Peer-to-Peer Networks

09 Random Graphs for Peer-to-Peer-Networks

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Peer-to-Peer Networking Facts

- Hostile environment
  - Legal situation
  - Egoistic users
  - Networking
    - ISP filter Peer-to-Peer Networking traffic
    - User arrive and leave
    - Several kinds of attacks
    - Local system administrators fight peer-to-peer networks
- Implication
  - Use stable robust network structure as a backbone
  - Napster: star
  - CAN: lattice
  - Chord, Pastry, Tapestry: ring + pointers for lookup
  - Gnutella, FastTrack: chaotic “social” network
- Idea: Use a Random d-regular Network

Why Random Networks ?

- Random Graphs ...
  - Robustness
  - Simplicity
  - Connectivity
  - Diameter
  - Graph expander
  - Security
- Random Graphs in Peer-to-Peer networks:
  - Gnutella
  - JXTApose

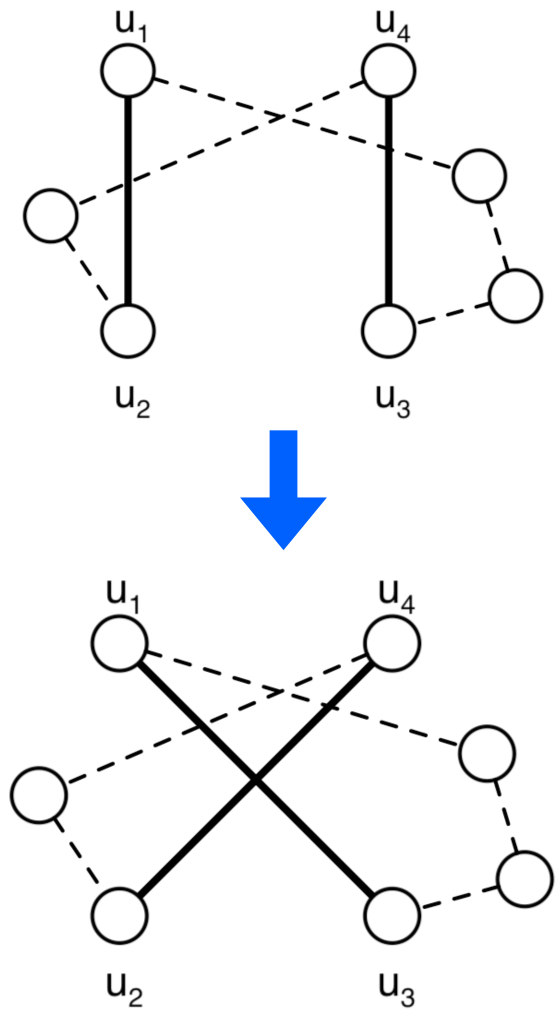
Dynamic Random Networks ...

- Peer-to-Peer networks are highly dynamic ...
  - maintenance operations are needed to preserve properties of random graphs
  - which operation can maintain (repair) a random digraph?

Desired properties:	
Soundness	Operation remains in domain (preserves connectivity and out-degree)
Generality	every graph of the domain is reachable does not converge to specific small graph set
Feasibility	can be implemented in a P2P-network
Convergence Rate	probability distribution converges quickly

Simple Switching

- Simple Switching
  - choose two random edges
    - $\{u_1,u_2\} \in E$, $\{u_3,u_4\} \in E$
  - such that $\{u_1,u_3\}$, $\{u_2,u_4\} \notin E$
    - add edges $\{u_1,u_3\}$, $\{u_2,u_4\}$ to E
    - remove $\{u_1,u_2\}$ and $\{u_3,u_4\}$ from E
- McKay, Wormald, 1990
  - Simple Switching converges to uniform probability distribution of random network
  - Convergence speed:
    - $O(nd^3)$ for $d \in O(n^{1/3})$
- Simple Switching cannot be used in Peer-to-Peer networks
  - Simple Switching disconnects the graph with positive probability
  - No network operation can re-connect disconnected graphs



Necessities of Graph Transformation

	Simple-Switching
Graphs	Undirected Graphs
Soundness	?
Generality	↯
Feasibility	✔
Convergence	✔

