Peer-to-Peer Networks

6. Analysis of DHT

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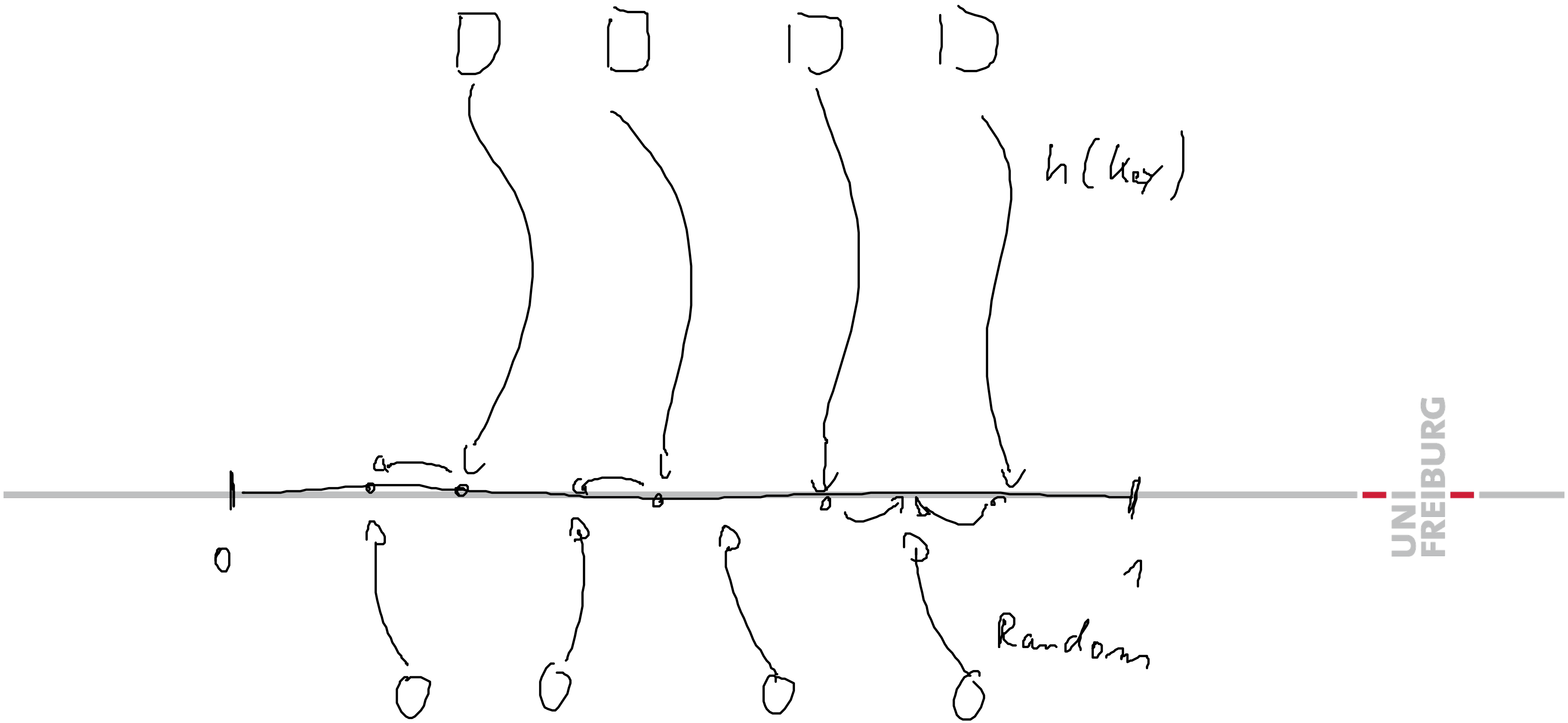
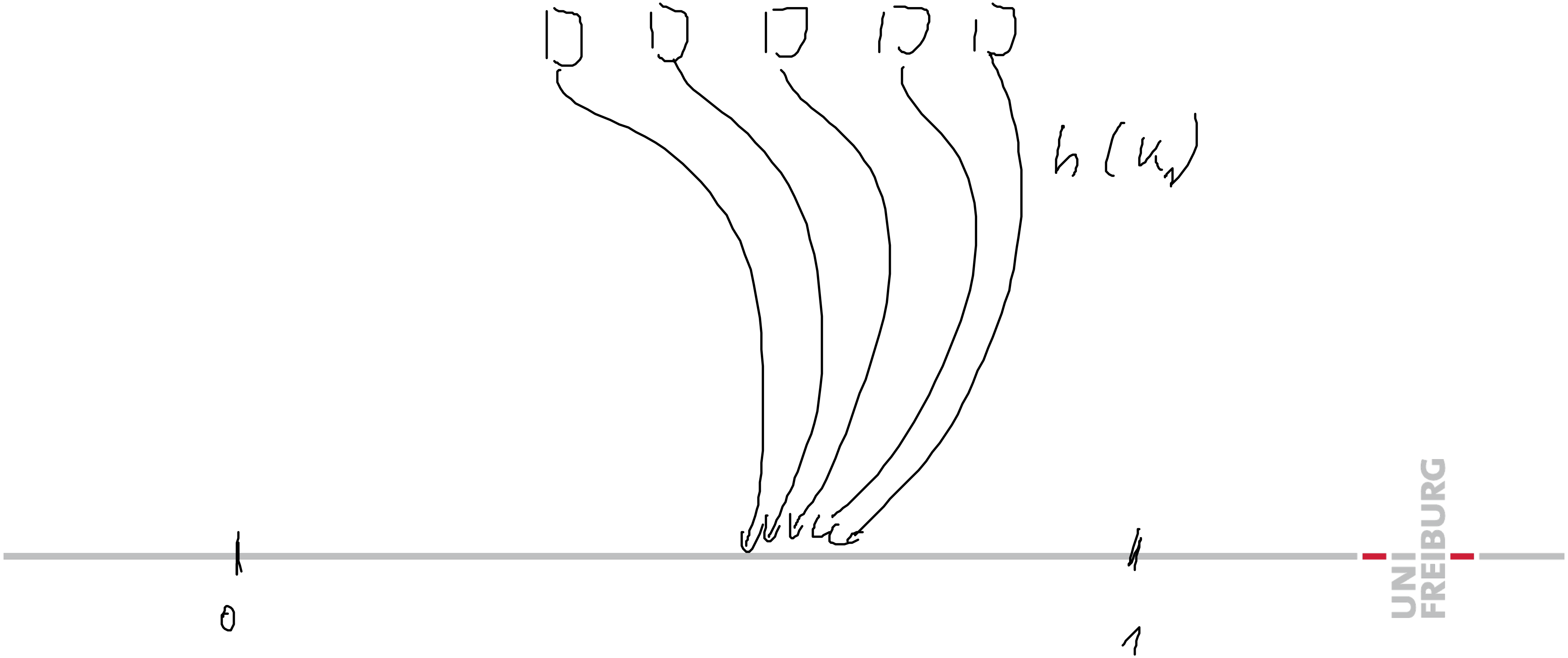
DHT



