**
  - $f : V \times V \rightarrow \mathbf{N}$, where f(u,v) packets have to be sent from u to v, for all $u,v \in V$
  - Find a path for each packet of this routing problem in the complete graph
- **The union of all path systems is called the Link Network or Communication Network**

Formal Definition of Interference

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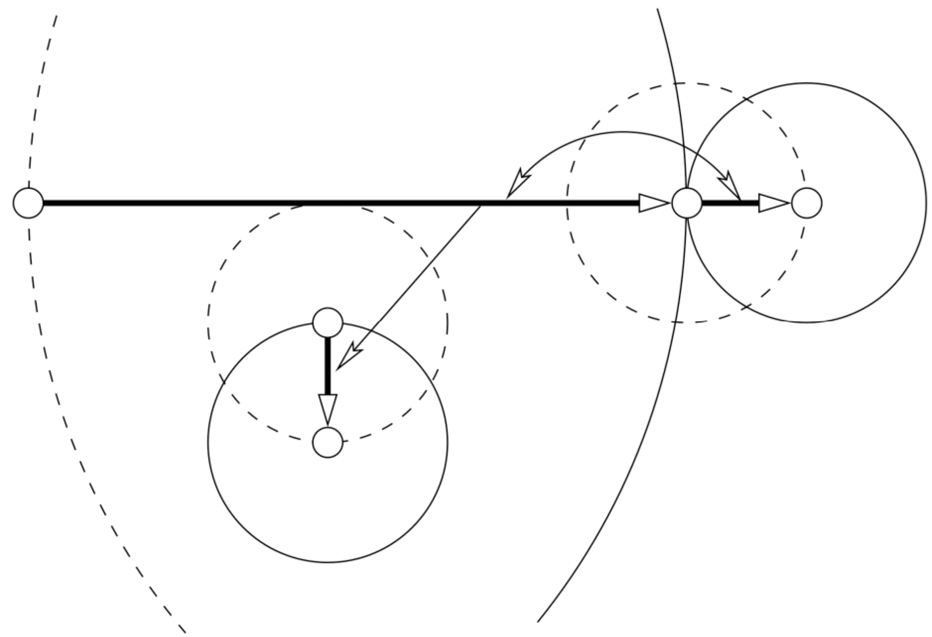
- Let $D_r(u)$ the disk of radius u with center u in the plane
- Define for an edge e={u,v} $D(e) = D_r(u) \cup D_r(v)$



- The set of edges interfering with an edge e = {u,v} of a communication network N is defined as:

$$\mathrm{Int}(e) := \{e' \in E(N) \setminus \{e\} \mid u \in D(e') \text{ or } v \in D(e')\}$$

- The Interference Number of an edge is given by |Int(e)|
- The Interference Number of the Network is max{|Int(e)| | e ∈ E}

Formal Definition of Congestion

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- The Congestion of an edge e is defined as:

$$C_{\mathcal{P}}(e) := \ell(e) + \sum_{e' \in \mathrm{Int}(e)} \ell(e')$$

- The Congestion of the path system P is defined as

$$C_{\mathcal{P}}(V) := \max_{e \in E_{\mathcal{P}}} \{C_{\mathcal{P}}(e)\}$$

- The Dilation D(P) of a path system is the length of the longest path.

Energy

- **The energy for transmission of a message can be modeled by a power over the distance d between sender and transceiver**
- **Two energy models:**
  - **Unit energy** accounts only the energy for upholding an edge
    - Idea: messages can be aggregated and sent as one packet

$$\text{U-Energy}_{\mathcal{P}}(V) := \sum_{e \in E_{\mathcal{P}}(N)} |e|^2$$

  - **Flow Energy** Model: every message is counted separately

$$\text{F-Energy}_{\mathcal{P}}(V) := \sum_{e \in E_{\mathcal{P}}(N)} \ell(e)|e|^2$$

