Peer-to-Peer Networks

03 CAN (Content Addressable Network)

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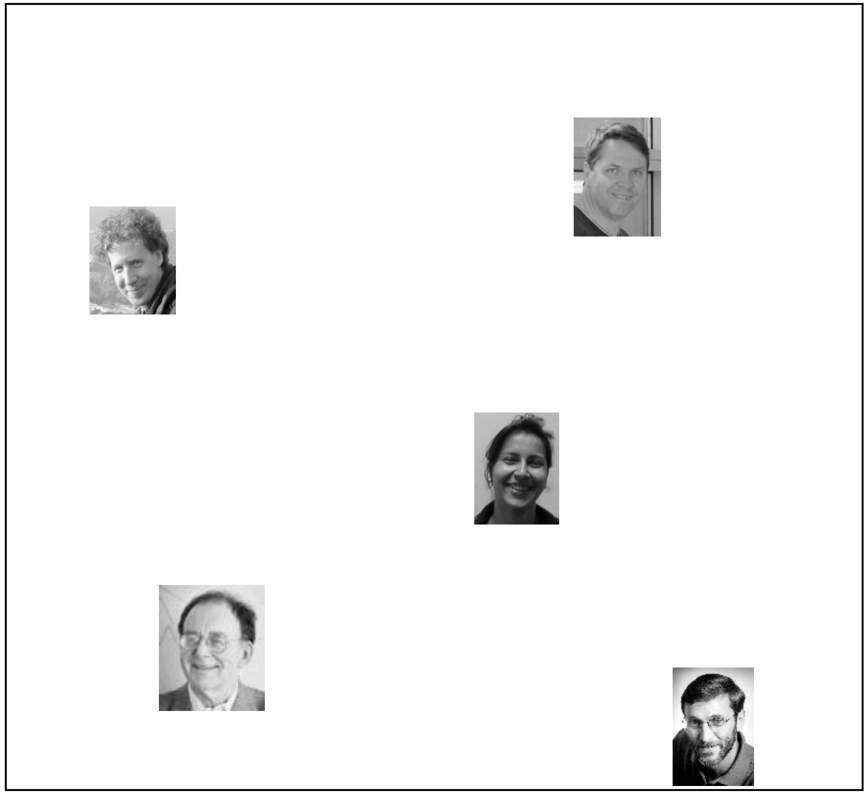
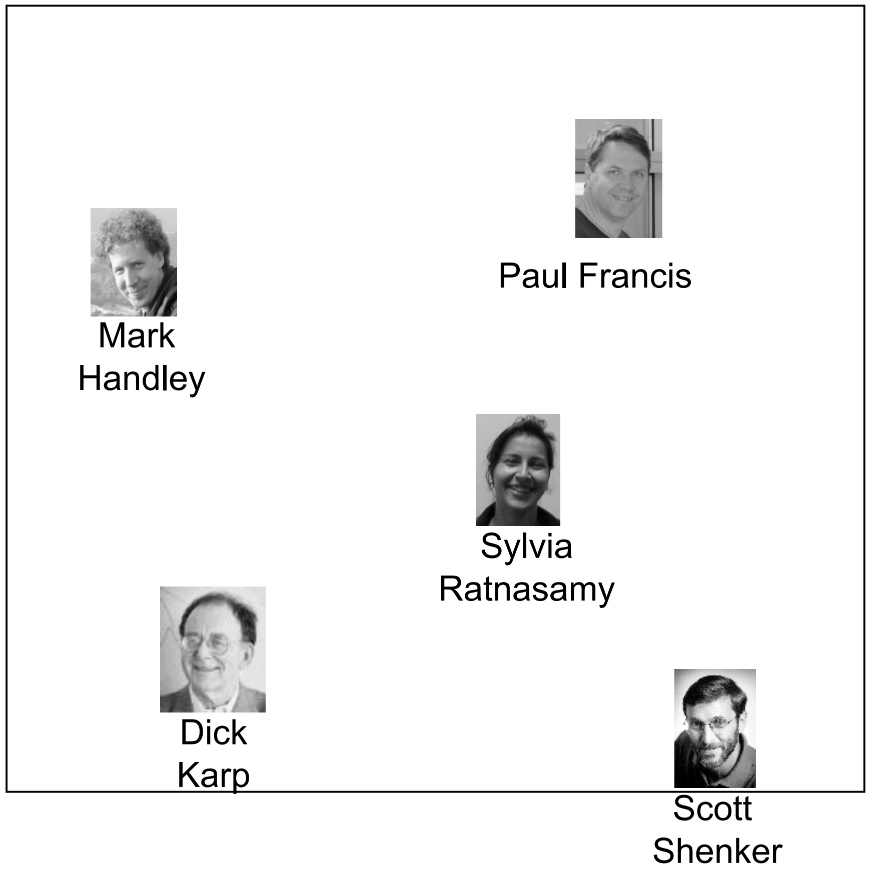
CAN Playground

- Index entries are mapped to the square $[0,1]^2$
  - using two hash functions to the real numbers
  - according to the search key
- Assumption:
  - hash functions behave a like a random mapping



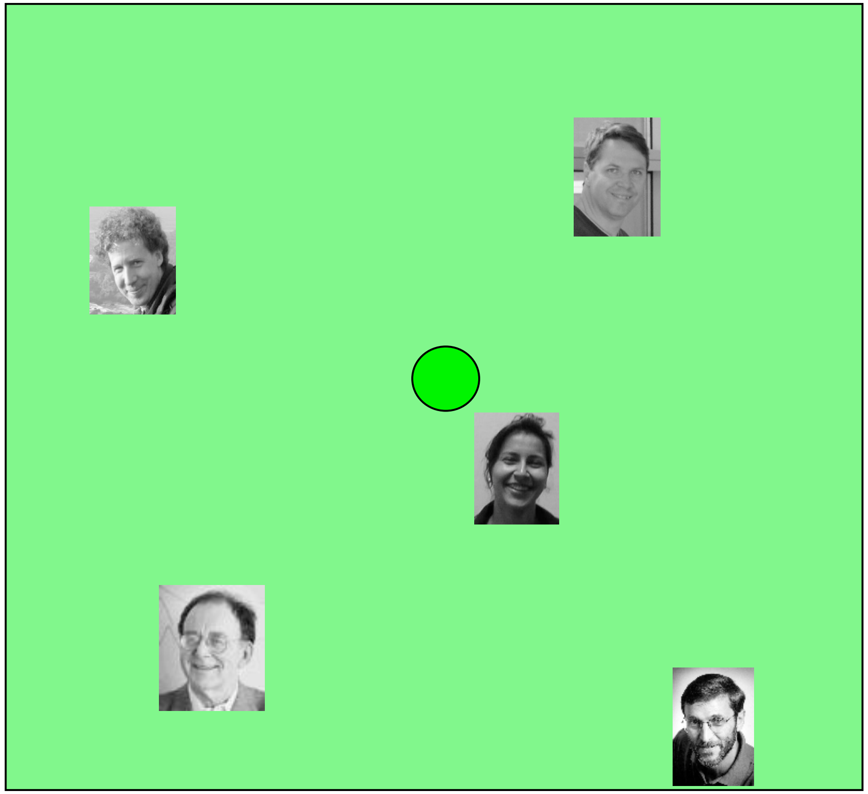
CAN Index Entries

- Index entries are mapped to the square [0,1]2
  - using two hash functions to the real numbers
  - according to the search key
- Assumption:
  - hash functions behave a like a random mapping
- Literature
  - Ratnasamy, S., Francis, P., Handley, M., Karp, R., Shenker, S.: A scalable content-addressable network. In: Computer Communication Review. Volume 31., Dept. of Elec. Eng. and Comp. Sci., University of California, Berkeley (2001) 161–172



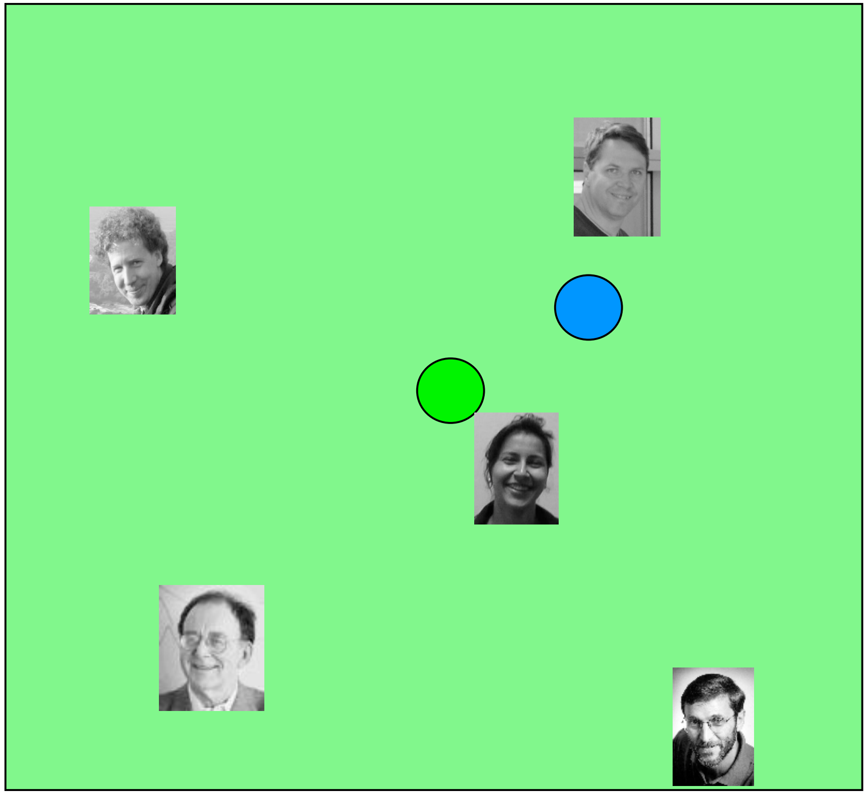
First Peer in CAN

- In the beginning there is one peer owning the whole square
- All data is assigned to the (green) peer



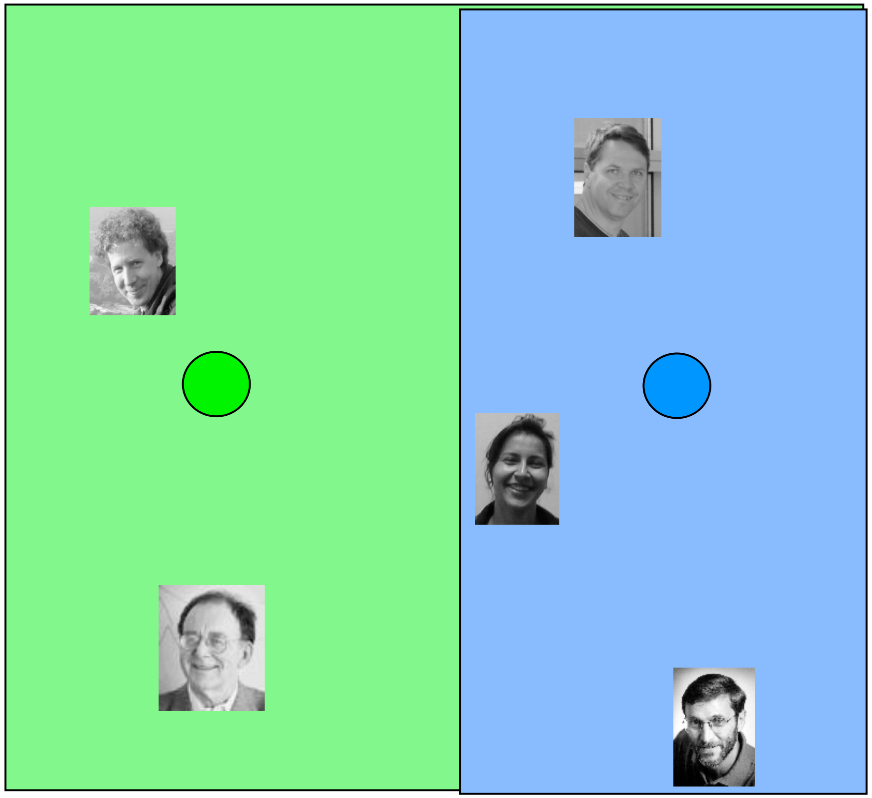
CAN: The 2nd Peer Arrives

- The new peer chooses a random point in the square
  - or uses a hash function applied to the peers Internet address
- The peer looks up the owner of the point
  - and contacts the owner