Hash has to appear RANDOM



Bernoulli

event prob.

$$X = \begin{cases} 0 & 1/2 \\ 1 & 1/2 \end{cases}$$

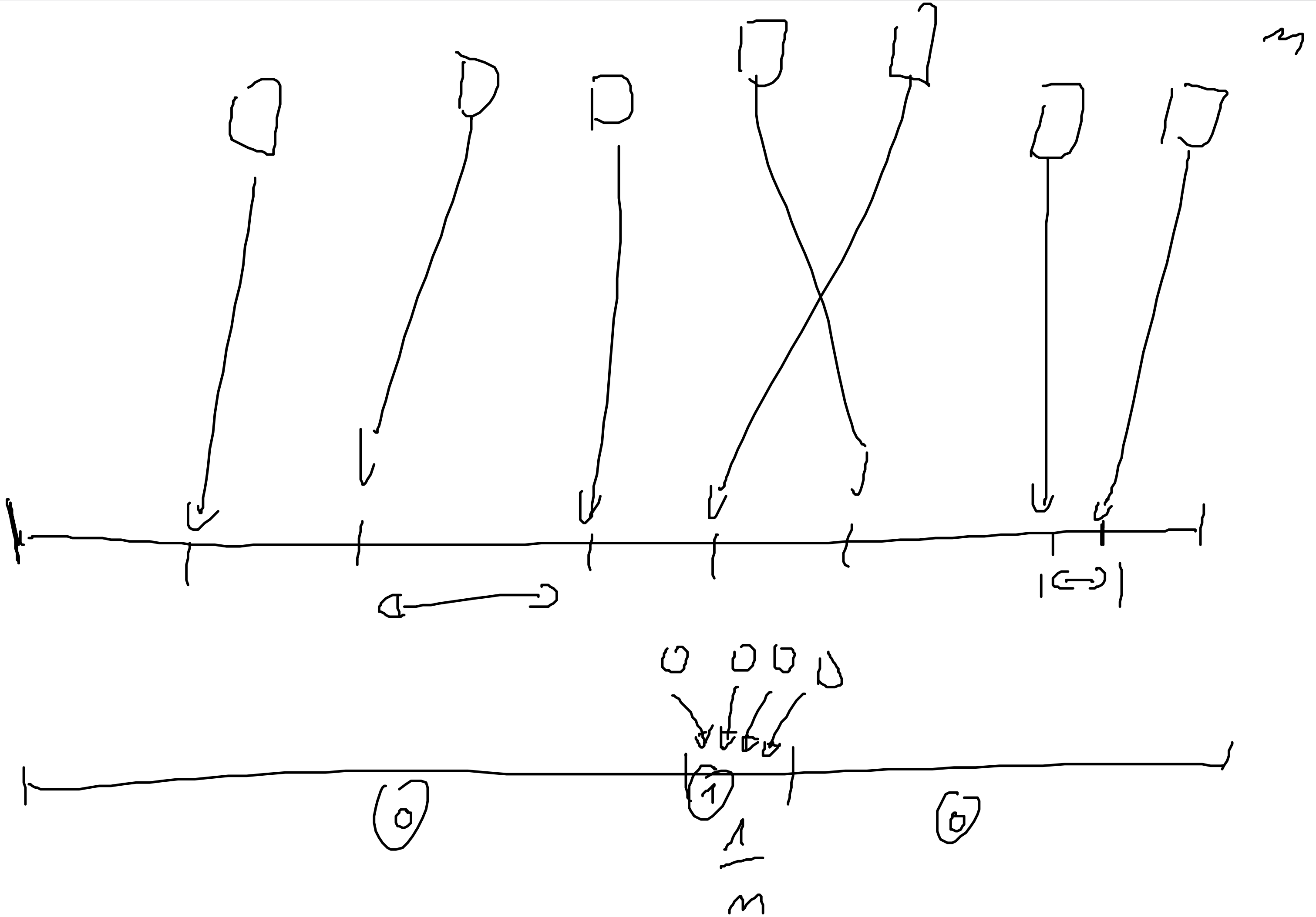
1 0

$$\underbrace{\overset{\leq 1}{P[X=0]}}_{\geq 0} + \underbrace{\overset{\leq 1}{P[X=1]}}_{\geq 0} + \underbrace{P[X \notin \{0,1\}]}_{0} = 1$$