- Problem: Simple Switching does not preserve connectivity
- Soundness
  - Graph transformation remains in domain
  - Map connected d-regular graphs to connected d-regular graphs
- Generality
  - Works for the complete domain and can lead to any possible graph
- Feasibility
  - Can be implemented in P2P network
- Convergence Rate
  - The probability distribution converges quickly

Directed Random Graphs

- Peter Mahlmann, Christian Schindelhauer
  - Distributed Random Digraph Transformations for Peer-to-Peer Networks, 18th ACM Symposium on Parallelism in Algorithms and Architectures, Cambridge, MA, USA. July 30 - August 2, 2006

Directed Graphs

Push Operation:

1.Choose random node *u*

2.Set *v* to *u*

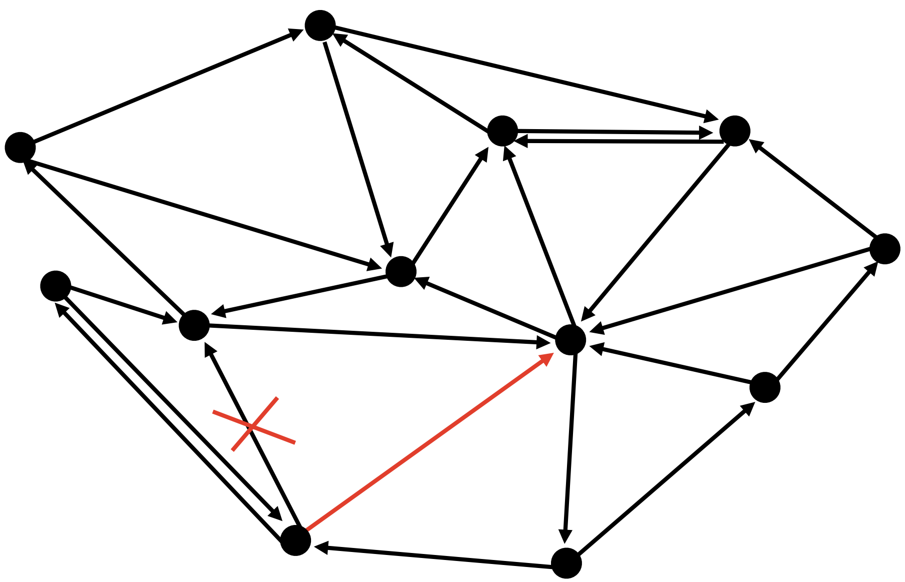
3.While a random event with p= 1/*h* appears

  a) Choose random edge starting at *v* and ending at *v*‘

  b) Set *v* to *v*‘

3.Insert edge (*u*,*v*)

4.Remove random edge starting at *v*



Pull Operation:

1.Choose random node *u*

2.Set *v* to *u*

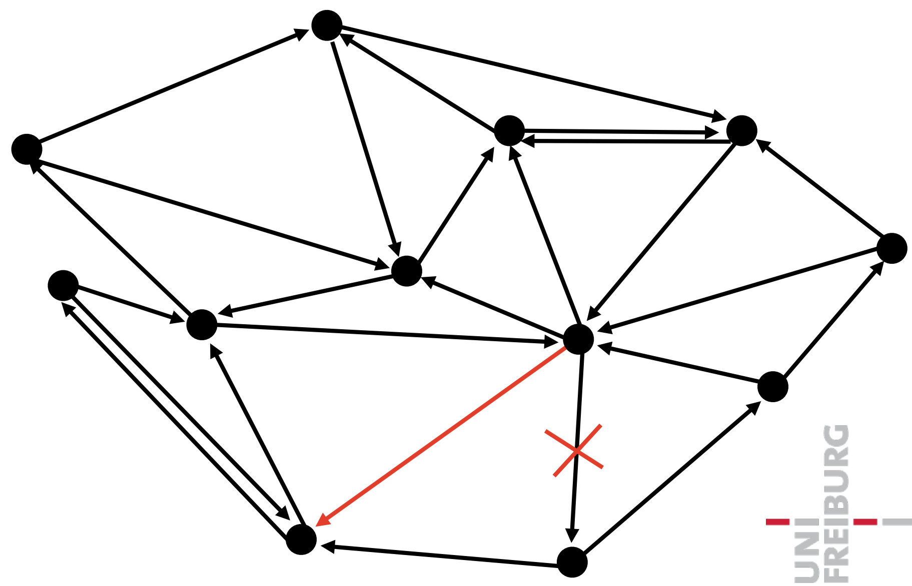
3.While a random event with p= 1/*h* appears

  a) Choose random edge starting at *v* and ending at *v*‘

  b) Set *v* to *v*‘

3.Insert edge (*v*,*u*)