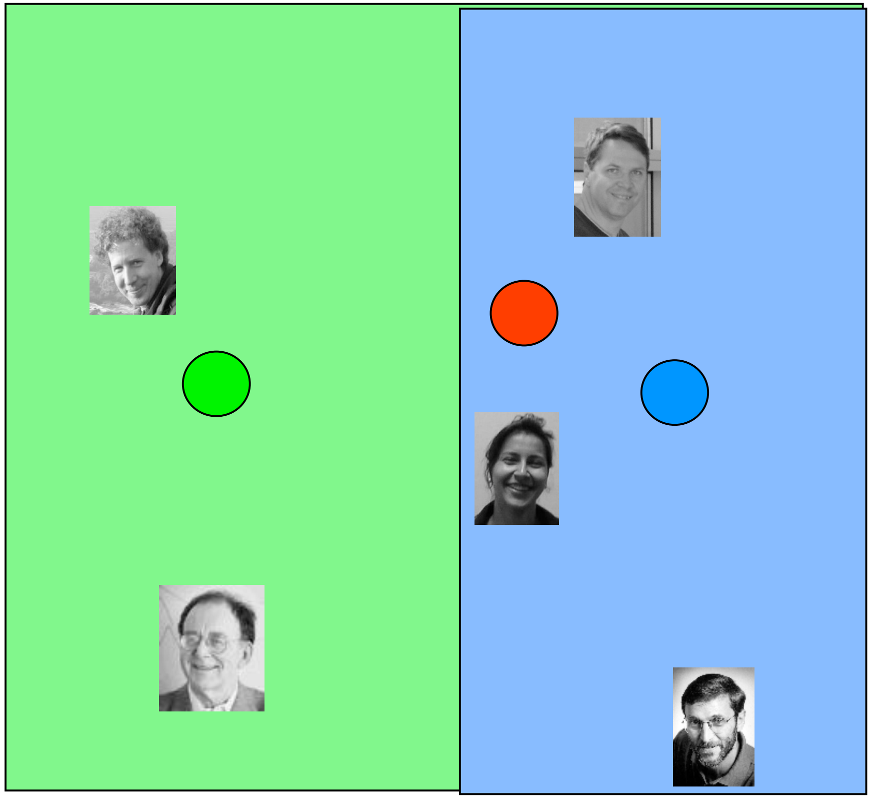


CAN: 2nd Peer Has Settled Down

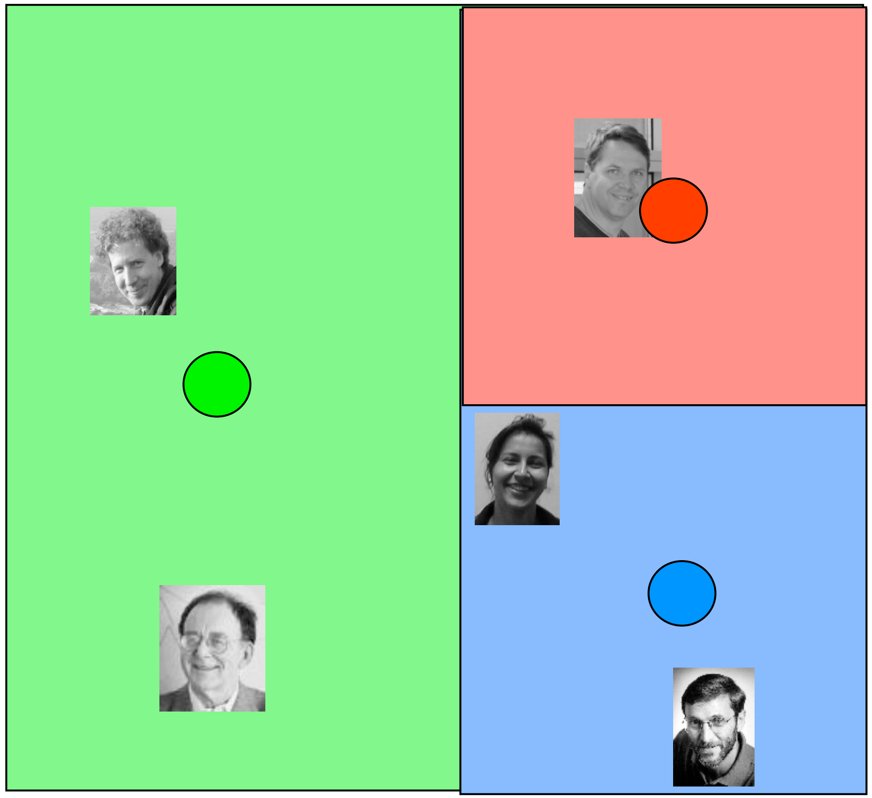
- The new peer chooses a random point in the square
  - or uses a hash function applied to the peers Internet address
- The peer looks up the owner of the point
  - and contacts the owner
- The original owner divides his rectangle in the middle and shares the data with the new peer



