Peer-to-Peer Networks

14 Network Coding

Christian Schindelhauer
Technical Faculty
Computer-Networks and Telematics
University of Freiburg

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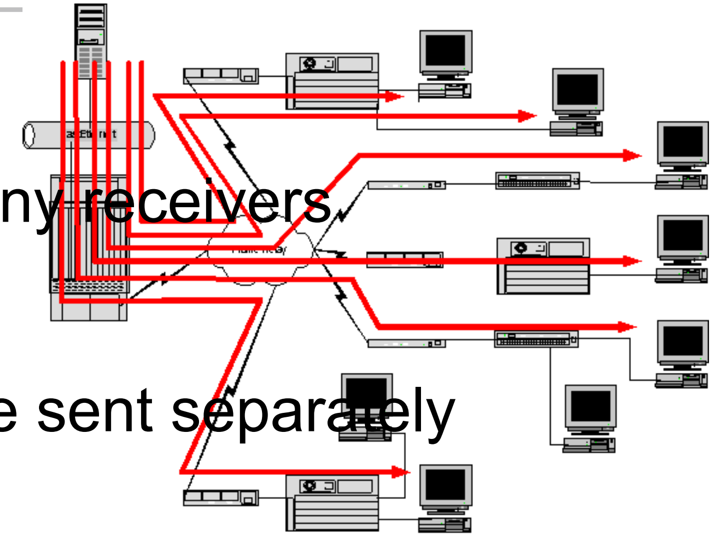
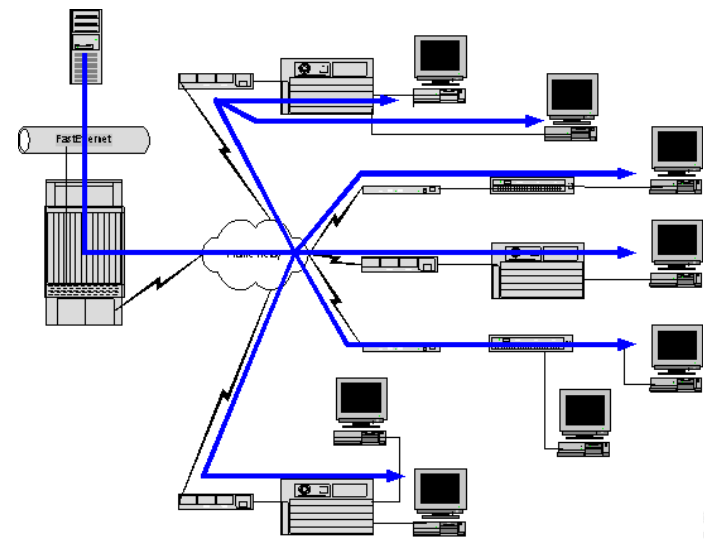
IP Multicast

- Motivation
  - Transmission of a data stream to many receivers
- Unicast
  - For each stream message have to be sent separately
  - Bottleneck at sender
- Multicast
  - Stream multiplies messages
  - No bottleneck





Peter J. Welcher
www.netcraftsmen.net/.../ papers/multicast01.html

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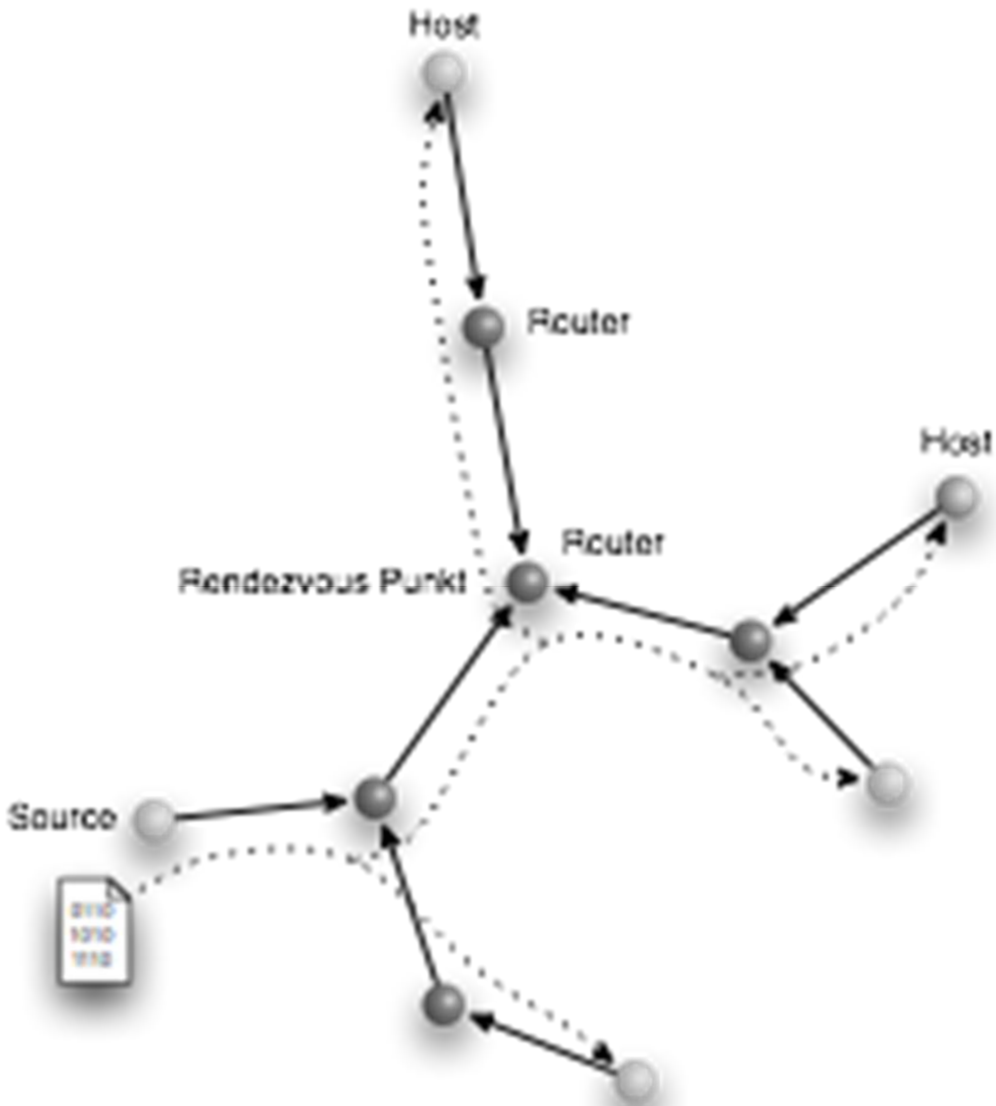
Working Principle

- IPv4 Multicast Addresses
  - class D
    - outside of CIDR (Classless Interdomain Routing)
  - 224.0.0.0 - 239.255.255.255
- Hosts register via IGMP at this address
  - IGMP = Internet Group Management Protocol
  - After registration the multicast tree is updated
- Source sends to multicast address
  - Routers duplicate messages
  - and distribute them into sub-trees
- All registered hosts receive these messages
  - ends after Time-Out
  - or when they unsubscribe
- Problems
  - No TCP only UDP

Routing Protocols

- Distance Vector Multicast Routing Protocol (DVMRP)
  - used for years in MBONE
    - particularly in Freiburg
  - own routing tables for multicast
- Protocol Independent Multicast (PIM)
  - in Sparse Mode (PIM-SM)
  - current (de facto) standard
  - prunes multicast tree
  - uses Unicast routing tables
  - is more independent from the routers
- Prerequisites of PIM-SM:
  - needs Rendezvous-Point (RP) in one hop distance
  - RP must provide PIM-SM
  - or tunneling to a proxy in the vicinity of the RP