4.Remove random edge starting at *v*‘



Simulation of Push-Operations

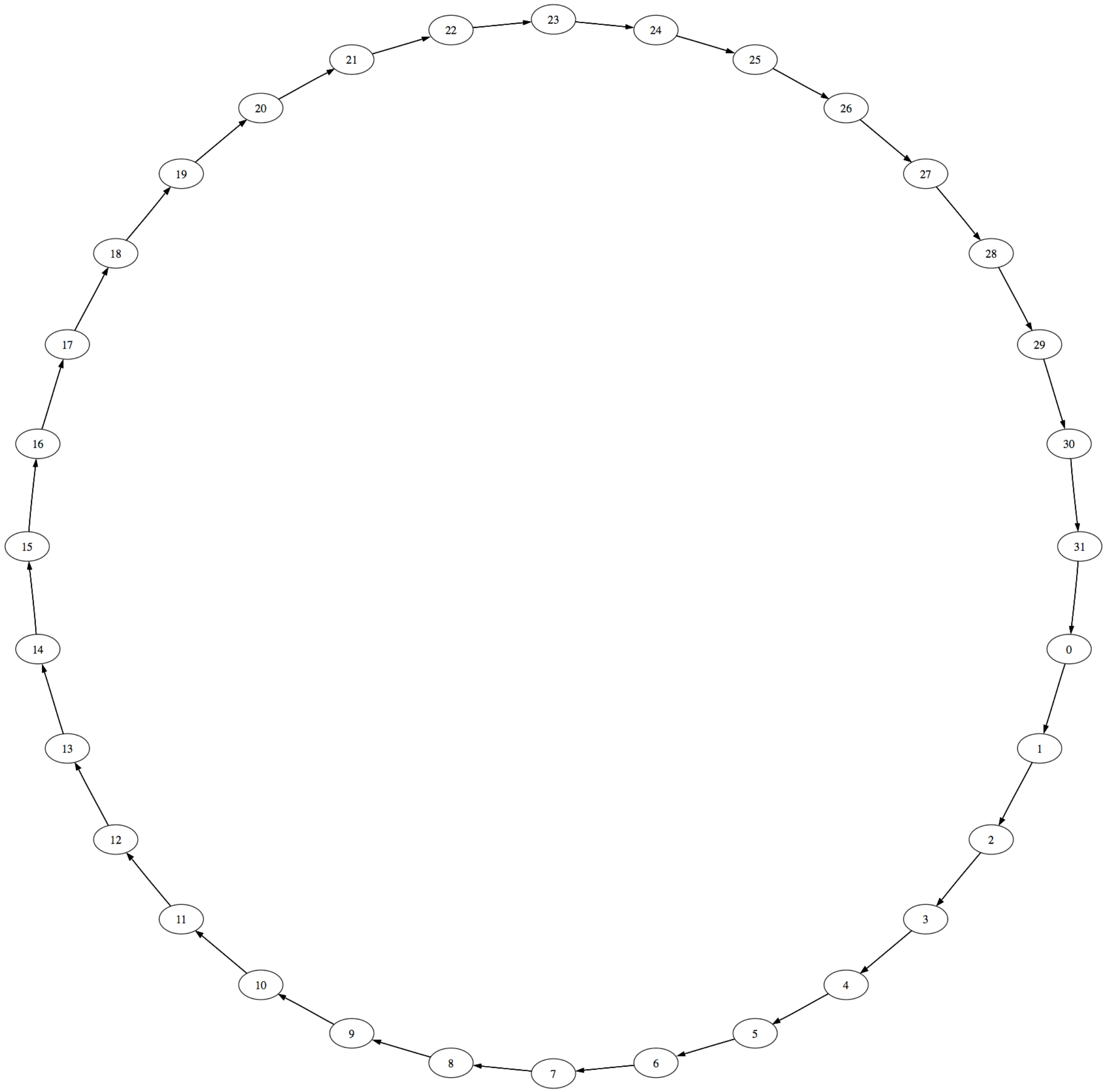
Start situation

Parameter:

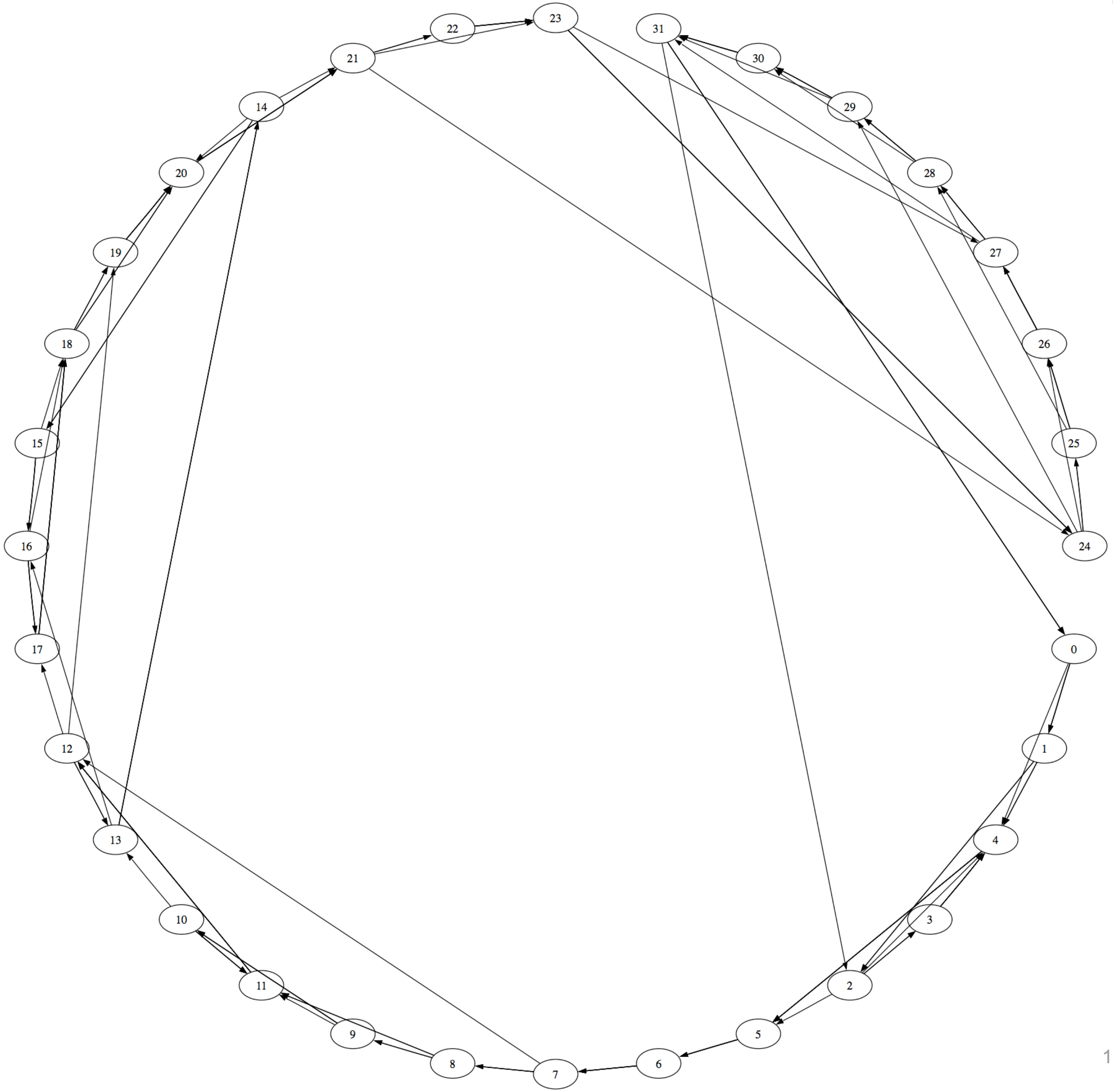
n = 32 nodes

out-degree d = 4