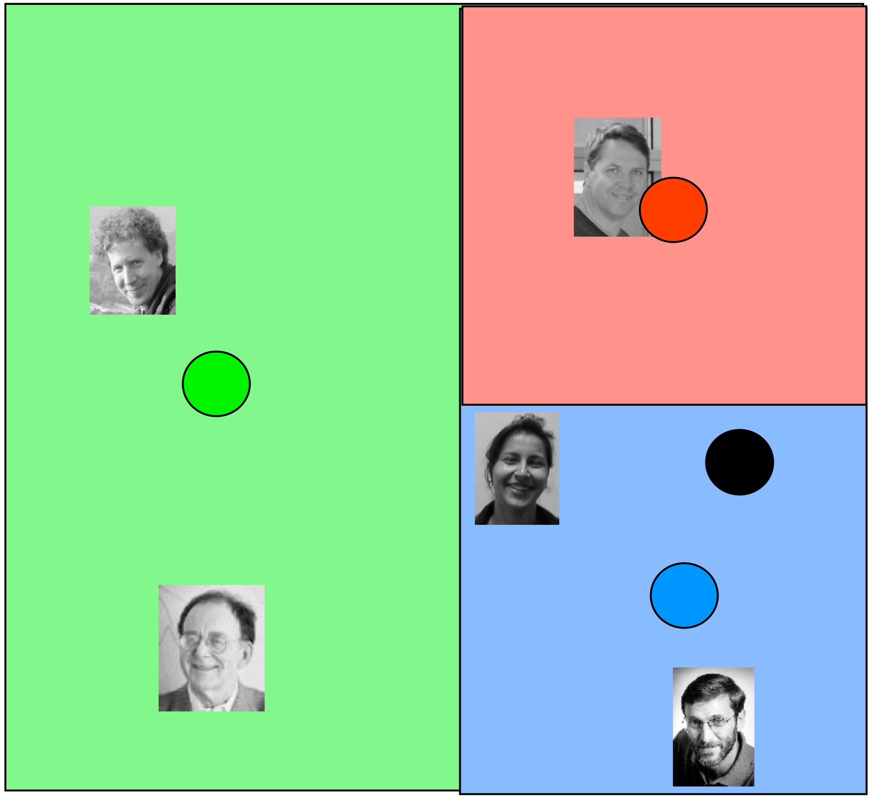
3rd Peer



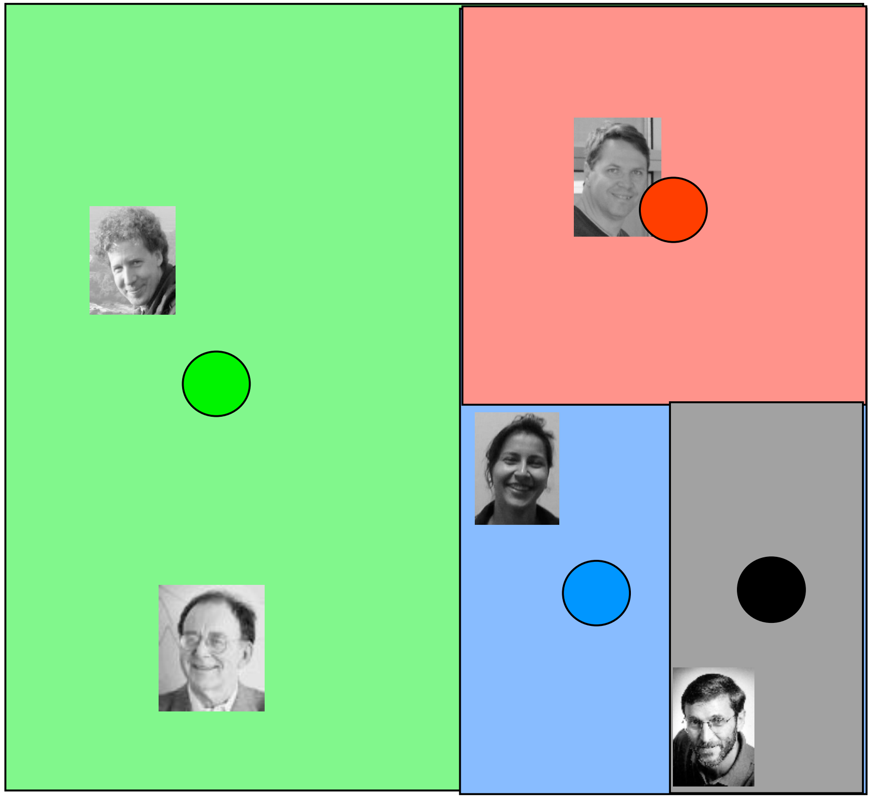
CAN: 3rd Peer



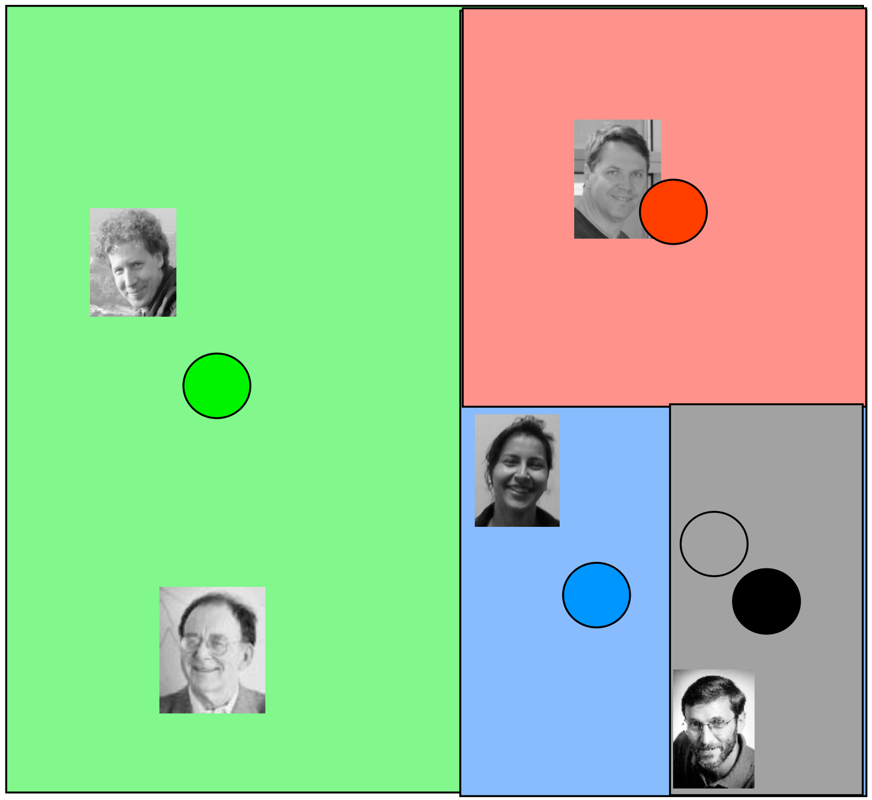
CAN: 4th Peer



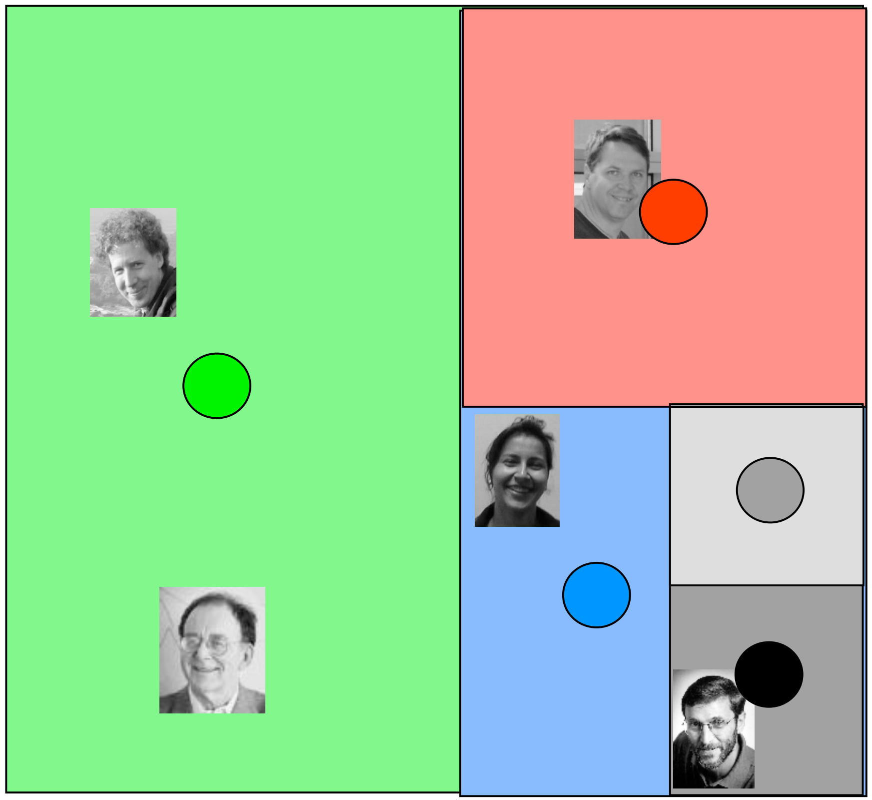
CAN: 4th Peer Added



CAN: 5th Peer



CAN: All Peers Added



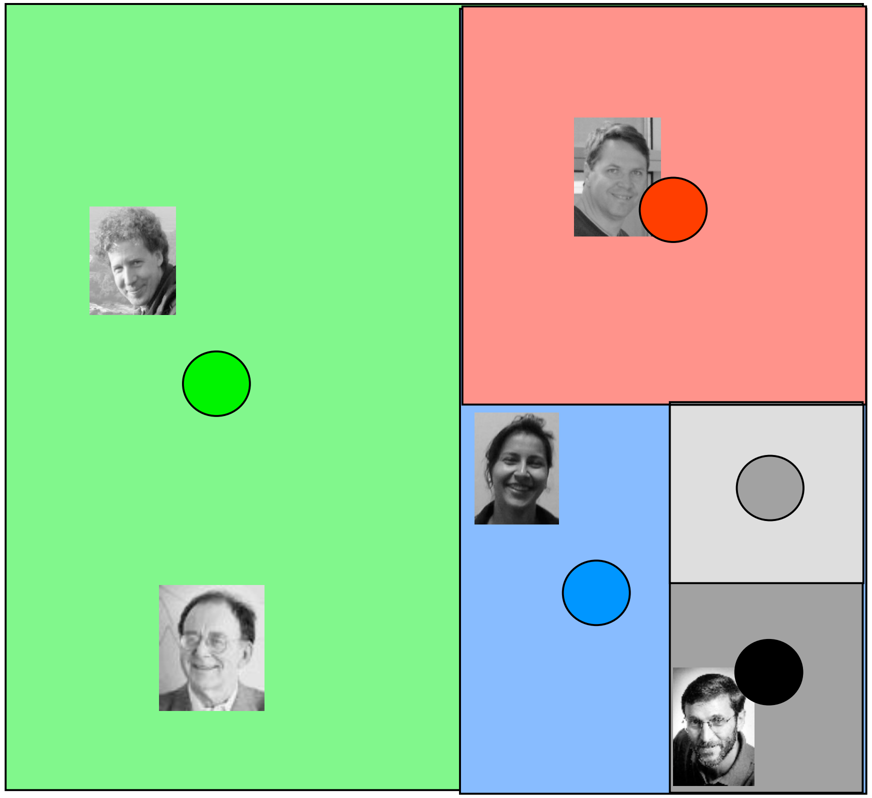
On the Size of a Peer‘s Area

- R(p): rectangle of peer p
- A(p): area of the R(p)
- n: number of peers
- area of playground square: 1
- Lemma
  - For all peers we have

$$E[A(p)] = \frac{1}{n}$$

- Lemma
  - Let $P_{R,n}$ denote the probability that no peers falls into an area R. Then we have

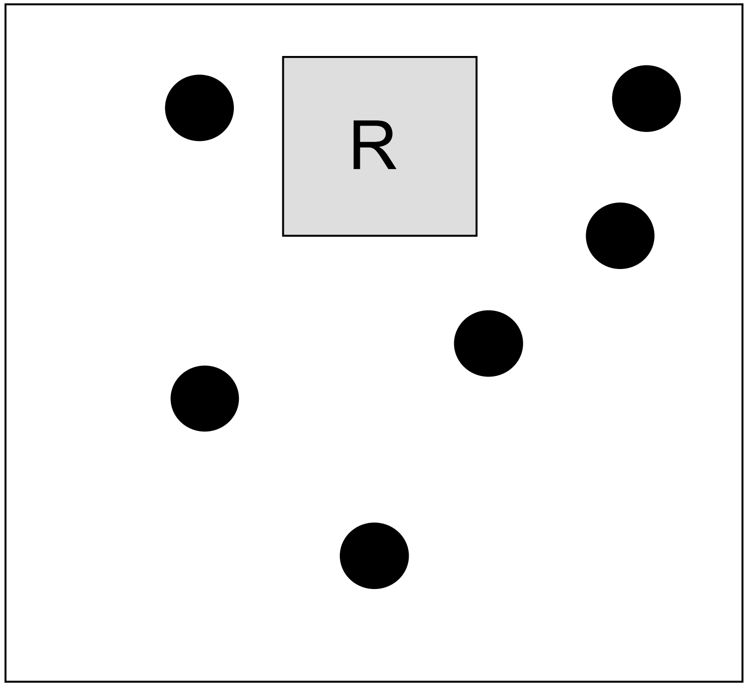
$$P_{R,n} \leq e^{-n\mathrm{Vol}(R)}$$



An Area Not being Hit

- Lemma
  - Let $P_{R,n}$ denote the probability that no peers falls into an area R. Then $$P_{R,n} \leq e^{-n\mathrm{Vol}(R)}$$
- Proof
  - Let x=Vol(R)
  - The probability that a peer does not fall into R is $1-x$
  - The probability that n peers do not fall into R is $(1-x)^n$
  - So, the probability is bounded by
$$m > 1 \;:\; \left(1 - \frac{1}{m}\right)^m \leq \frac{1}{e}$$
  - because

$$(1-x)^n = ((1-x)^{\frac{1}{x}})^{nx} \leq e^{-nx}$$



How Fair Are the Data Balanced

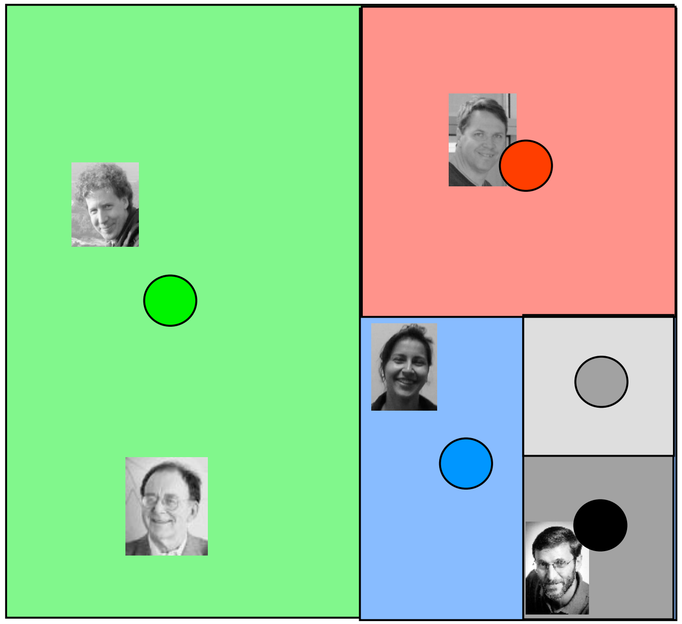
- Lemma
  - With probability n^{-c} a rectangle of size (c ln n)/n is not further divided
- Proof
  - Let $P_{R,n}$ denote the probability that no peers falls into an area R. Then we have

$$P_{R,n} \leq e^{-n\mathrm{Vol}(R)}$$

- Every peer receives at most c (ln n) m/n elements
  - if all m elements are stored equally distributed over the area
- While the average peer stores m/n elements

$$P_{R,n} \leq e^{-n\frac{c\ln n}{n}} = e^{-c\ln n} = n^{-c}$$

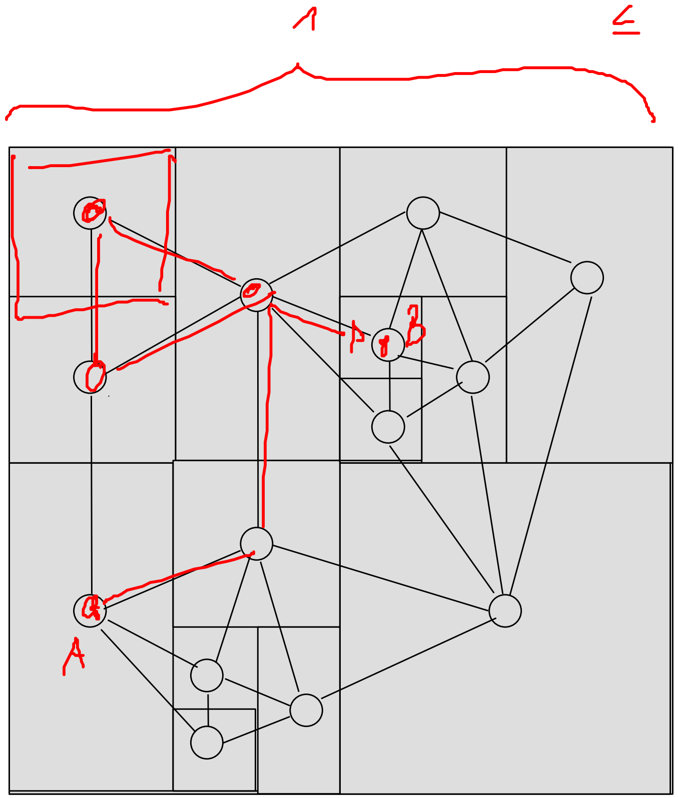
- So, the number of data stored on a peer is bounded by c (ln n) times the average amount