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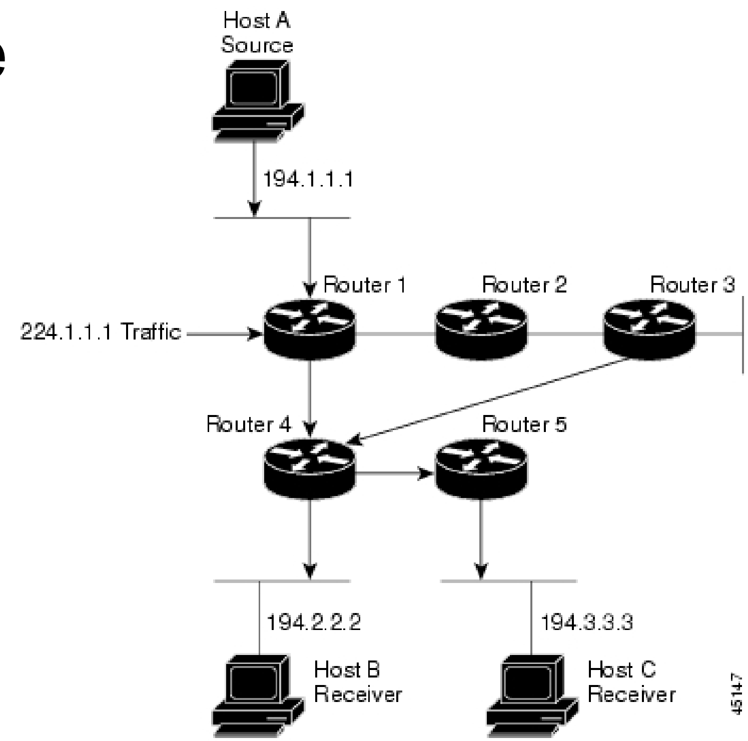
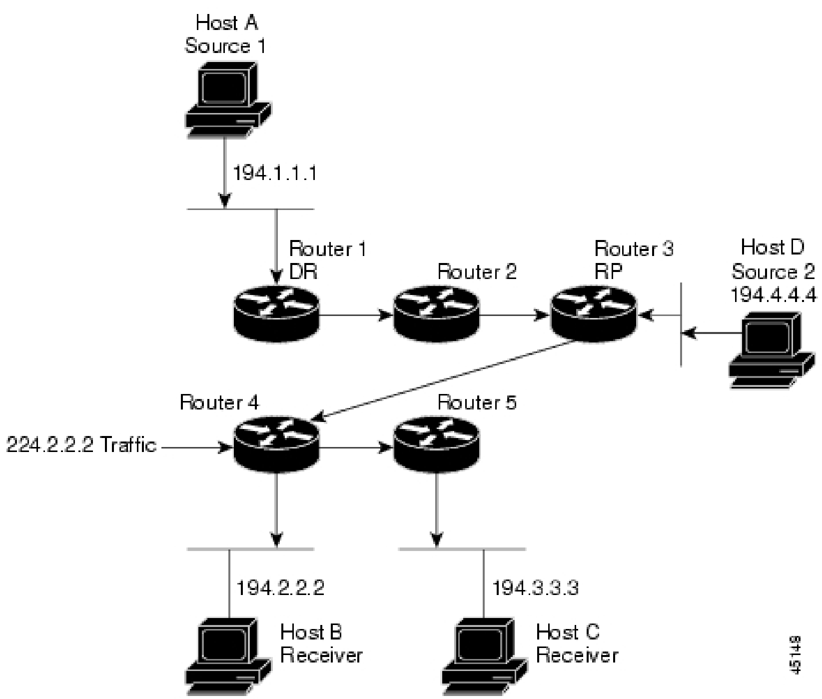
PIM-SM
Tree Construction

- Host a Shortest-Path-Tree



- Shared Distribution Tree



From Cisco:
http://www.cisco.com/en/US/products/hw/switches/ps646/products_configuration_guide_chapter09186a008014f350.html

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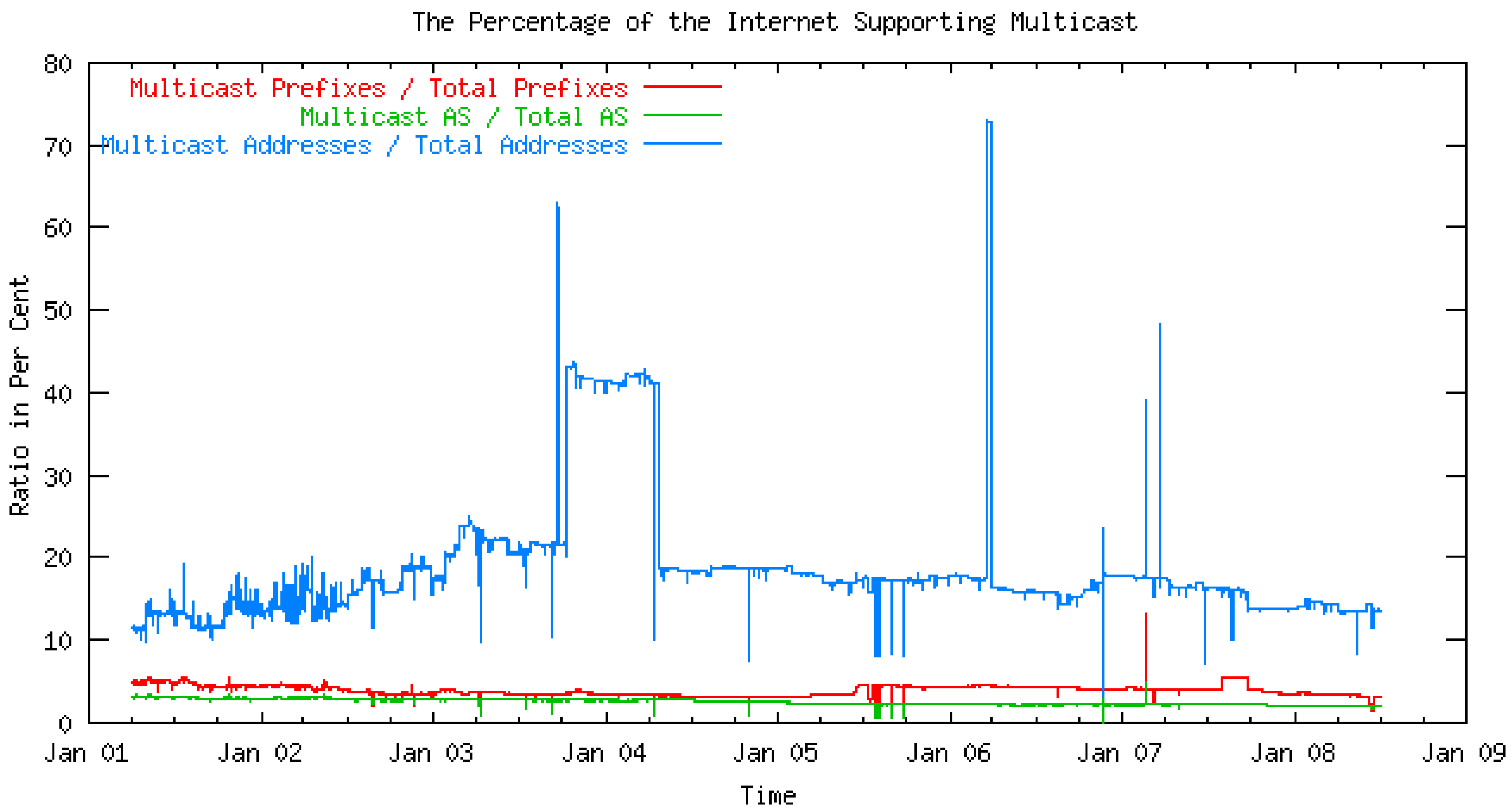
IP Multicast Seldomly Available

- IP Multicast is the fastest download method
- Yet, not many routers support IP multicast
  - –http://www.multicasttech.com/status/



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Why so few Multicast Routers?