$$\sum_{x} P[X=x] = 1$$

$$E[X] = \sum_{x \in \ldots} x \cdot P[X=x]$$

$\left(\frac{1}{n}\right)^n$



$$P[X_1 = 1] = p = \frac{1}{n}$$

$$P[X_1 = 0] = (1-p) = 1 - \frac{1}{n}$$

$X_1 \ldots X_n$

$$E[X_1] = p \cdot 1 + 0 \cdot (1-p)$$
$$= p$$

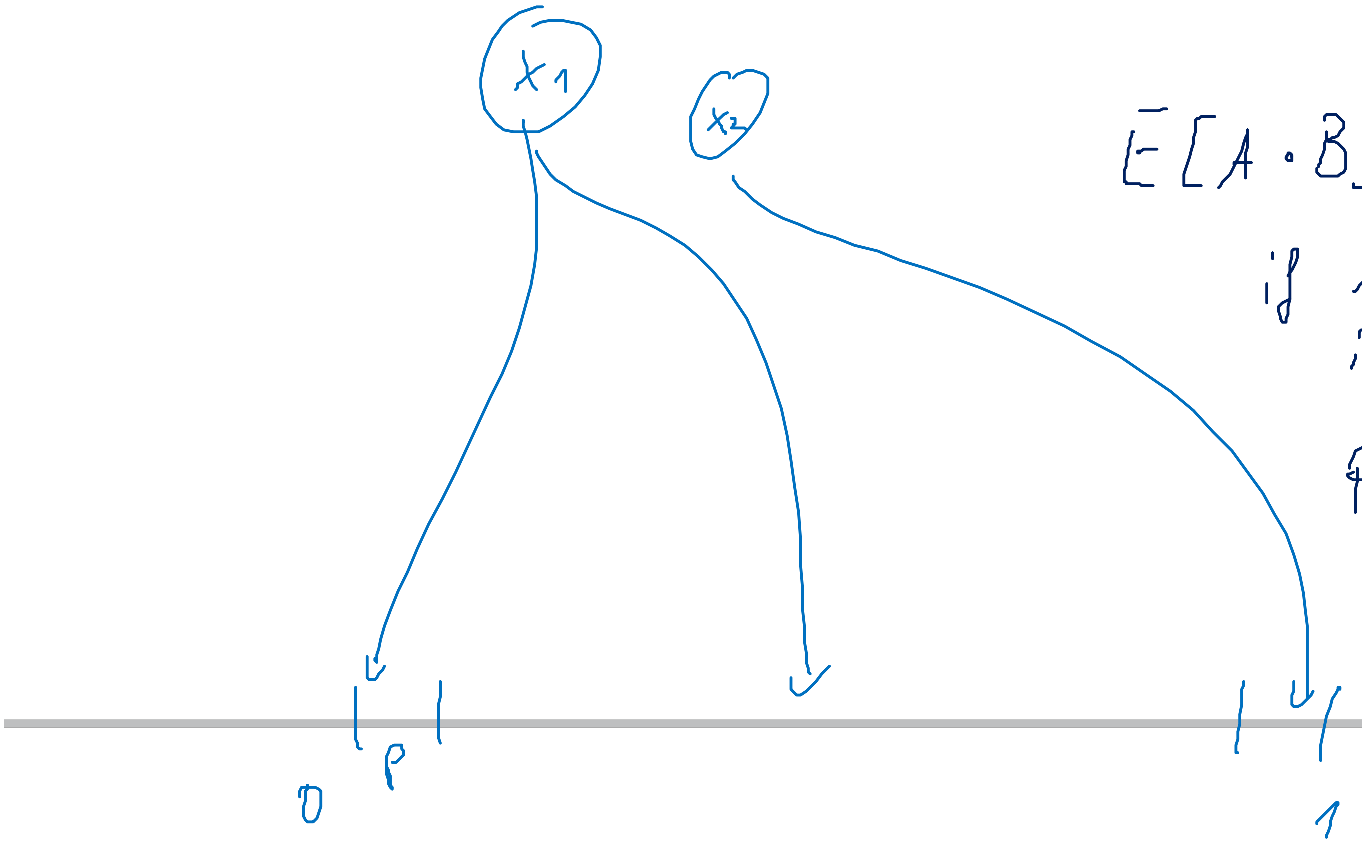
$$E[X_1 + X_2 + X_3 + \ldots X_n]$$
$$\left(= \sum_{x_1, x_2, \ldots, x_n \in \{0,1\}^n} P[X_1 + X_2 \ldots X_n = x_1 + \ldots] \right)$$