Network Structure of CAN

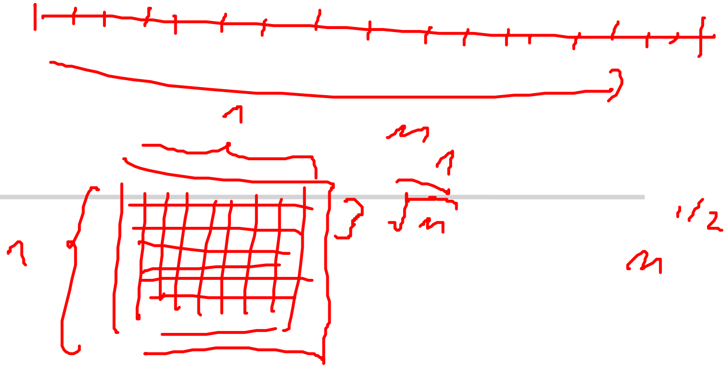
$\sim \frac{1}{n}$

- Let d be the dimension of the square, cube, hyper-cube
  - 1: line
  - 2: square
  - 3: cube
  - 4: ...
- Peers connect
  - if the areas of peers share a (d-1)-dimensional area
  - e.g. for d=2 if the rectangles touch by more than a point

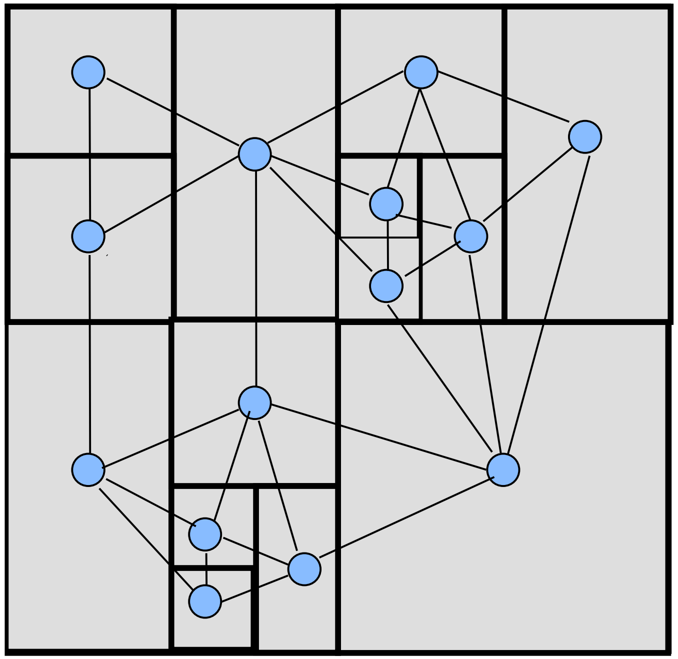


$\leq \frac{c \cdot \ln n}{n}$ w.h.p.

Lookup in CAN

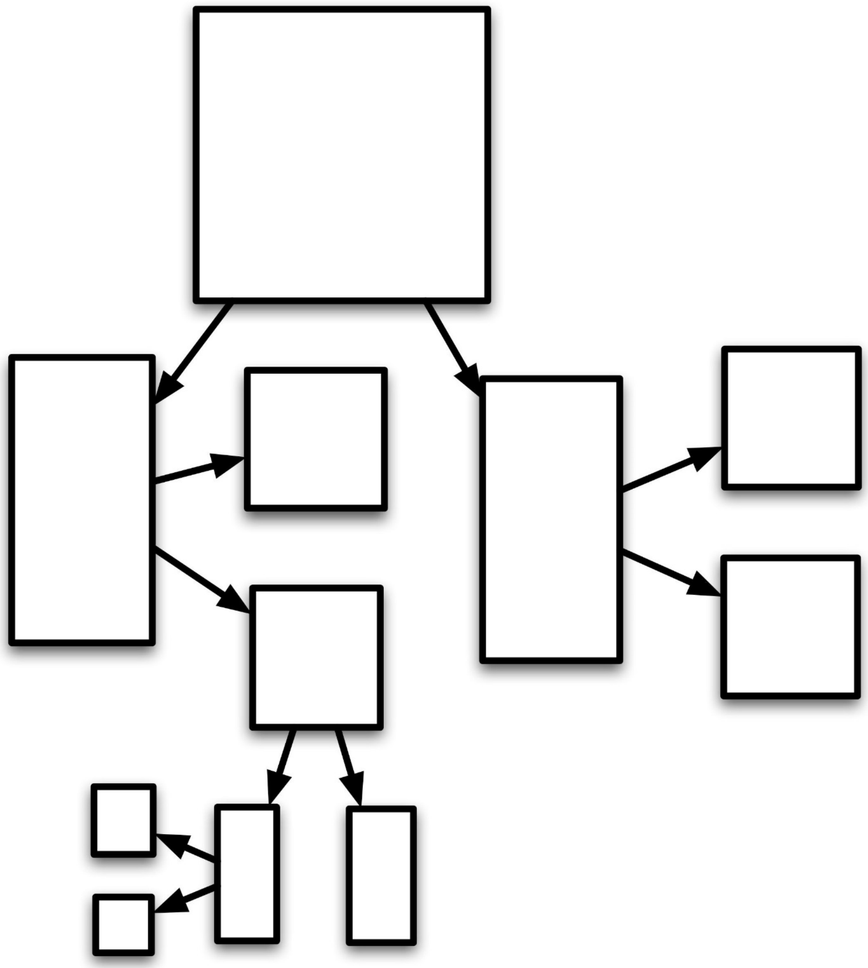


- Compute the position of the index using the hash function on the key value
- Forward lookup to the neighbored peer which is closer to the index
- Expected number of hops for CAN in d dimensions:
  - $O(n^{1/d})$
- Average degree of a node
  - $O(d)$



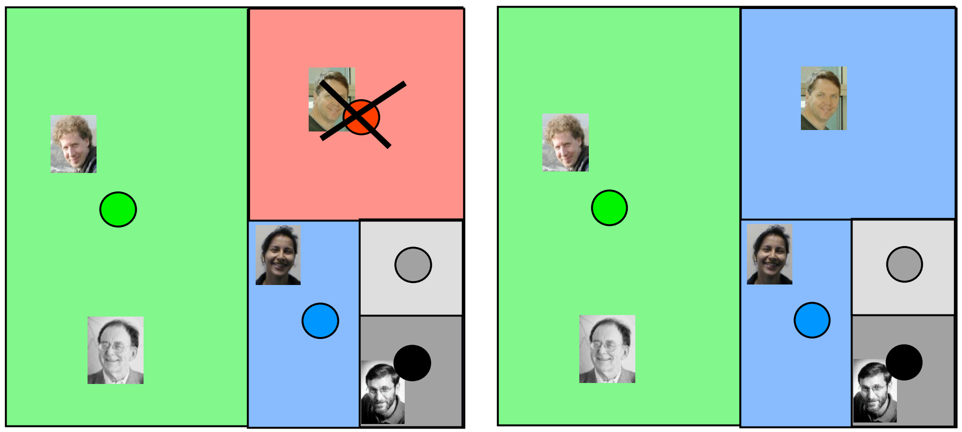
Insertions in CAN = Random Tree

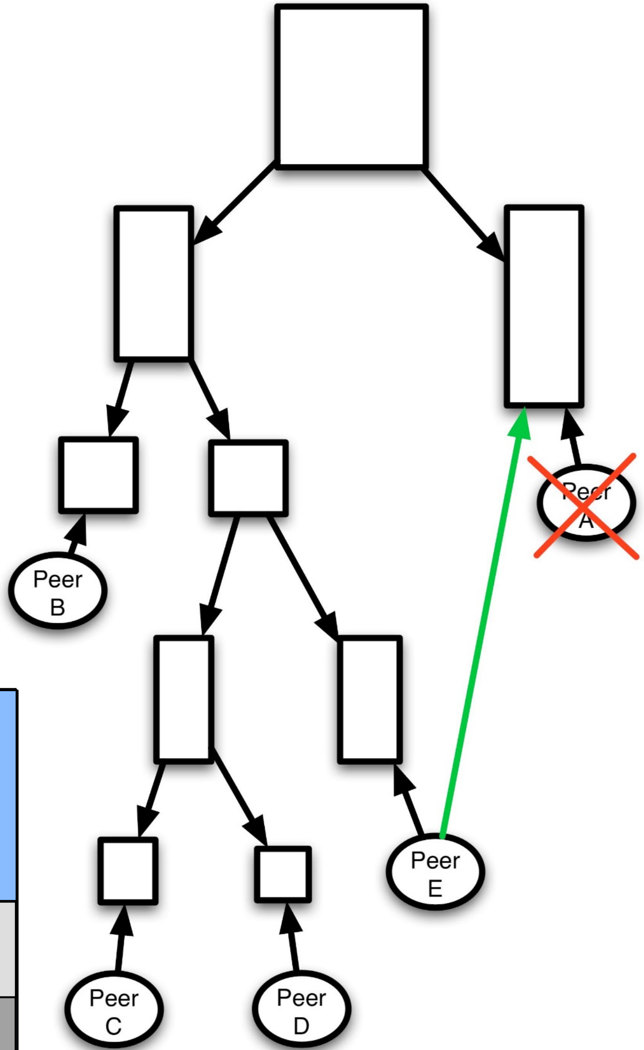
- Random Tree
  - new leaves are inserted randomly
  - if node is internal then flip coin to forward to left or right sub-tree
  - if node is leaf then insert two leafs to this node
- Depth of Tree
  - in the expectation: O(log n)
  - Depth O(log n) with high probability, i.e. $1-n^{-c}$
- Observation
  - CAN inserts new peers like leafs in a random tree