- Despite successful use
  - in video transmission of IETF-meetings
  - MBONE (Multicast Backbone)
- Only few ISPs provide IP Multicast
- Additional maintenance
  - difficult to configure
  - competing protocols
- Enabling of Denial-of-Service-Attacks
  - Implications larger than for Unicast
- Transport protocol
  - only UDP
    - Unreliable
  - Forward error correction necessary
    - or proprietary protocols at the routers (z.B. CISCO)
- Market situation
  - consumers seldomly ask for multicast

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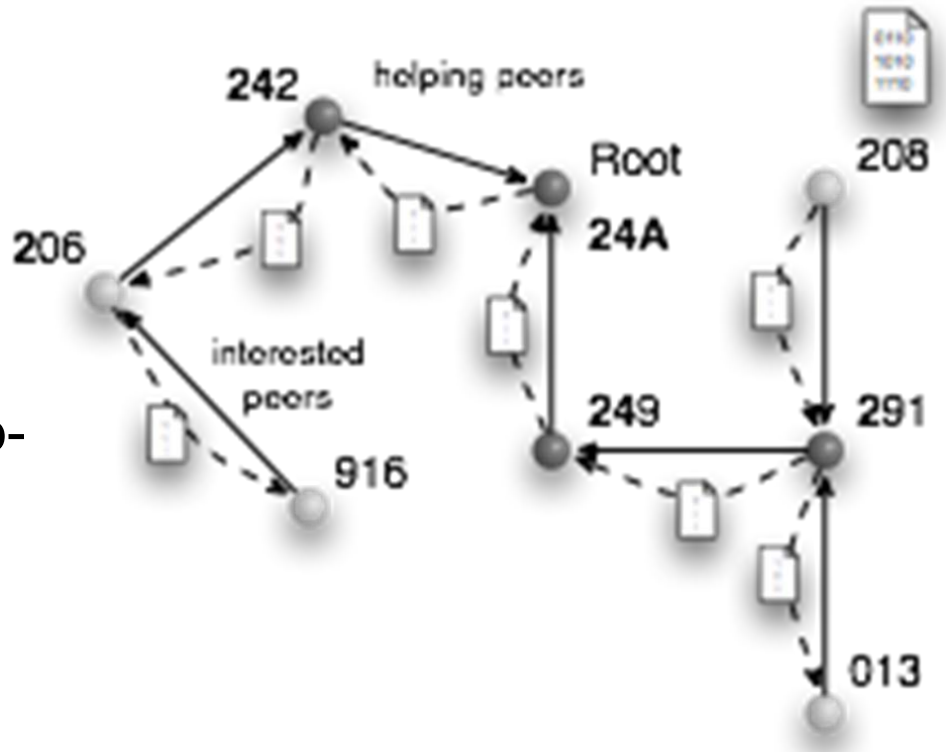
Scribe & Friends

- Multicast-Tree in the Overlay Network
- Scribe [2001] is based on Pastry
  - Castro, Druschel, Kermarrec, Rowstron
- Similar approaches
  - CAN Multicast [2001] based on CAN
  - Bayeux [2001] based on Tapestry
- Andere Ansätze
  - Overcast [´00] and Narada [´00]
  - construct multi-cast trees using unicast connections
  - do not scale

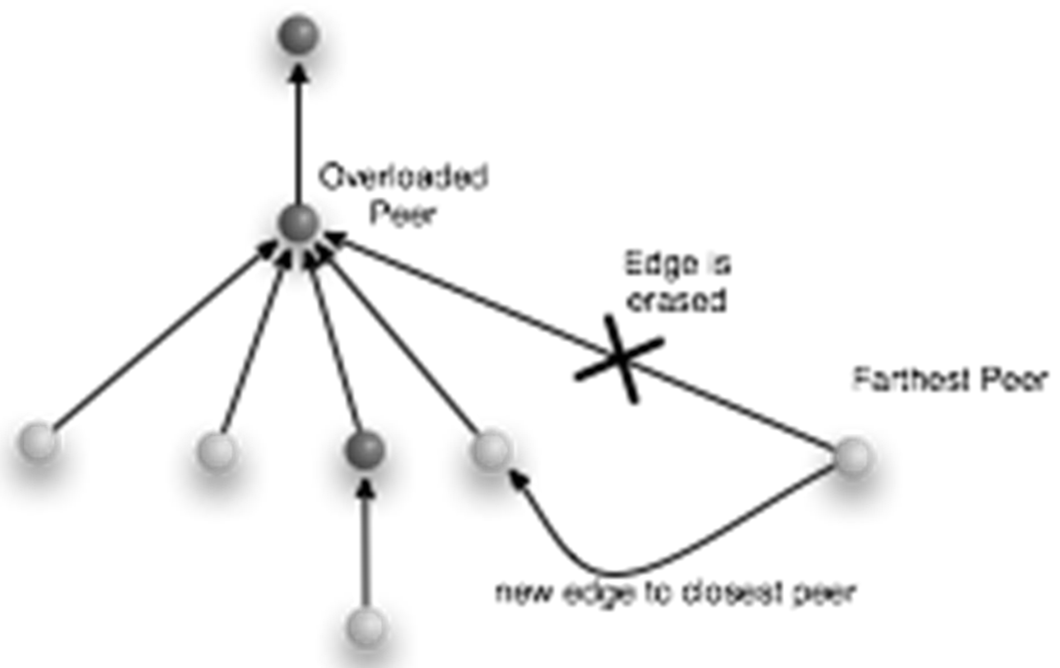
How Scribe Works

- Create
  - GroupID is assigned to a peer according to Pastry index
- Join
  - Interested peer performs lookup to group ID
  - When a peer is found in the Multicast tree then a new sub-path is inserted
- Download
  - Messages are distributed using the multicast tree
  - Nodes duplicate parts of the file



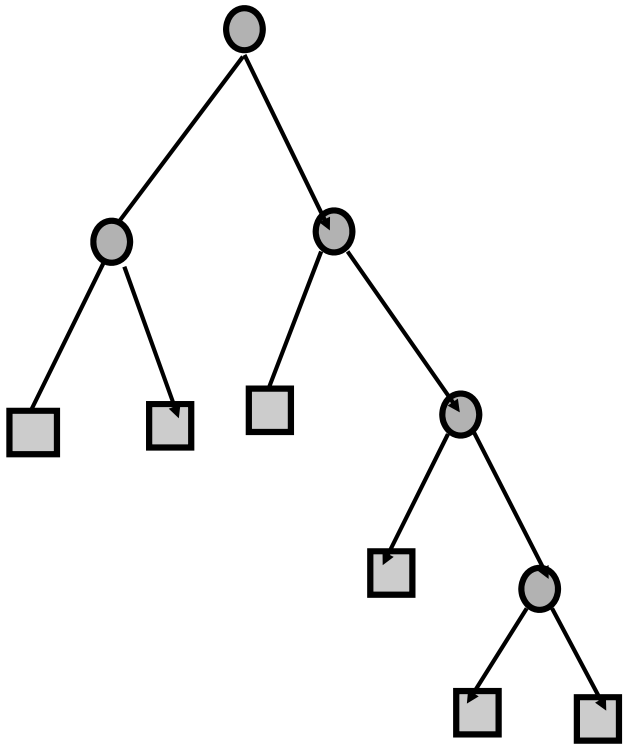
Scribe Optimization

- Bottleneck-Remover
  - If a node is overloaded then from the group of peers he sends messages
  - Select the farthest peer
  - This node measures the delay between it and the other nodes
  - and rebalances itself under the next (then former) brother