Congestion versus Time

- **Theorem 1**

  Consider a radio network N in d-dimensional space ($d \in \{2,3\}$) and a path system $\mathcal{P}$ for a routing problem f with dilation D and congestion C. Let T be its optimal routing time. Then it holds for $c_2 = 6$ and $c_3 = 20$ that

$$T \;\geq\; \max\left\{\frac{C}{2c_d}, D\right\} = \Omega(C + D)\ .$$

- **Theorem 2**

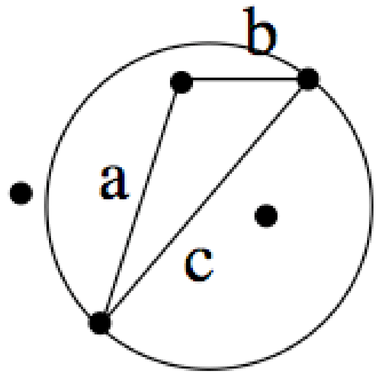
  Consider a radio network $N = (V, E)$ and a path system $\mathcal{P}$ of size n for some routing problem f with maximum interference number I, dilation D, and congestion C. Let T be its optimal routing time, when the path system $\mathcal{P}$ is used. There is an online routing protocol that needs routing time $O(C + D \cdot I \cdot \log(n \cdot I))$, with probability at least $1 - n^{-c}$ for any constant c.

Minimizing Energy

- **Theorem**

  The unique paths defined by a minimum spanning tree result in an optimal path system for a radio network $N = (V, E), V \subseteq \mathbb{R}^d$ for any d, with respect to the unit energy.

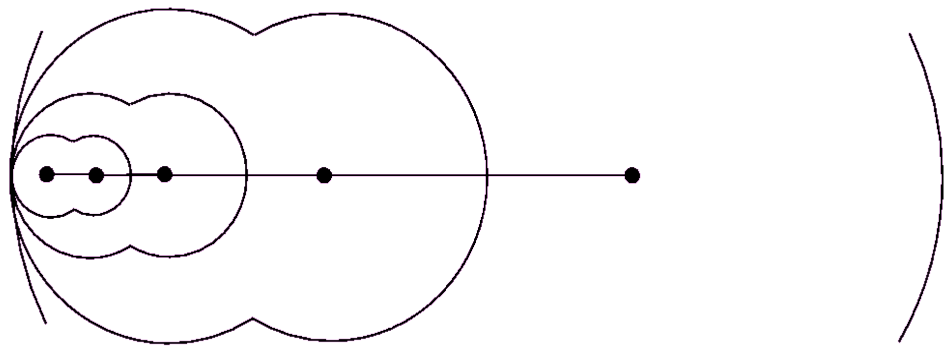
- **Definition Gabriel Graph**



- **Theorem**

For a given vertex set V and a routing problem f, the shortest paths between vertices $u, v \in V$ with $f(u, v) \neq 0$ of the Gabriel Graph of V form an optimal path system for a radio network with respect to the flow energy.

Worst Case Construction for Interferences



- Interference Number for n nodes = n-1

A Measure for the Ugliness of Positions

- **For a network G=(V,E) define the Diversity as**

$$g(V) := |\{m \mid \exists u,v \in V : \lfloor \log |u,v| = m \rfloor\}|$$

- **Properties of the diversity:**
  - g(V)=Ω(log *n*)
  - g(V)=O(*n*)

Diversity (II)

Fact: For k dimensions and every vertex set V:

$$\log_k(n) \leq g(V) \leq O(n)$$

V	chain of pearls	grid	random locations
diversity g(V)	log n	1/2 log n	c log n with prob. $1-n^{-k}$

$$\frac{\text{max transmission range}}{\text{size of a radio station}} = O(n^c) \Rightarrow g(V) = O(\log n)$$

Congestion

Maximum number of packets interfering at an edge

$$C_{\mathcal{P}}(V) := \max_{e \in E_{\mathcal{P}}} \left\{ \ell(e) + \sum_{e' \in \mathrm{Int}(e)} \ell(e') \right\} .$$

Energy

Sum of energy consumed in all routes

$$\mathrm{Energy}_{\mathcal{P}}(V) := \sum_{e \in E_{\mathcal{P}}(N)} \ell(e) |e|^2 .$$

Dilation

Maximum number of hops (diameter of the network)

Thank you!

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