Leaving Peers in CAN

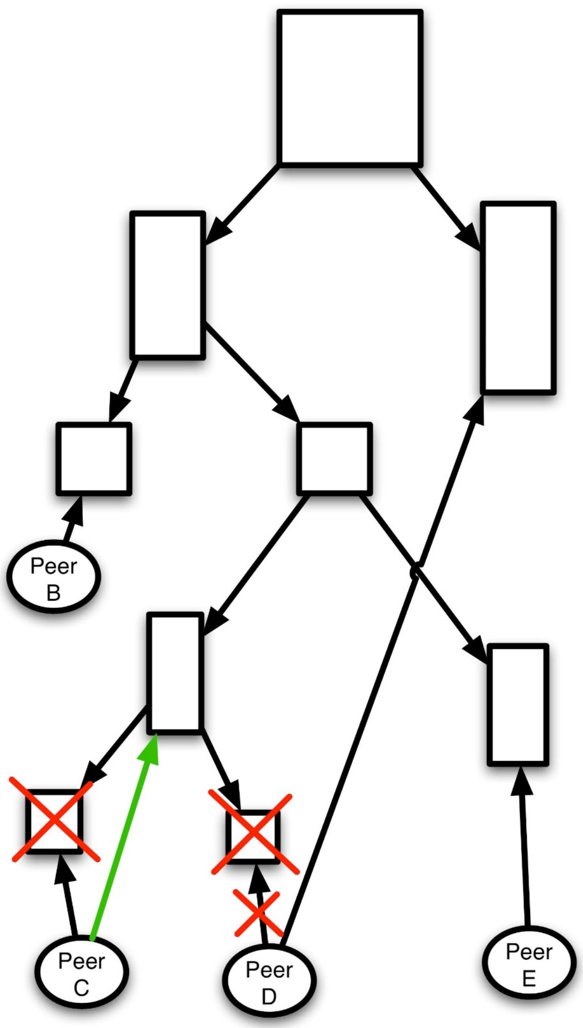
- If a peer leafs
  - he does not announce it
- Neighbors continue testing on the neighborhood
  - to find out whether peer has left
  - the first neighboir who finds a missing neighbor takes over the area of the missing peer
- Peers can be responsible for many rectangles
- Repeated insertions and deletions of peers leed to fragmentation





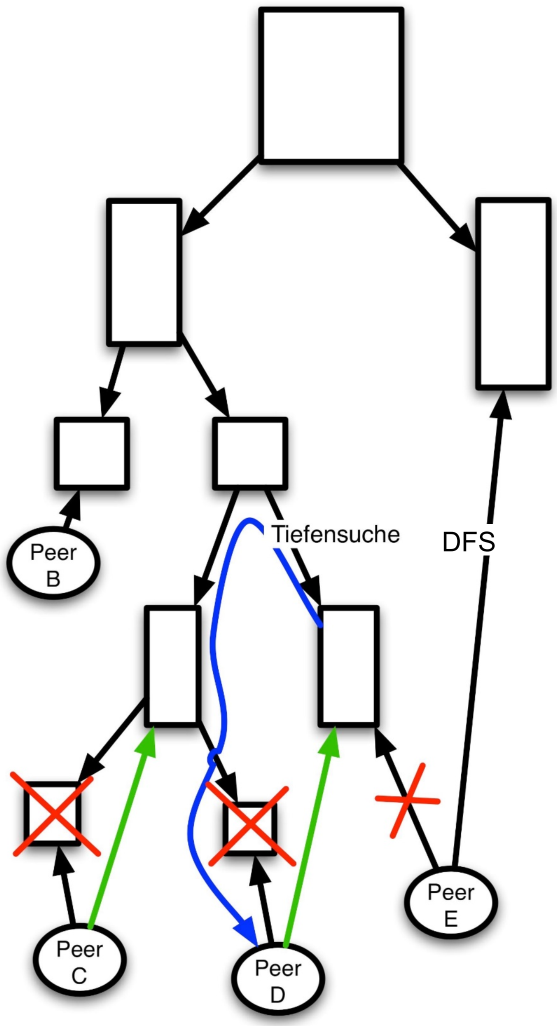
Defragmentation — The Simple Case

- To heal fragmented areas
  - from time to to time areas are freshly assigned
- Every peer with at least two zones
  - erases smalles zone
  - finds replacement peer for this zone
- 1st case: neighboring zone is undivided
  - both peers are leafs in the random tree
  - transfer zone to the neighbor



Defragmentation — The Difficult Case

- Every peer with at least two zones
  - erases smalles zone
  - finds replacement peer for this zone
- 2nd case: neighboring zone is further divided
  - Perform DFS (depth first search) in neighbor tree until two neighbored leafs are found
  - Transfer the zone to one leaf which gives up his zone
  - Choose the other leaf to receive the latter zone



Improvements for CAN

- More dimensions
- Multiples realities
- Distance metric for routing
- Overloading of zones
- Multiple hasing
- Topology adapted network construction
- Fairer partitioning
- Caching, replication and hot-spot management

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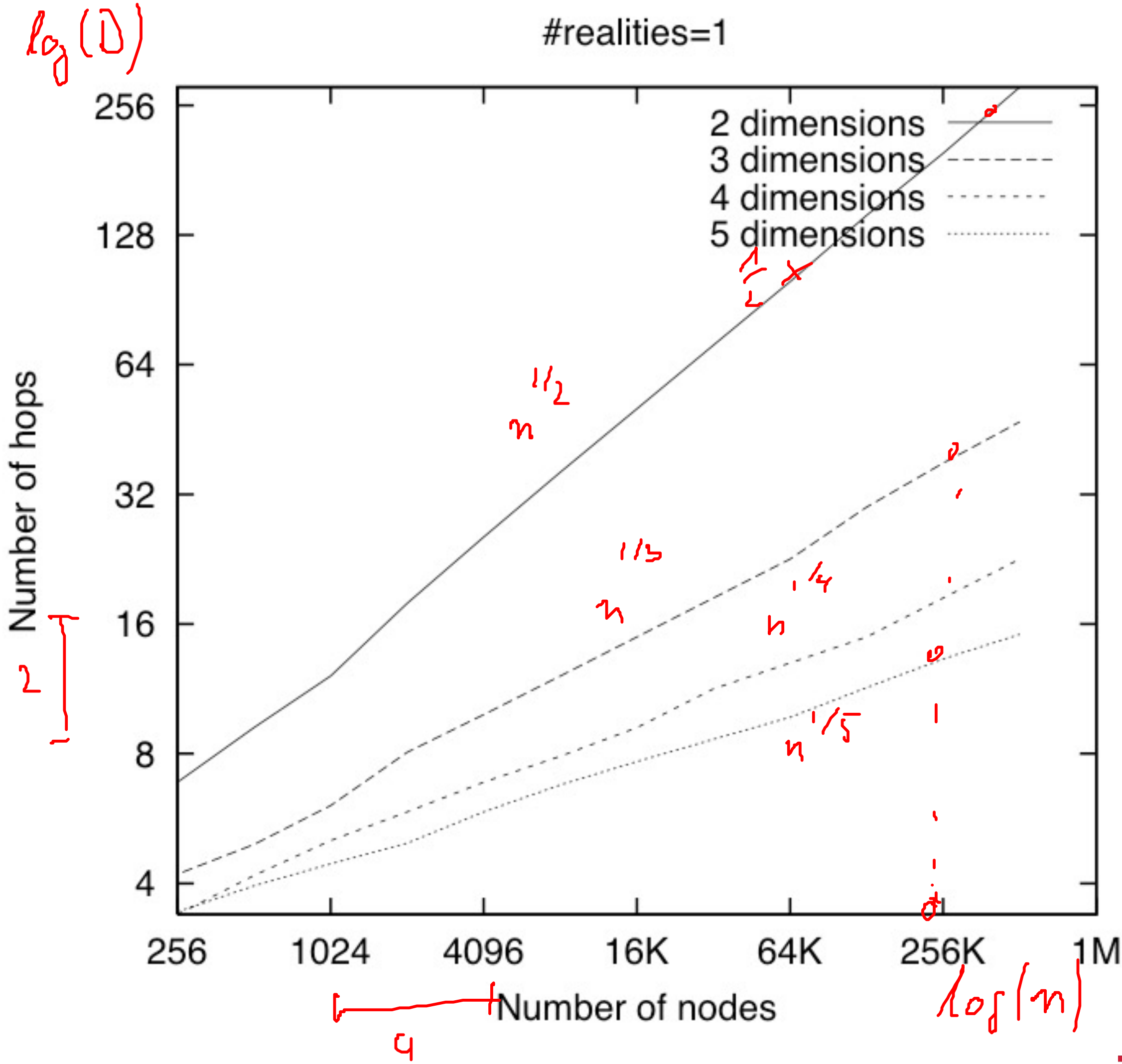
Higher Dimensions

$$\underbrace{\log(D)}_{y} = \log(n^{1/2}) = \frac{1}{2}\underbrace{\log n}_{x}$$

- Let d be the dimension of the square, cube, hyper-cube
  - 1: line
  - 2: square
  - 3: cube
  - 4: ...
- The expected path length is $O(n^{1/d})$
- Average number of neighbors $O(d)$



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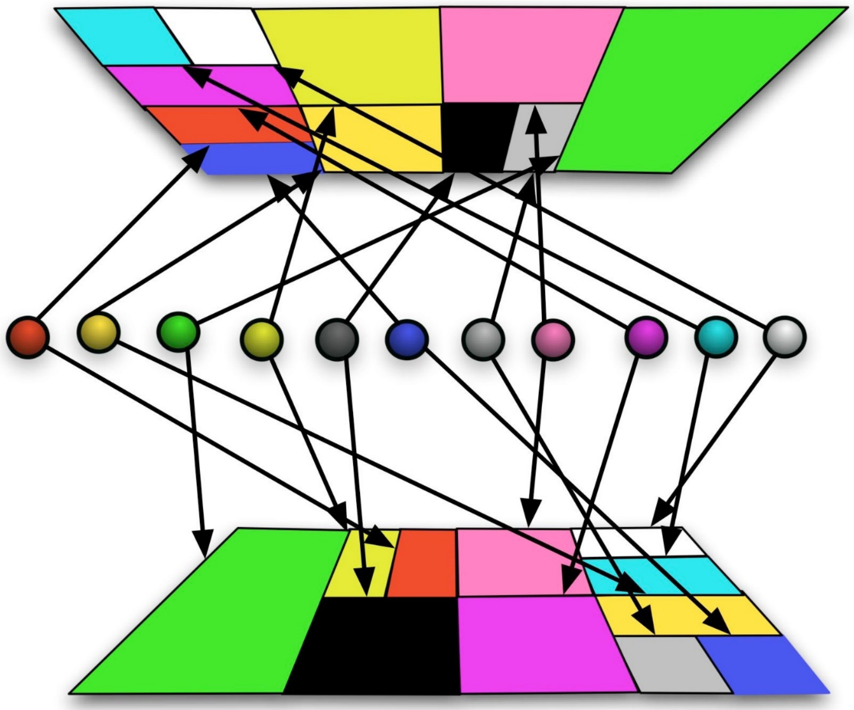
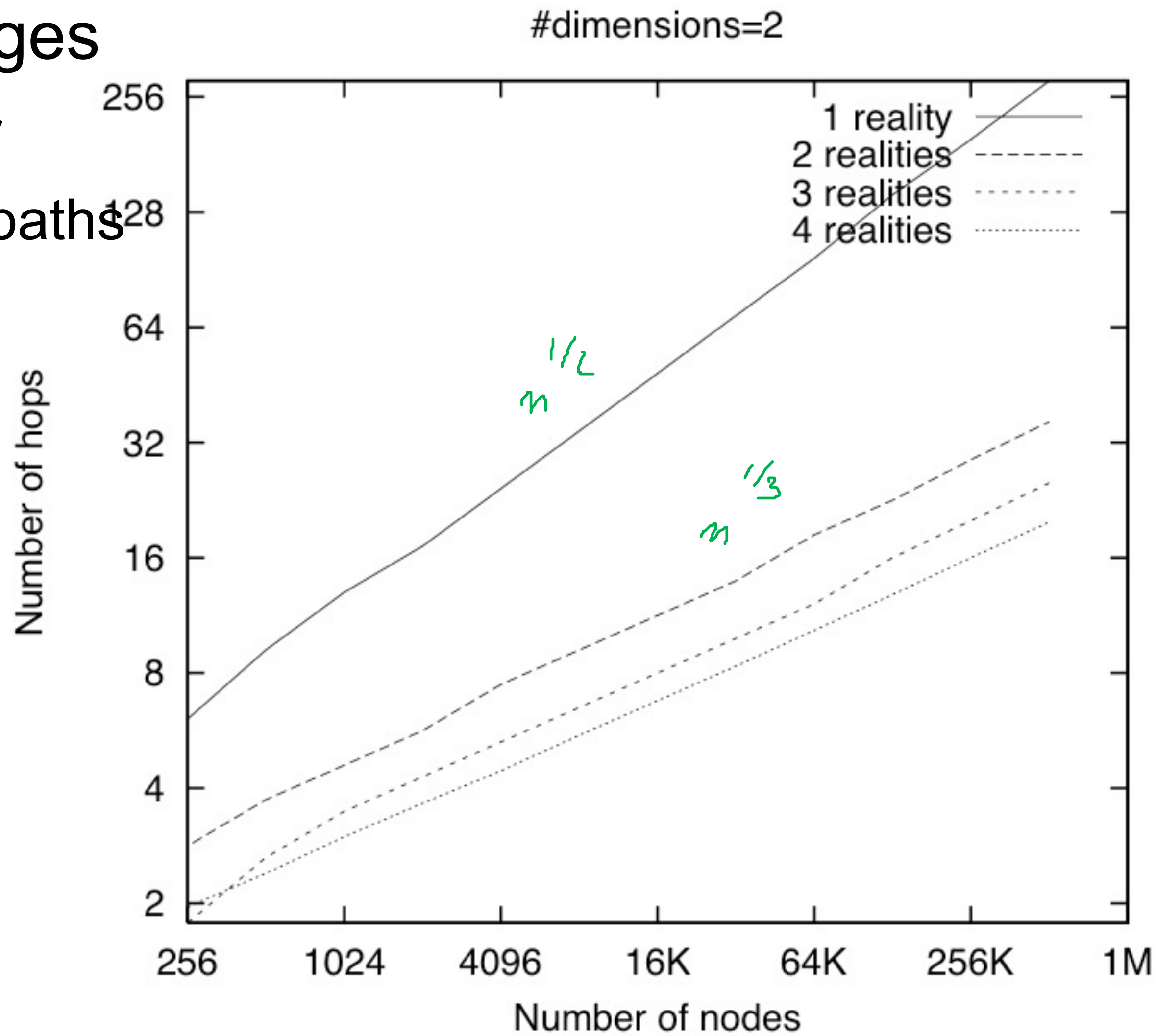
More Realities