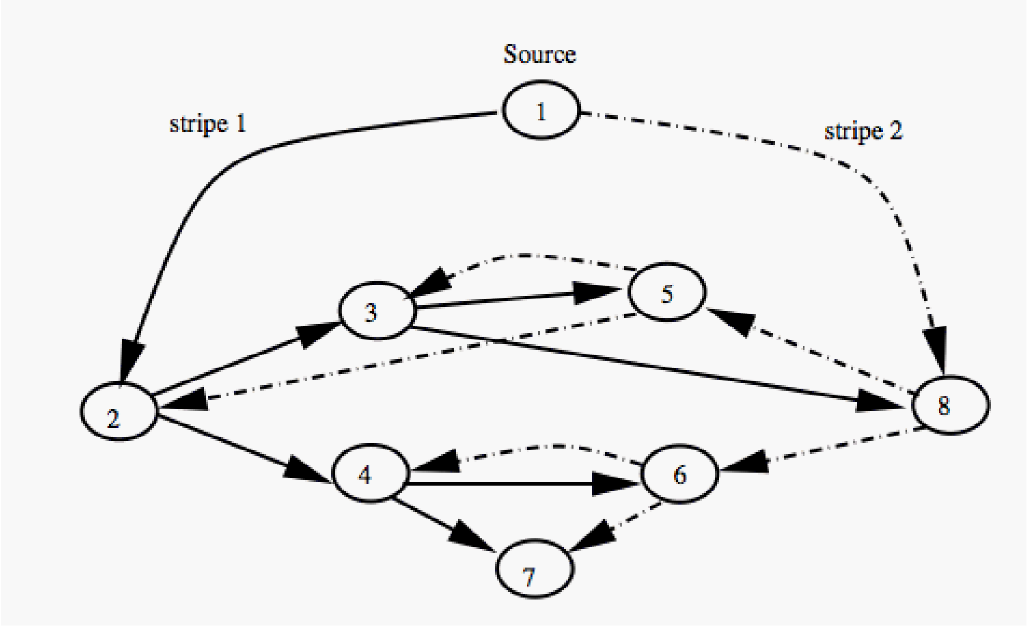
Split-Stream Motivation

- Multicast trees discriminate certain nodes
- Lemma
  - In every binary tree the number of leaves = number of internal nodes +1
- Conclusion
  - Nearly half of the nodes distribute data
  - While the other half does not distribute any data
  - An internal node has twice the upload as the average peer
- Solution: Larger degree?
- Lemma
  - In every node with degree d the number of internal nodes k und leaves b we observe
    - $(d-1)\, k = b - 1$
- Implication
  - Less peers have to suffer more upload



Split-Stream

- Castro, Druschel, Kermarrec, Nandi, Rowstron, Singh 2001
- Idea
  - Partition a file of size into k small parts
  - For each part use another multicast tree
  - Every peer works as leave and as distributing internal tree node
    - except the source
- Ideally, the upload of each node is at most the download



Bittorrent

- Bram Cohen
- Bittorrent is a real (very successful) peer-to-peer network
  - concentrates on download
  - uses (implicitly) multicast trees for the distribution of the parts of a file
- Protocol is peer oriented and not data oriented
- Goals
  - efficient download of a file using the uploads of all participating peers
  - efficient usage of upload
    - usually upload is the bottleneck
    - e.g. asymmetric protocols like ISDN or DSL
  - fairness among peers
    - seeders against leeches
  - usage of several sources

Bittorrent Coordination and File

- Central coordination
  - by tracker host
  - for each file the tracker outputs a set of random peers from the set of participating peers
    - in addition hash-code of the file contents and other control information
  - tracker hosts to not store files
    - yet, providing a tracker file on a tracker host can have legal consequences
- File
  - is partitions in smaller pieces
    - as describec in tracker file
  - every participating peer can redistribute downloaded parts as soon as he received it
  - Bittorrent aims at the Split-Stream idea
- Interaction between the peers
  - two peers exchange their information about existing parts
  - according to the policy of Bittorrent outstanding parts are transmitted to the other peer

Bittorrent
Part Selection

- Problem
  - The Coupon-Collector-Problem is the reason for a uneven distribution of parts
    - if a completely random choice is used
- Measures
  - Rarest First
    - Every peer tries to download the parts which are rarest
      - density is deduced from the comunication with other peers (or tracker host)
    - in case the source is not available this increases the chances the peers can complete the download
  - Random First (exception for new peers)
    - When peer starts it asks for a random part
    - Then the demand for seldom peers is reduced
      - especially when peers only shortly join
  - Endgame Mode
    - if nearly all parts have been loaded the downloading peers asks more connected peers for the missing parts
    - then a slow peer can not stall the last download

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Bittorrent Policy

- Goal
  - self organizing system
  - good (uploading, seeding) peers are rewarded
  - bad (downloading, leeching) peers are penalized
- Reward
  - good download speed
  - un-choking
- Penalty
  - Choking of the bandwidth
- Evaluation
  - Every peers Peers evaluates his environment from his past experiences

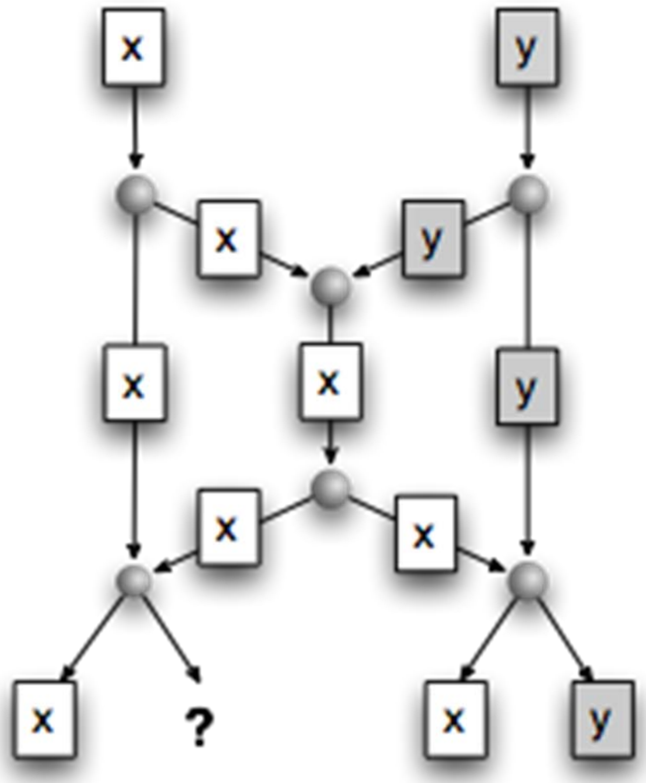
Bittorrent
Choking

- Every peer has a choke list
  - requests of choked peers are not served for some time
  - peers can be unchoked after some time
- Adding to the choke list
  - Each peer has a fixed minimum amount of choked peers (e.g. 4)
  - Peers with the worst upload are added to the choke list
    - and replace better peers
- Optimistic Unchoking
  - Arbitrarily a candidate is removed from the list of choking candidates
    - the prevents maltreating a peer with a bad bandwidth

Network Coding

- R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network Information Flow", (IEEE Transactions on Information Theory, IT-46, pp. 1204-1216, 2000)

- Example
  - Bits x and y need to be transmitted
  - Every line transmits one bit
  - If only bits are transmitted
    - then only x or y can be transmitted in the middle?
  - By using X we can have both results at the outputs



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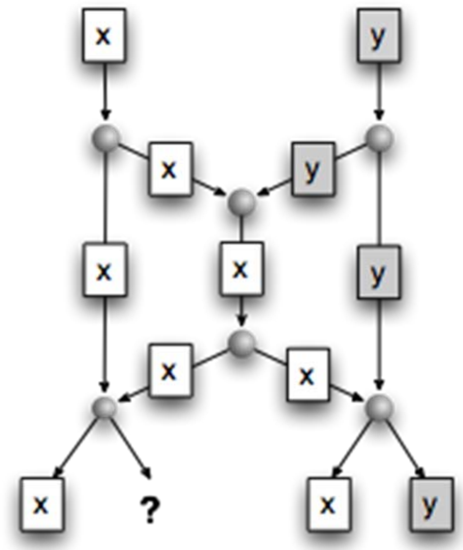
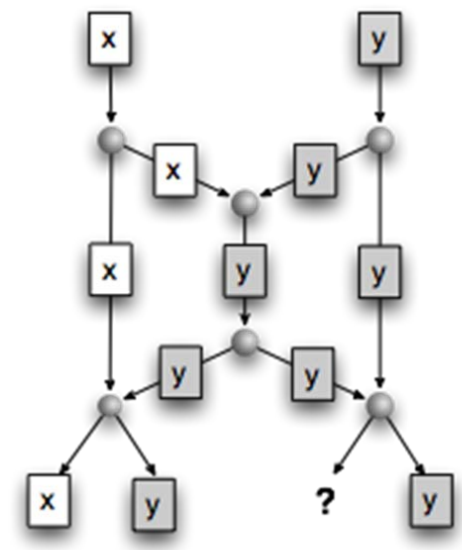
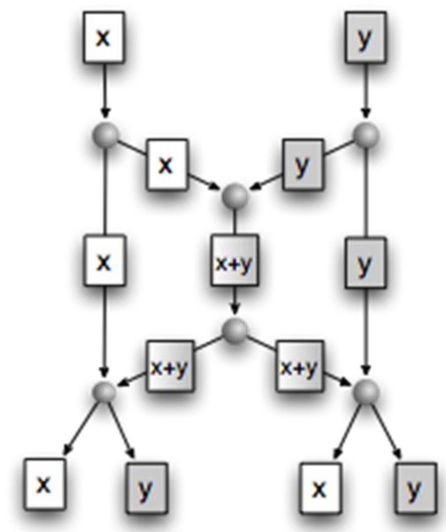