$p = \frac{1}{n}$

$1 - p$

$$E[A+B] = E[A] + E[B]$$

$$E[X_1 + X_1] = 2 \cdot E[X_1]$$



$$E[A \cdot B] = E[A] \cdot E[B]$$

if A and B are independent

$$\Updownarrow$$

$$P[A = a_1, B = b_1] = P[A = a_1] \cdot P[B = b_1]$$

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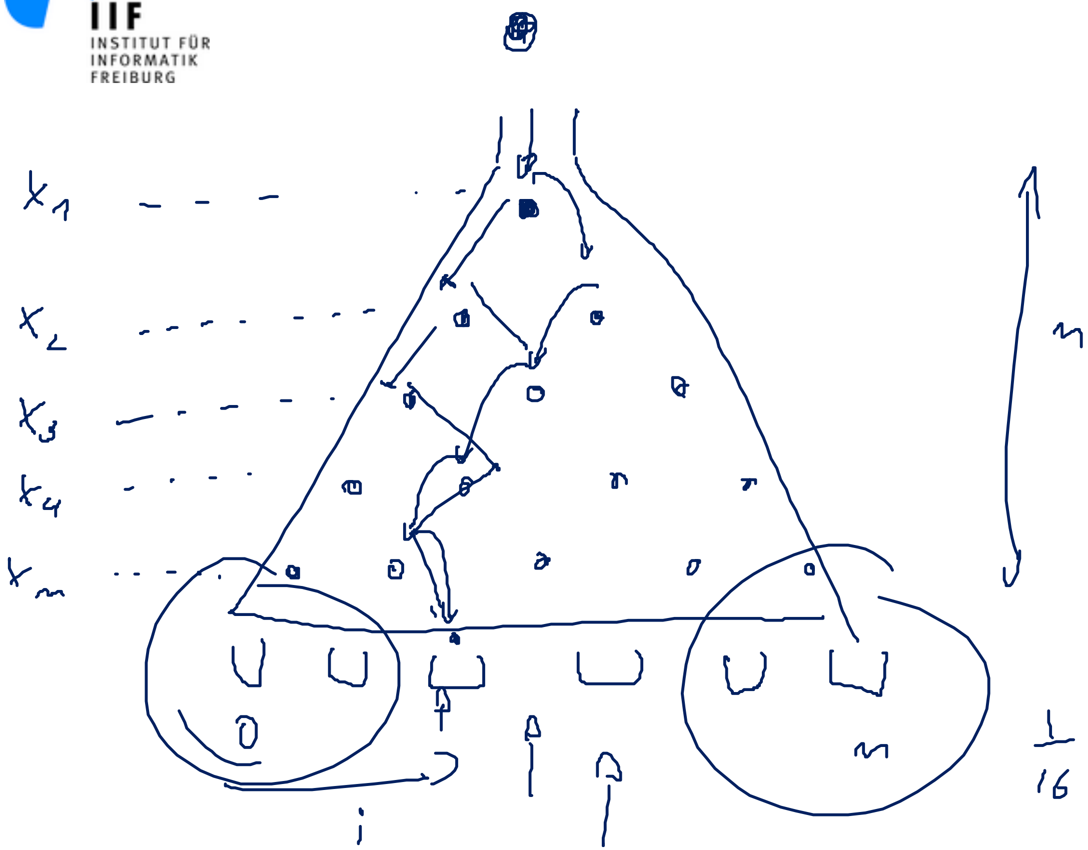