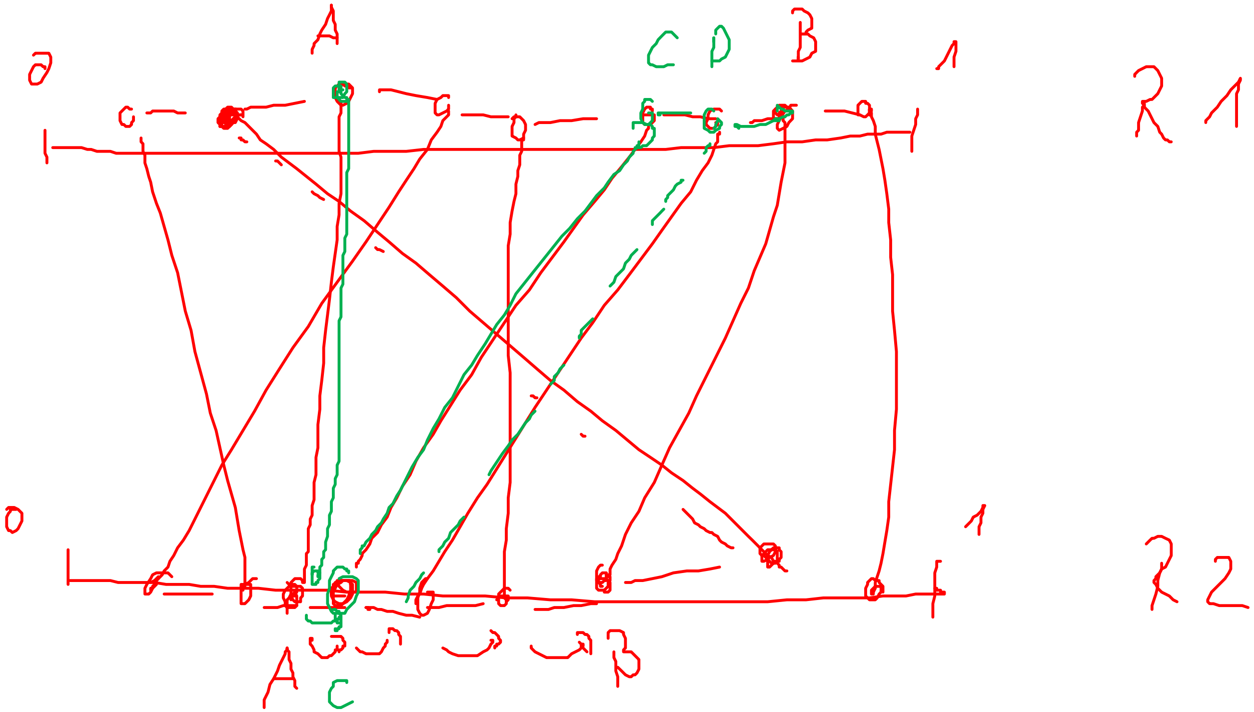
- Build simultanously r CANs with the same peers
- Each CAN is called a *reality*
- For lookup
  - greedily jump between realities
  - choose reality with the closest distance to the target
- Advantanges
  - robuster network
  - faster search



More Realities

- Advantages
  - robuster
  - shorter paths





A B

$\boxed{\Theta(\log n)}$

$\Theta(n^{1/d})$

asymptotically large

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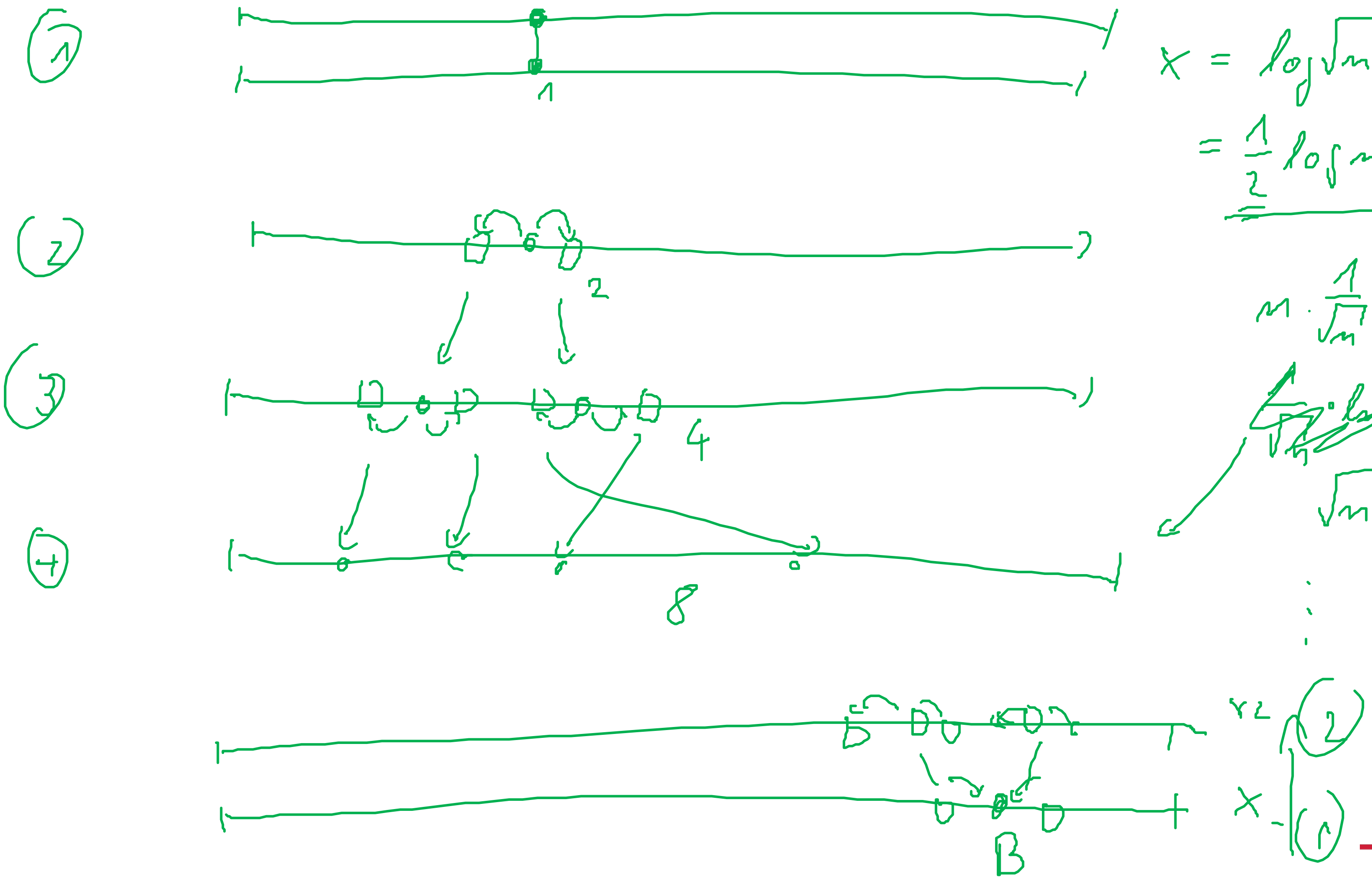
$2^x = \sqrt{n} \cdot \ln n$



$$x = \log \sqrt{n} + \log \ln n$$
$$= \frac{1}{2} \log n + \log \ln n$$

$$n \cdot \frac{1}{\sqrt{n}} \cdot \ln n$$

$$\sqrt{n} \cdot \ln n$$

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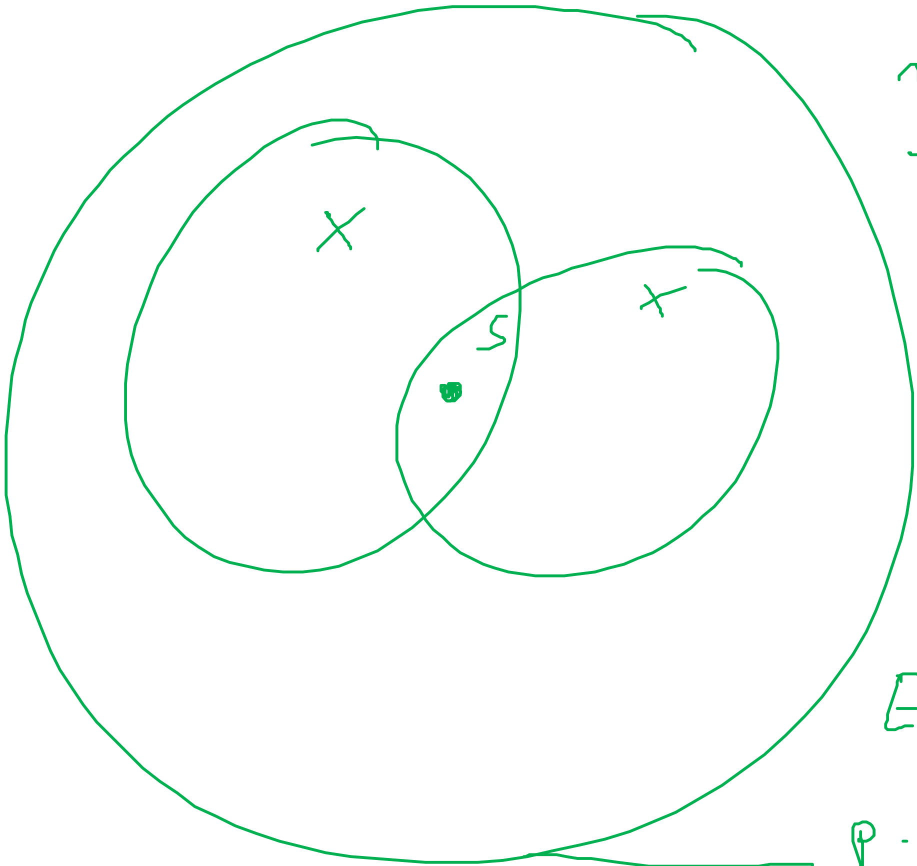