- Theorem [Ahlswede et al.]
  - There is a network code for each graph such that each node receives as much information as the maximum flow of the corresponding flow problem



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Galois Fields

- $GF(2^w)$ = Finite Field over 2^w elements
  - Elements are all binary strings of length w
  - $0 = 0^w$ is the neutral element for addition
  - $1 = 0^{w-1}1$ is the neutral element for multiplication
- u + v = bit-wise Xor of the elements
  - e.g. 0101 + 1100 = 1001
- a b= product of polynomials modulo 2 and modulo an irreducible polynomial q
  - i.e. $(a_{w-1} \ldots a_1\, a_0)\,(b_{w-1} \ldots b_1\, b_0) =$

$$((a_0 + a_1 x + \ldots + a_{w-1} x^{w-1})(b_0 + b_1 x + \ldots + b_{w-1} x^{w-1}) \bmod q(x)) \bmod 2)$$

Example: GF(2^2)

Generated Element of $GF(4)$	Polynomial Element of $GF(4)$	Binary Element b of $GF(4)$	Decimal Representation of b
0	0	00	0
x^0	1	01	1
x^1	x	10	2
x^2	$x+1$	11	3

$q(x) = x^2+x+1$

+	0 = 00	1 = 01	2 = 10	3 = 11
0 =00	0	1	2	3
1 =01	1	0	3	2
2 =10	2	3	0	1
3 =11	3	2	1	0

*	0 = 0	1 = 1	2 = x	3 = x+1
0 = 0	0	0	0	0
1 = 1	0	1	2	3
2 = x	0	2	3	1
3 = x+1	0	3	1	2

$2 \cdot 3 = x(x+1) = x^2+x = 1 \bmod x^2+x+1 = 1$

$2 \cdot 2 = x^2 = x+1 \bmod x^2+x+1 = 3$

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Irreducible Polynomials

- Irreducible polynomials cannot be factorized
  - counter-example: $x^2+1 = (x+1)^2 \bmod 2$
- Examples:
  - w=2: x^2+x+1
  - w=4: x^4+x+1
  - w=8: $x^8+x^4+x^3+x^2+1$
  - w=16: $x^{16}+x^{12}+x^3+x+1$
  - w=32: $x^{32}+x^{22}+x^2+x+1$
  - w=64: $x^{64}+x^4+x^3+x+1$

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Fast Multiplication

- Powers laws
  - Consider: $\{2^0, 2^1, 2^2,...\}$
  - $= \{x^0, x^1, x^2, x^3, ...$
  - = exp(0), exp(1), ...
- exp(x+y) = exp(x) exp(y)
- Inverse: log(exp(x)) = x
  - $\log(x \cdot y) = \log(x) + \log(y)$
- x y = exp(log(x) + log(y))
  - Warning: integer addition!!!
- Use tables to compute exponential and logarithm function

Example: GF(16)

$q(x) = x^4+x+1$

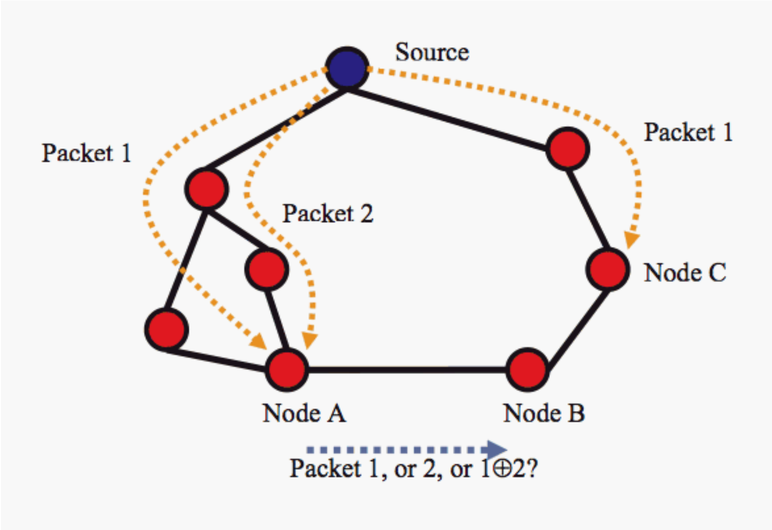
x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
exp(x)	1	x	x^2	x^3	$1+x$	$x+x^2$	x^2+x^3	$1+x+x^3$	$1+x^2$	$x+x^3$	$1+x+x^2$	$x+x^2+x^3$	$1+x+x^2+x^3$	$1+x^2+x^3$	$1+x^3$	1
exp(x)	1	2	4	8	3	6	12	11	5	10	7	14	15	13	9	1

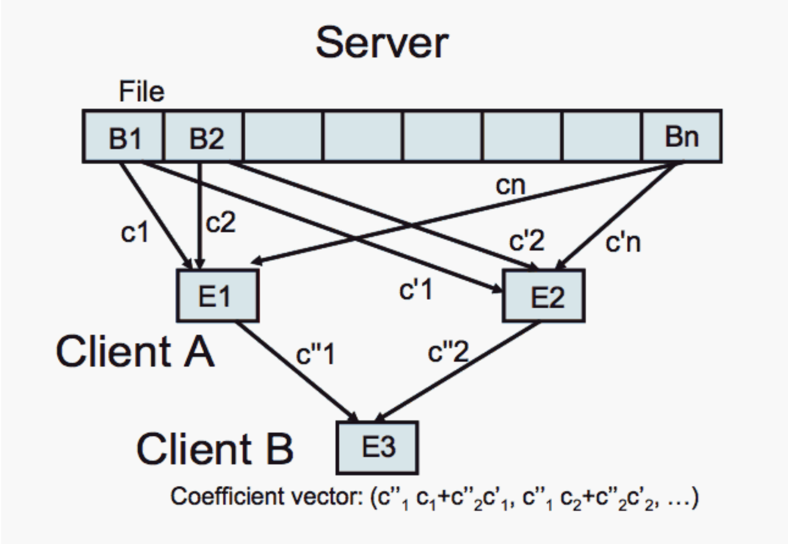
x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
log(x)	0	1	4	2	8	5	10	3	14	9	7	6	13	11	12

- $5 \cdot 12 = \exp(\log(5)+\log(12)) = \exp(8+6) = \exp(14) = 9$
- $7 \cdot 9 = \exp(\log(7)+\log(9)) = \exp(10+14) = \exp(24) = \exp(24-15)$
  $= \exp(9) = 10$

Practical Network Coding Avalanche

- Christos Gkantsidis, Pablo Rodriguez Rodriguez, 2005
- Goal
  - Overcoming the Coupon-Collector-Problem
    - a file of m parts can be always reconstructed if at least m network codes have been received
  - Optimal transmission of files within the available bandwidth
- Method
  - Use codes as linear combinations of a file
    - Produced code contains the vector and the variables
  - During the distribution the linear combination are re-combined to new parts
  - The receiver collects the linear combinations
  - and reconstructs the original file using matrix operations