1

$\frac{1}{2}$ $\frac{1}{2}$

$\frac{1}{4}$ $\frac{2}{4}$ $\frac{1}{4}$

$\frac{1}{8}$ $\frac{3}{8}$ $\frac{3}{8}$ $\frac{1}{8}$

$\frac{1}{16}$ $\frac{4}{16}$ $\frac{6}{16}$ $\frac{6}{16}$ $\frac{1}{16}$

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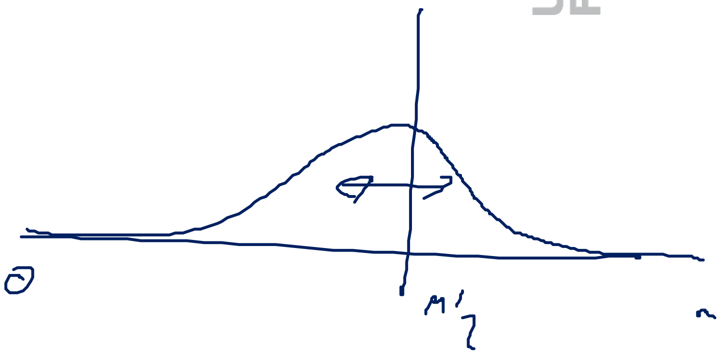
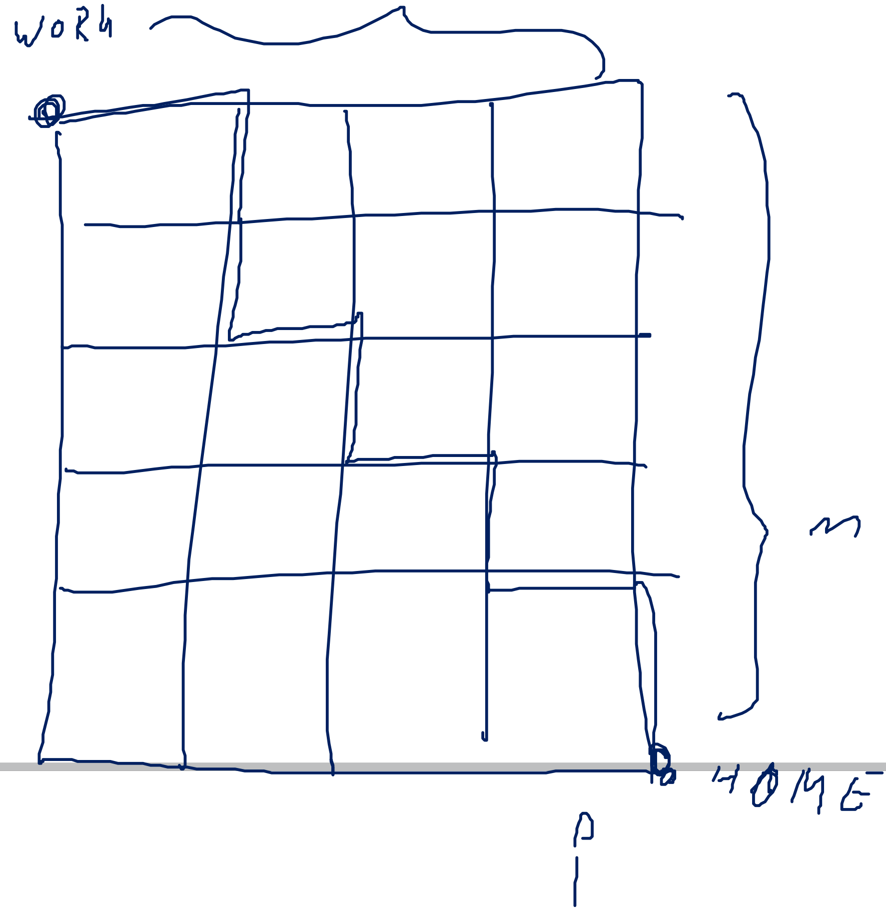
$$\left(\frac{1}{2}\right)^{i} \cdot \left(\frac{1}{2}\right)^{n-i} \cdot \underbrace{\binom{n}{i}}_{\left(\frac{n!}{i!\,(n-i)!}\right)}$$