$n = 10.000$

$x = \frac{n}{2}$

Prob. that an element is in S

$\frac{x}{n} \cdot \frac{x}{n} = p$

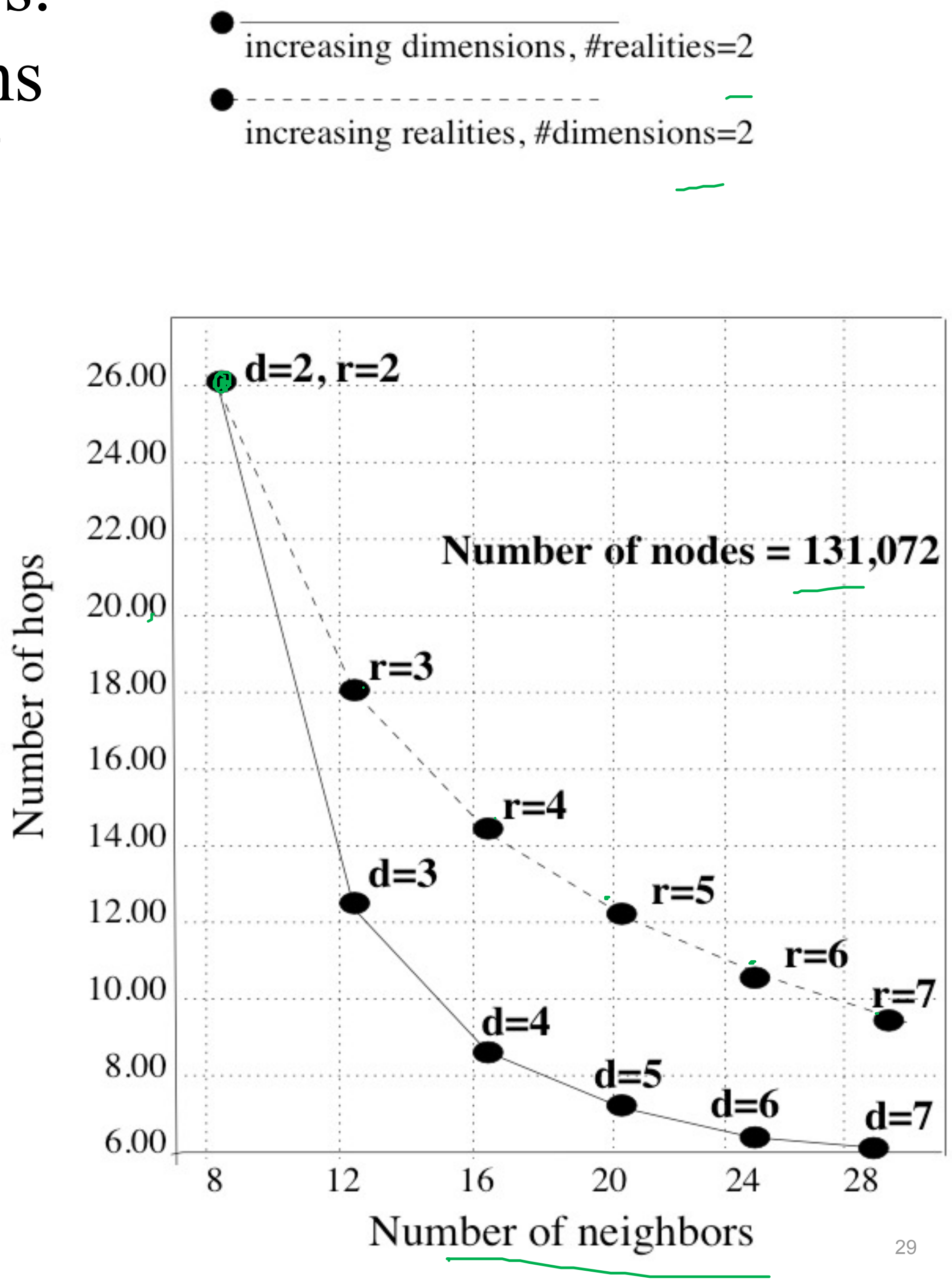
Exp. number

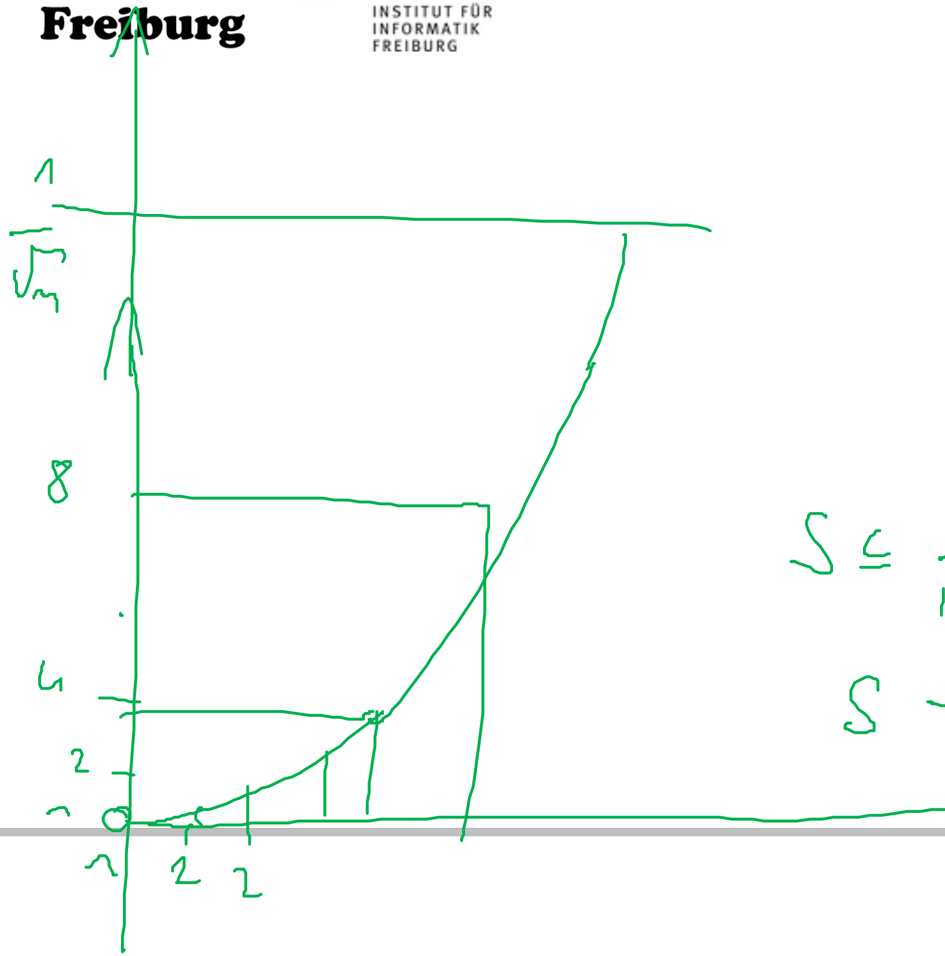
$p \cdot n = \frac{x^2}{n} \geq 1$

$x^2 \geq n \Rightarrow \boxed{x \geq \sqrt{n}}$

Realities vs. Dimensions

- Dimensionens reduce the lookup path length more effciently
- Realities produce more robust networks





$$S \le \frac{1}{10} \cdot n \qquad \ge \sqrt{n}$$

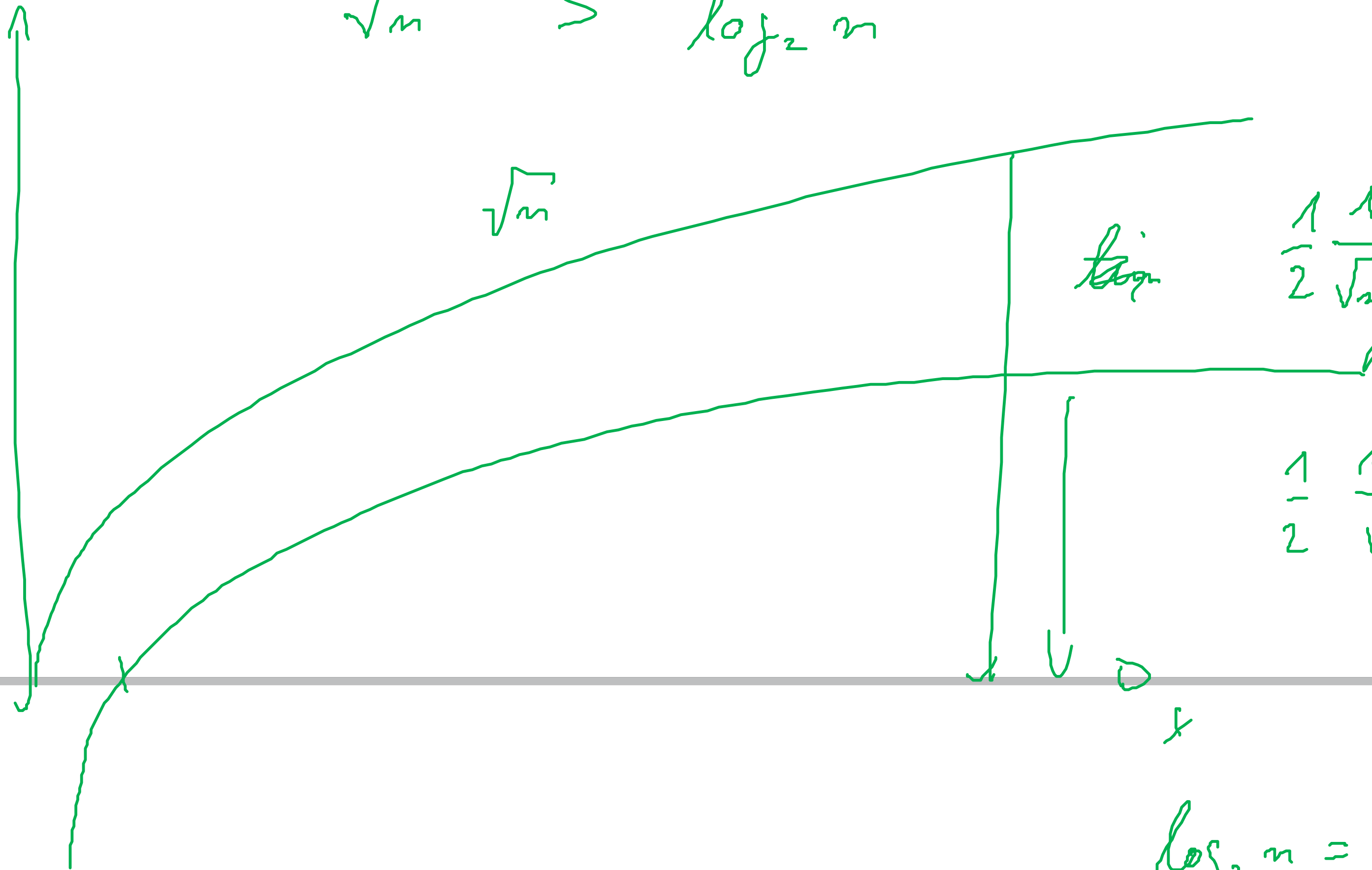
$$S \to \left(2 - \frac{1}{10}\right) \cdot n$$