Coding and Decoding

- File: $x_1, x_2, \ldots, x_m$
- Codes: $y_1, y_2, \ldots, y_m$
- Random Variables r_{ij}

$$(r_{i1} r_{i2} \ldots r_{im}) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = y_i$$

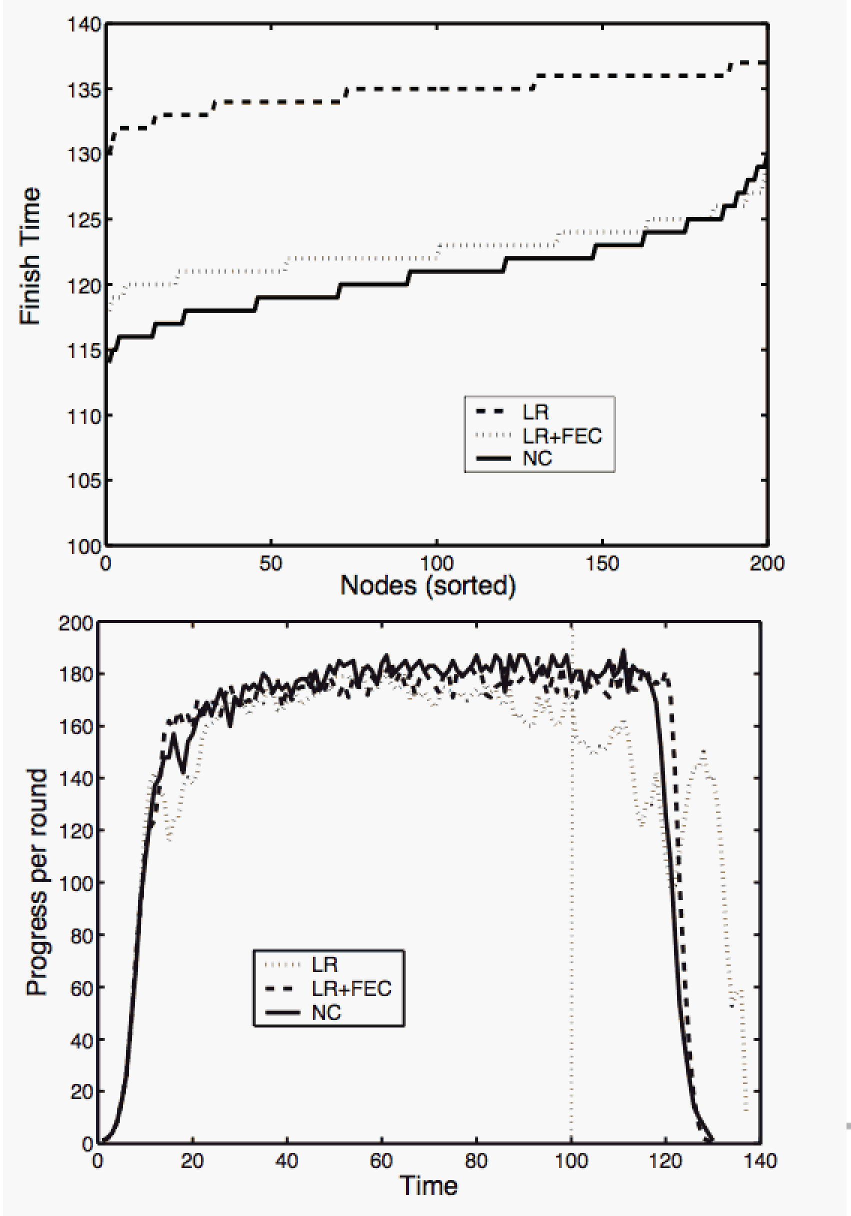
$$\begin{pmatrix} r_{11} & \cdots & r_{1m} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mm} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$$

- If the matrix is invertable then

$$\begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} r_{11} & \cdots & r_{1m} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mm} \end{pmatrix}^{-1} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$$

Speed of Network-Coding

- Comparison
  - Network-Coding (NC) versus
  - Local-Rarest (LR) and
  - Local-Rarest+Forward-Error-Correction (LR+FEC)



Problems of Network-Coding

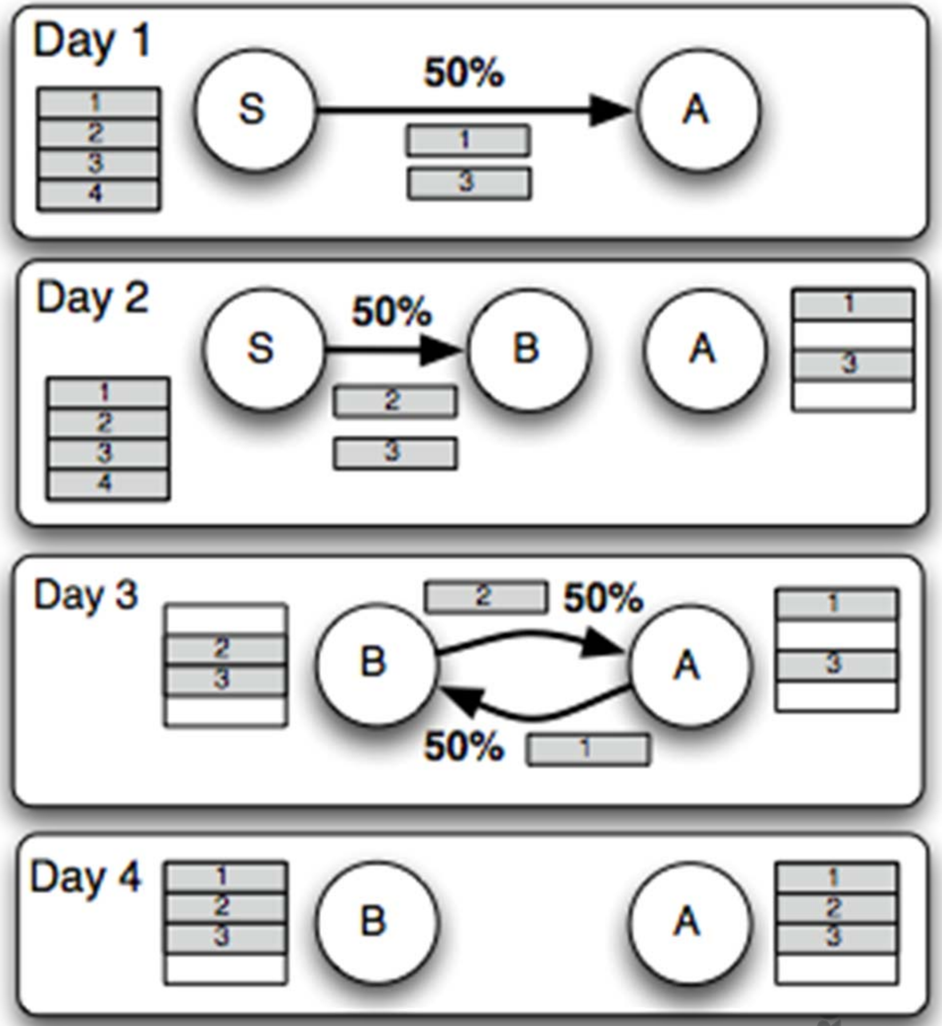
- Overhead of storing of variables
  - per block one variable vector
  - e.g. 4 GB file with 100 kB blocks
    - 4 GB/100 KB = 40 kB
    - Overhead of 40%
  - better: 4 GB und 1 MB-Block
    - 4kB Overhead = 0,4%
- Overhead of Decoding
  - Inversion of a m x m- Matrix needs time $O(m^3)$
- Read/Write Accesses
  - For writing m blocks each part must be read m times
  - Disk access is much slower than memory access

Pair-Coding

- Paircoding: Improving File Sharing Using Sparse Network Codes Christian Ortolf Christian Schindelhauer Arne Vater
- Model Description
  - Round model
    - complete information of the system can be described by file sharing state γ(p,t) of each peer p after round t.
      - It is defined as the set of all code blocks that are available at peer p after round t.
  - Progress of a peer
    - number of indepdendent code blocks at a peer at round t
  - Availability at a set of peers
    - number of independent code blocks at the peers of the set divided by the number of code blocks