$$\sum_{i=1}^{n} x_i$$

$$\boxed{n! = 1 \cdot 2 \cdot 3 \cdots n}$$

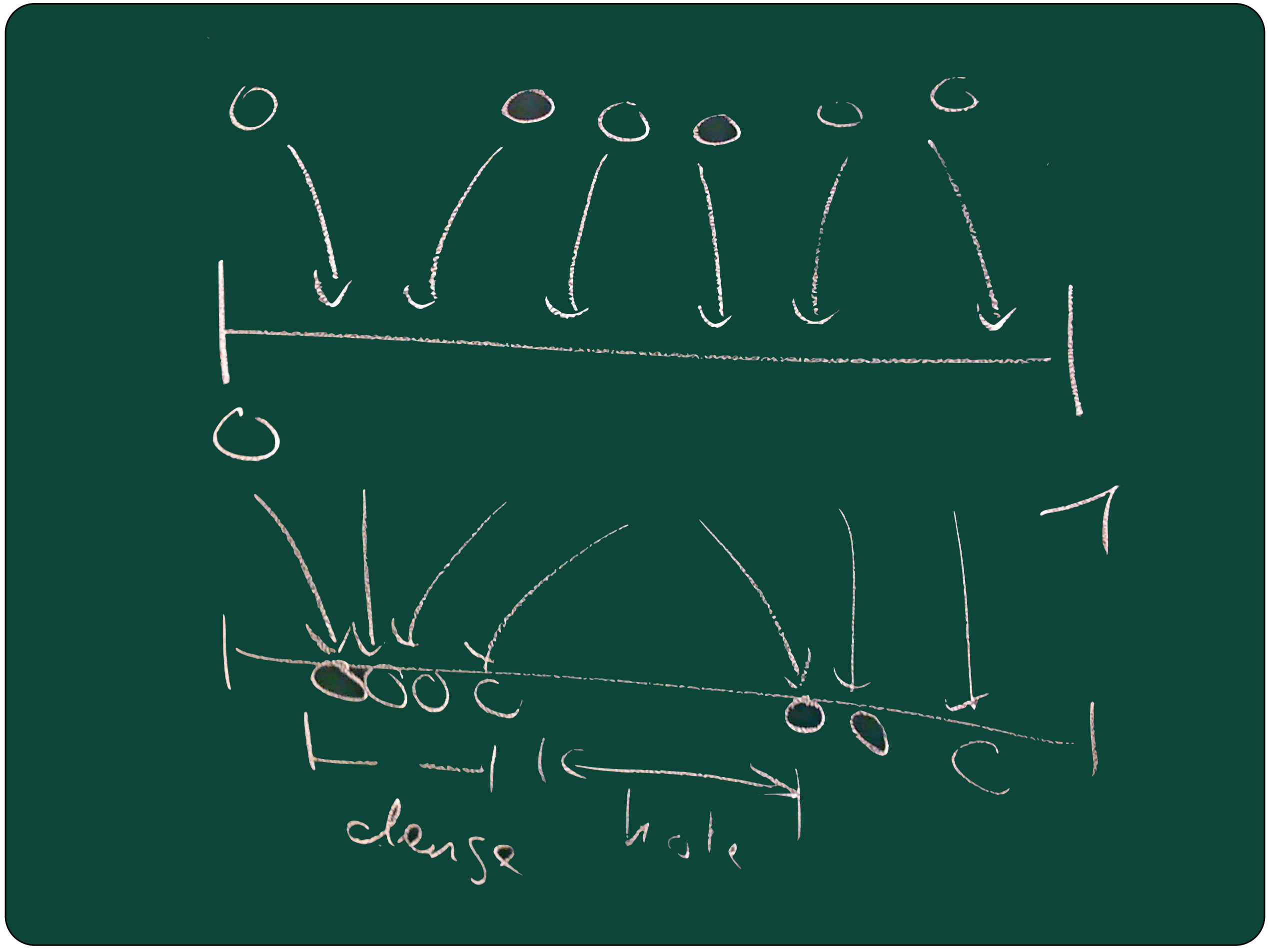


Berkeley / Manhattan



$$\begin{pmatrix} m \\ \vdots \\ \vdots \end{pmatrix}$$

Holes and Dense Areas



Dense Spots

- Theorem
  - If n elements are randomly inserted into an array [0,1[then with constant probability there is a dense interval of length 1/n with at least Ω(log n/ (log log n)) elements.
- Proof
  - The probability to place exactly i elements in to such an interval is $$\left(\frac{1}{n}\right)^i \left(1-\frac{1}{n}\right)^{n-i} \binom{n}{i}$$
  - for i = c log n / (log log n) this probability is at least $1/n^k$ for an appropriately chosen c and k<1
  - Then the expected number of intervals is at least 1

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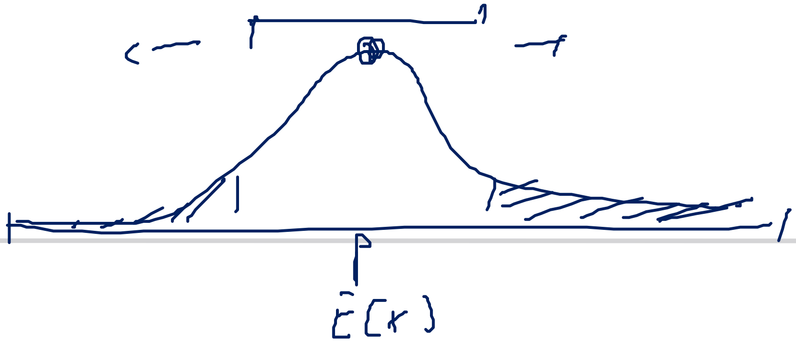
Excursion



- Markov-Inequality
  - For random variable X>0 with E[X] > 0:

$$\mathbf{P}[X \geq k \cdot \mathbf{E}[X]] \leq \frac{1}{k}$$

weak

- Chebyshev

$$\mathbf{P}[|X - \mathbf{E}[X]| \geq k] \leq \frac{\mathbf{V}[X]}{k^2}$$

  - for Variance

$$\mathbf{V}[X] = \mathbf{E}[X^2] - \mathbf{E}[X]^2$$

$\neq 0$

usually

$E[(X - E(X))^2]$

- Stronger bound: Chernoff

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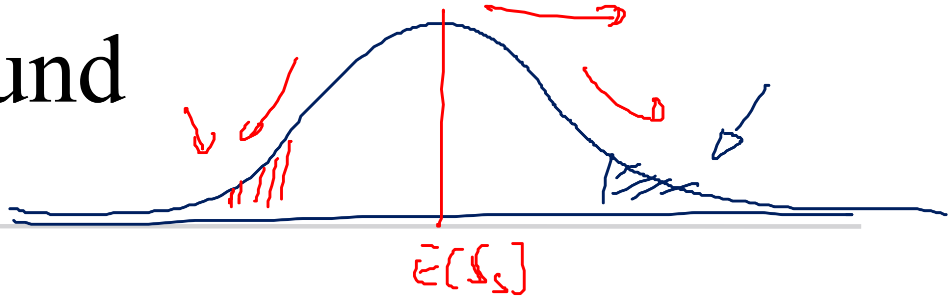
Chernoff-Bound

Hoeffding



Theorem Chernoff Bound

- Let $x_1,\dots,x_n$ independent Bernoulli experiments with
  - $P[x_i = 1] = p$
  - $P[x_i = 0] = 1-p$
- Let
$$S_n = \sum_{i=1}^{n} x_i$$

E[S_n]

p·n

- Then for all c>0
$$\mathbf{P}[S_n \geq (1+c)\cdot \mathbf{E}[S_n]] \quad \leq \quad e^{-\frac{1}{3}\min\{c,c^2\}pn}$$
- For 0≤c≤1
$$\mathbf{P}[S_n \leq (1-c)\cdot \mathbf{E}[S_n]] \quad \leq \quad e^{-\frac{1}{2}c^2pn}$$

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