$$\lim_{n \to \infty} \frac{\sqrt{n}}{\log_2 n} \to [c_1, c_2]$$

$\to \infty$

$\to 0$

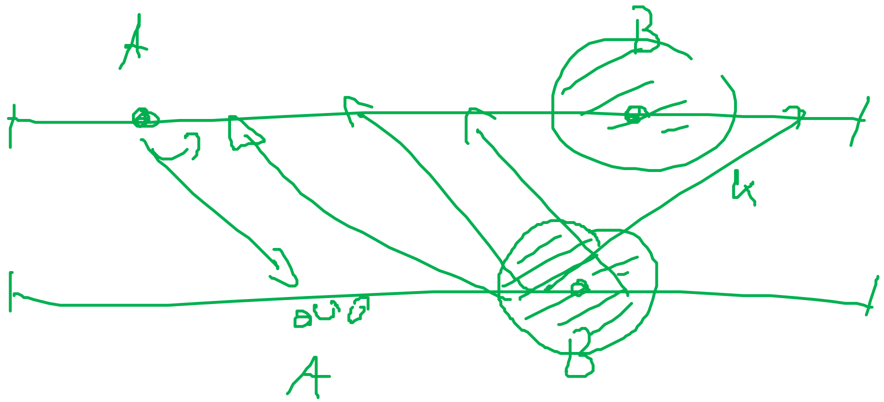
$$\sqrt{n} > \log_2 n$$

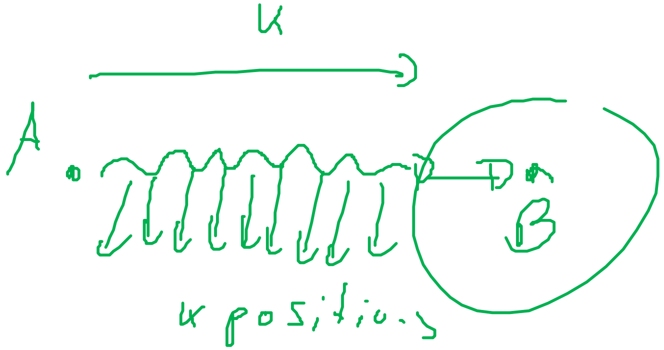


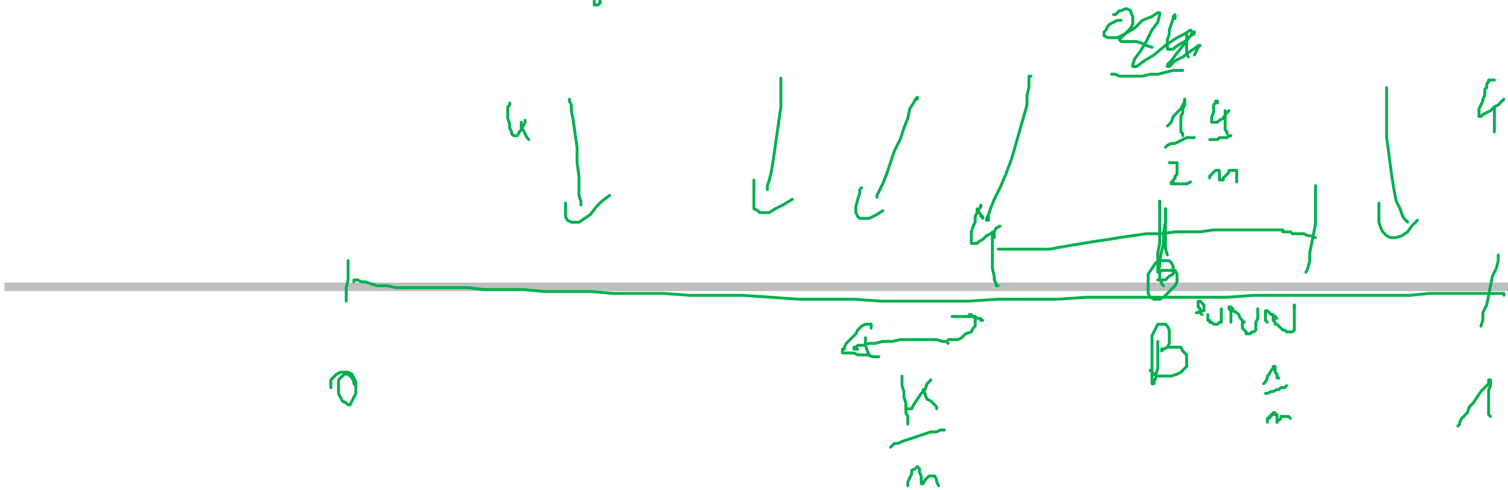
$$\cancel{\frac{1}{2}\frac{1}{\sqrt{n}} \Big/ \frac{1}{n \cdot \ln 2}}$$

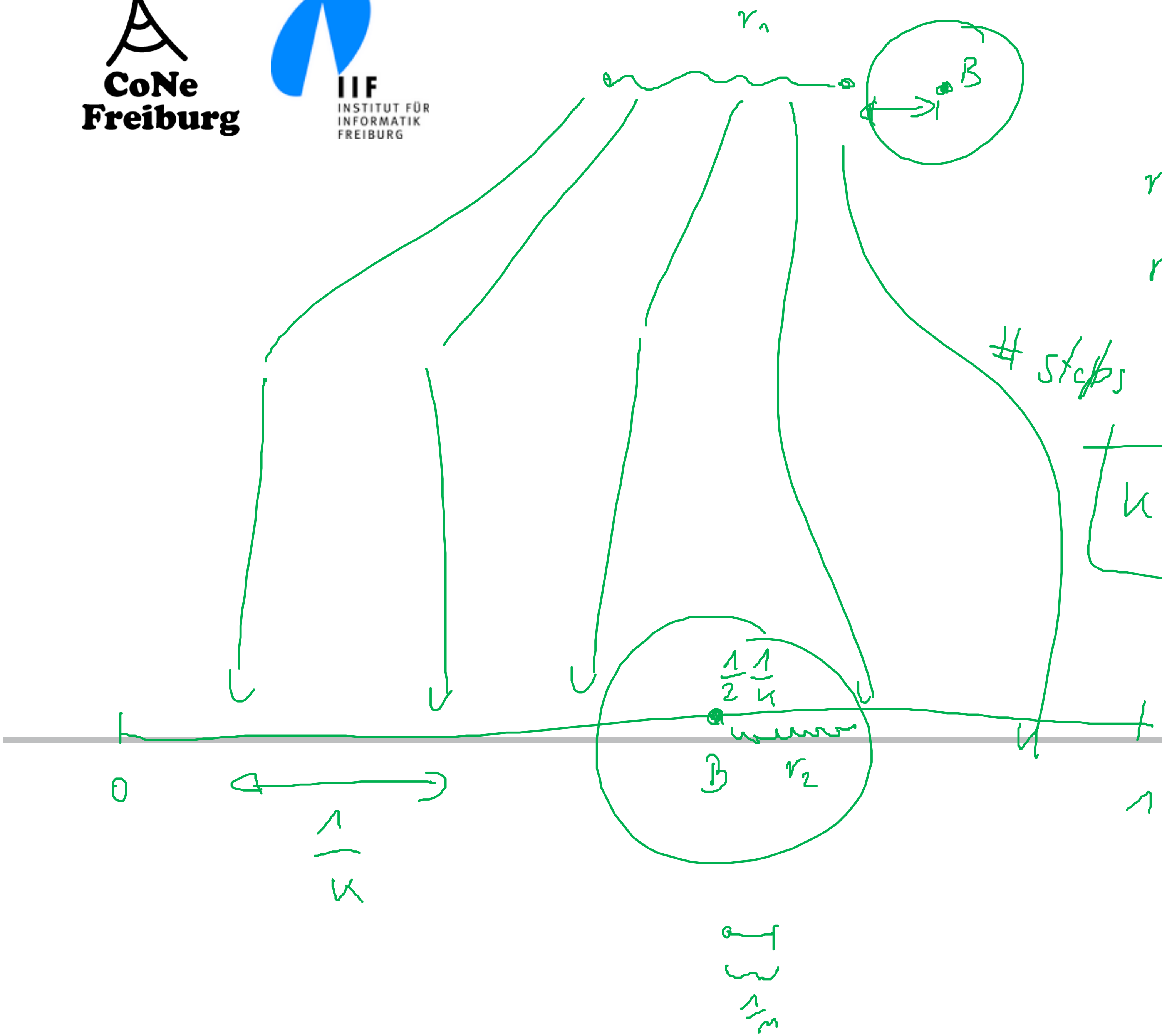
$$\frac{1}{2}\frac{n \cdot \ln 2}{\sqrt{n}} = \sqrt{n}\,\frac{\ln 2}{2}$$

$$\log_2 n = \frac{\ln n}{\ln 2}$$


A
B
k
BUG
A
B


k
A
B
k positions


2k
k
1/2m
4
B
0
k/m
1/m
1

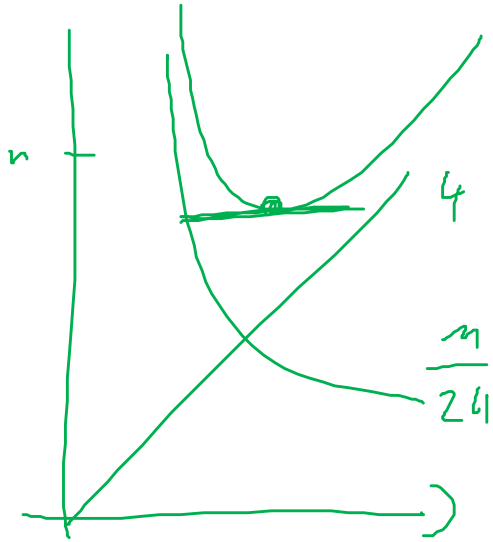


$r_1 = 4$

$r_2 = \frac{\frac{1}{2\cdot 4}}{\frac{1}{n}} = \frac{n}{2\cdot 4}$

~~# steps~~

$\boxed{k + \frac{n}{2\cdot 4}}$



$$\min_k \left\{ k + \frac{n}{2k} \right\}$$

$$\frac{d\left(k + \frac{n}{2k}\right)}{dk} = 1 - \frac{n}{2k^2} = 0$$

$$2k^2 = n$$

$$k = \sqrt{\frac{1}{2} n}$$

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