Scenario

- Round model
  - In each round each peer can upload and download a bounded number of blocks of the document
- Peers do not know the future
- Progress
  - number of (independent encoded) blocks that are available at the end of the rounds



Policy and Outperforming

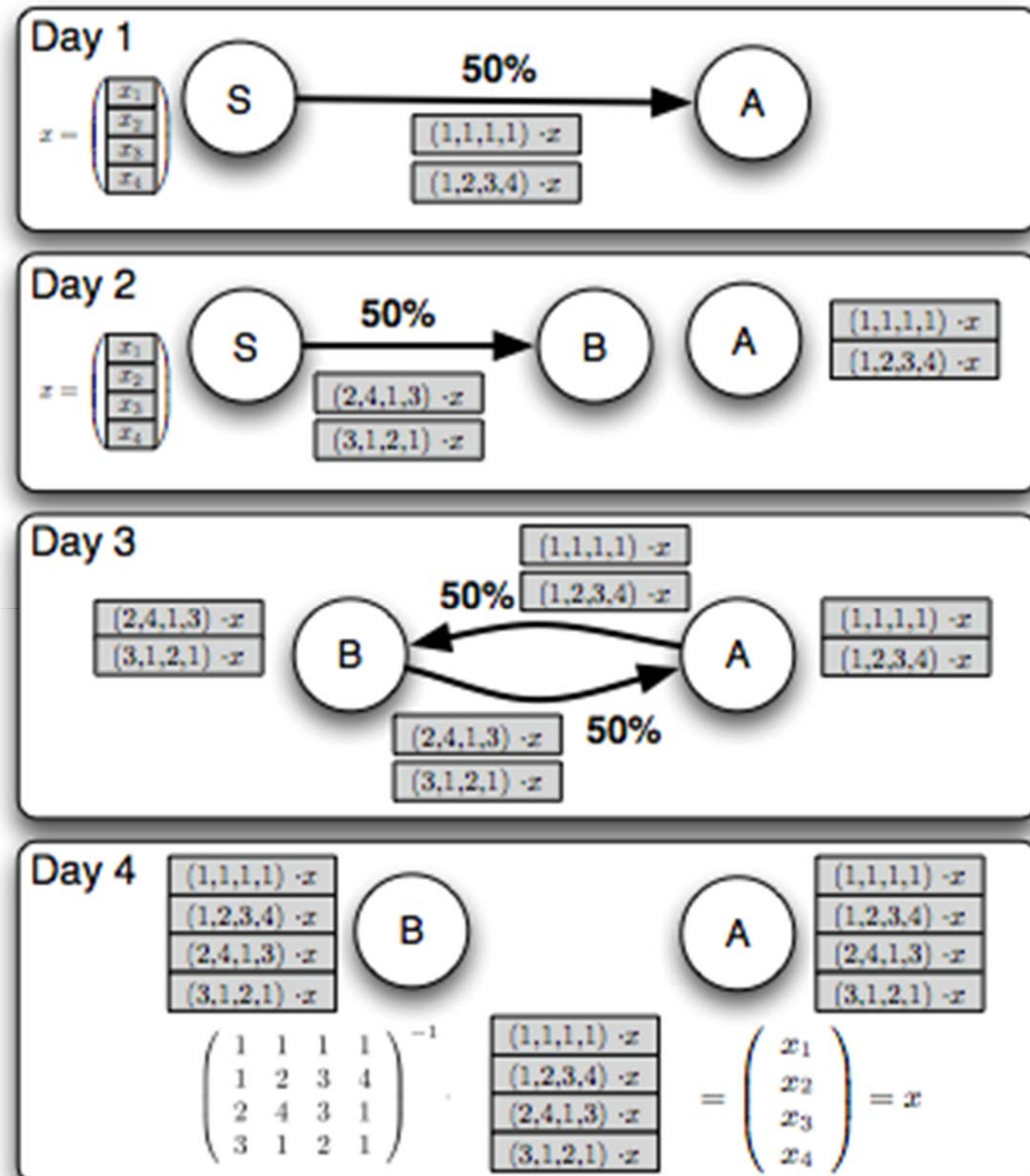
- Policy of a scheme
  - algorithmic choice of encoding of a block in a round
  - determine the efficiency of a scheme
- Policies of Bittorrent
  - chosen to optimize throughput and fairness
- A scheme A is at least as good as B

$$A \geq B$$

  - if for every scenario and every policy of B there is a policy in A such that A performs as well as B in all scenarios.

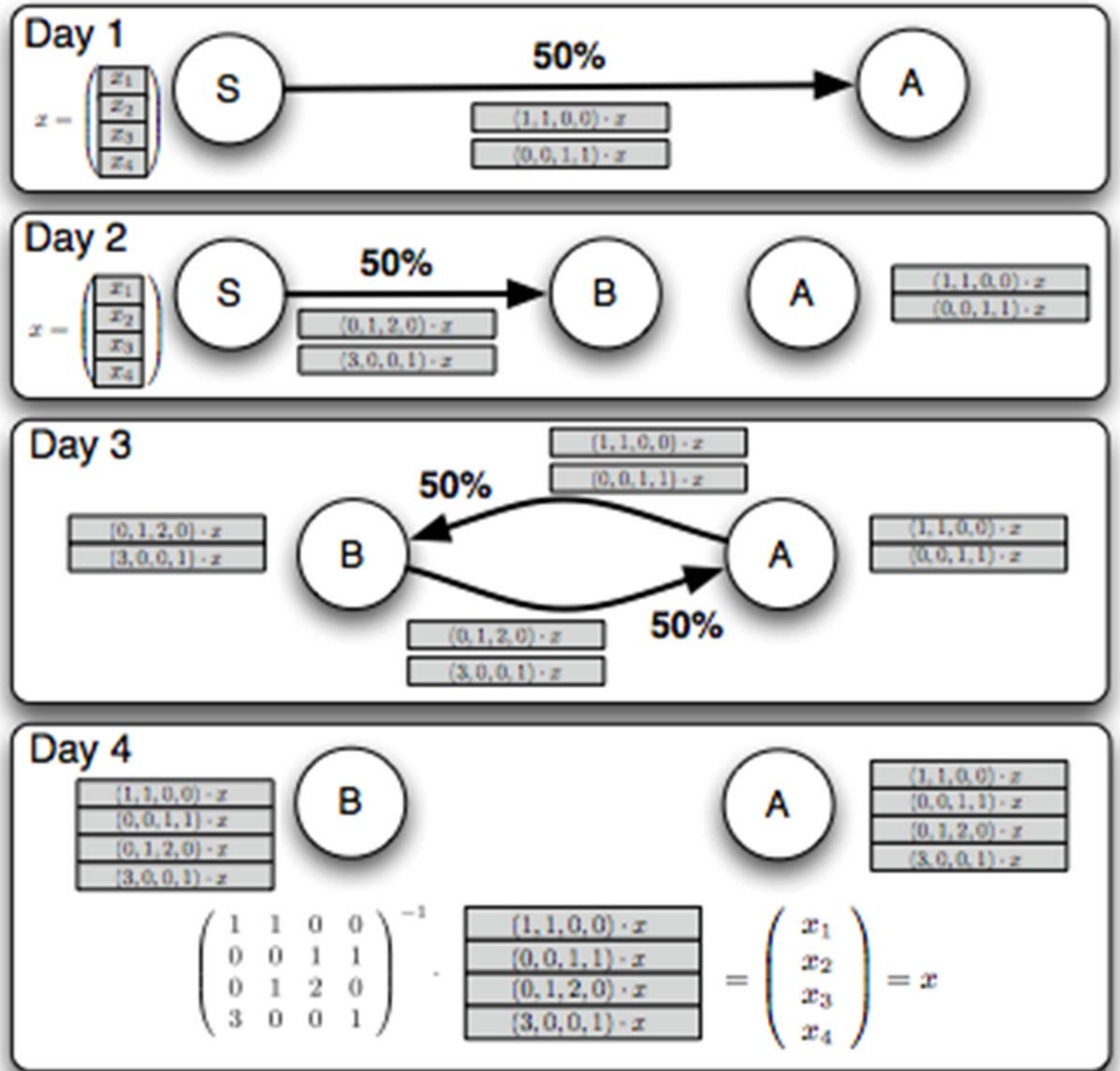
Network Coding

- Practical Network Coding
  - is the best possible method
  - as long as the underlying finite base is large enough
- But:
  - Decoding needs O(m) read/write operations



