Exercise 1 *Network Coding*

We want to transmit a vector \( d \) composed of 3 bits. In order to do this, we distribute the data to 3 nodes and each node stores 2 different linear combination of the bits \( k_1 b_1 + k_2 b_2 + k_3 b_3 \) and \( k_4 b_1 + k_5 b_2 + k_6 b_3 \).

1. Assume one peer leaves the network. The remaining peers have the following combinations:

\[
\begin{pmatrix}
0 \\
0 \\
1 \\
1
\end{pmatrix} = \begin{pmatrix}
0 & 1 & 0 \\
1 & 1 & 1 \\
1 & 1 & 0 \\
1 & 0 & 1
\end{pmatrix} d
\]  
(1)

What is the vector \( d \)?

2. If we remove the last row of the matrix, can we still recover the data?

3. What is the probability that the data cannot be recovered if we have two peers with two different linear combinations over \( GF[2^4] \)?

4. Assume we add an extra bit to \( d \). The message can be recovered if we have at least 3 bits of \( d \). What is the probability that we cannot recover the data?