Pair Coding

- Pair Coding
  - is a reduced form of Network Coding
  - Only two components are combined
- Theorem
  - For all scenarios Pair-Coding is at least as efficient as Bittorrent
  - For some scenarios Pair-Coding is more efficient than Bittorrent
  - Encoding and Decoding can be performed with (almost) linear number of Read/Write-Operations



The Random Policy

- Scenario
  - one seeder
  - one downloading peer
- Seeder sends a random block in each round

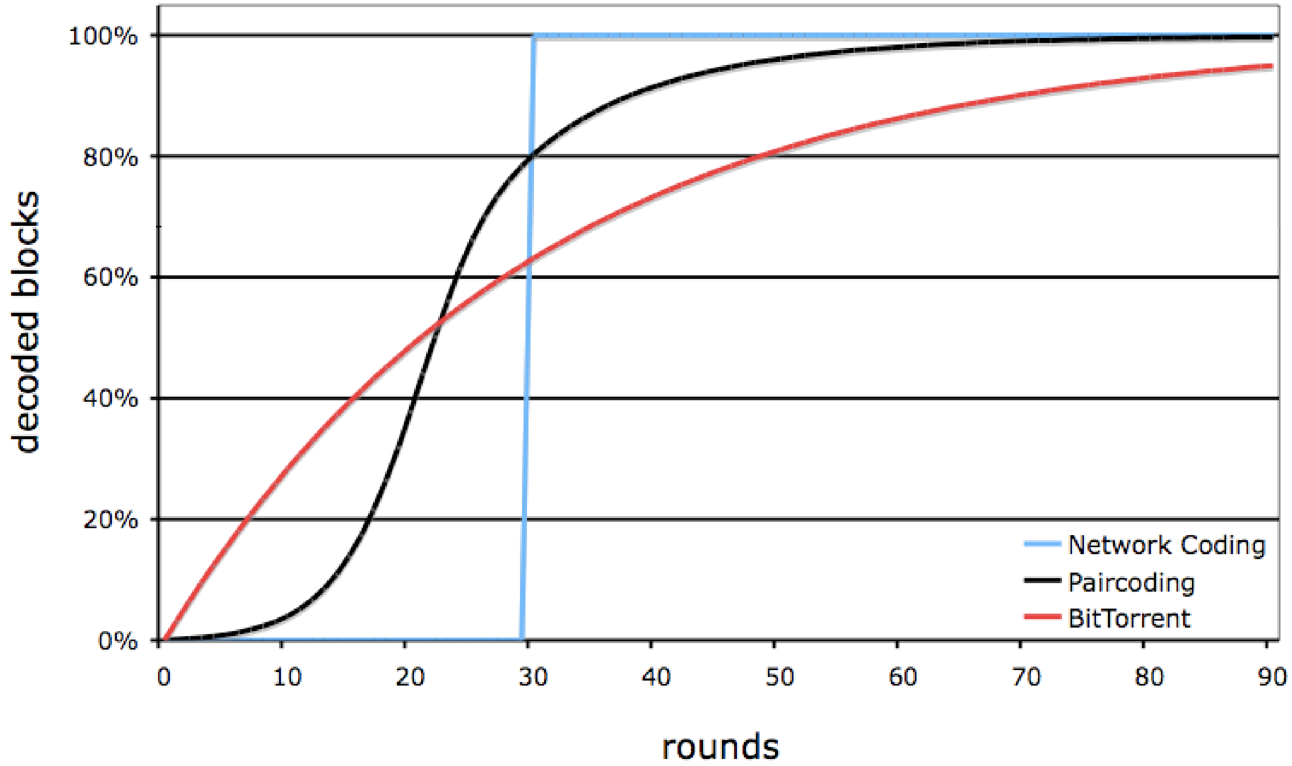


Figure 8. Simulation of decodability for one peer

Availability

- Scenario:
  - p peers
  - one seeder
  - every peer receives n/p+1 blocks from the seed
  - then the seed disappears

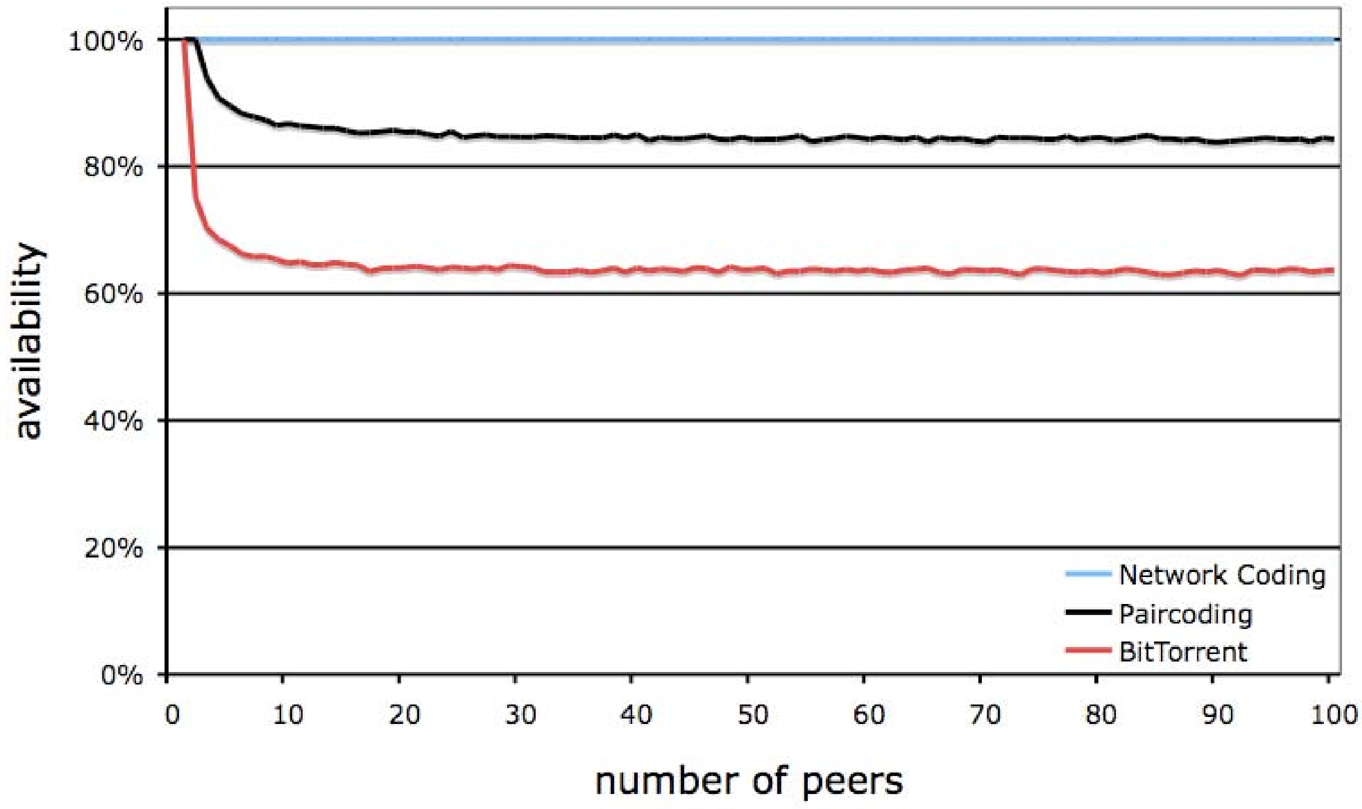


Figure 9. Simulation of availabilty for increasing number of peers

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Christian Schindelhauer
Technical Faculty
Computer-Networks and Telematics
University of Freiburg