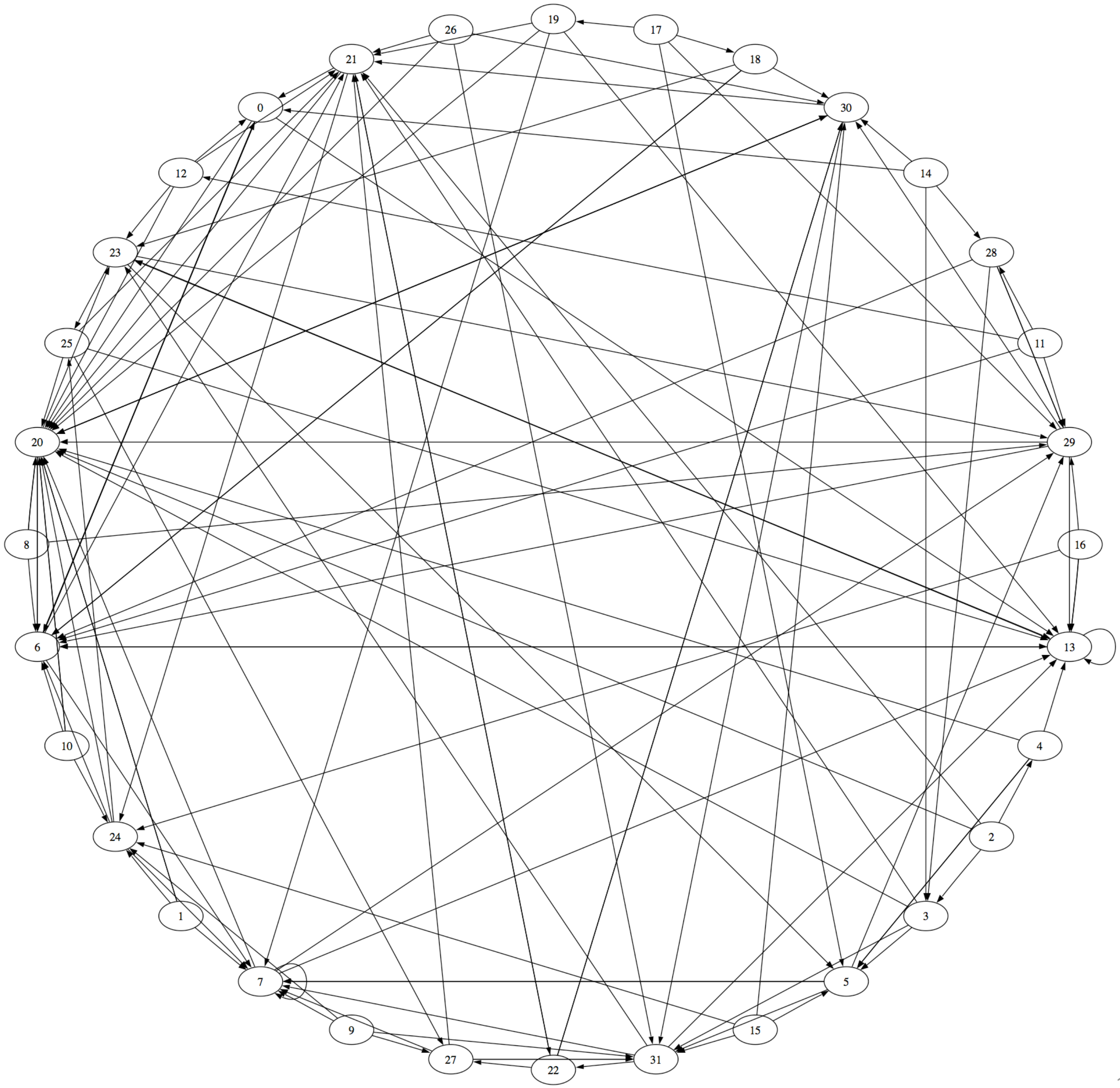
Hop-distance h = 3



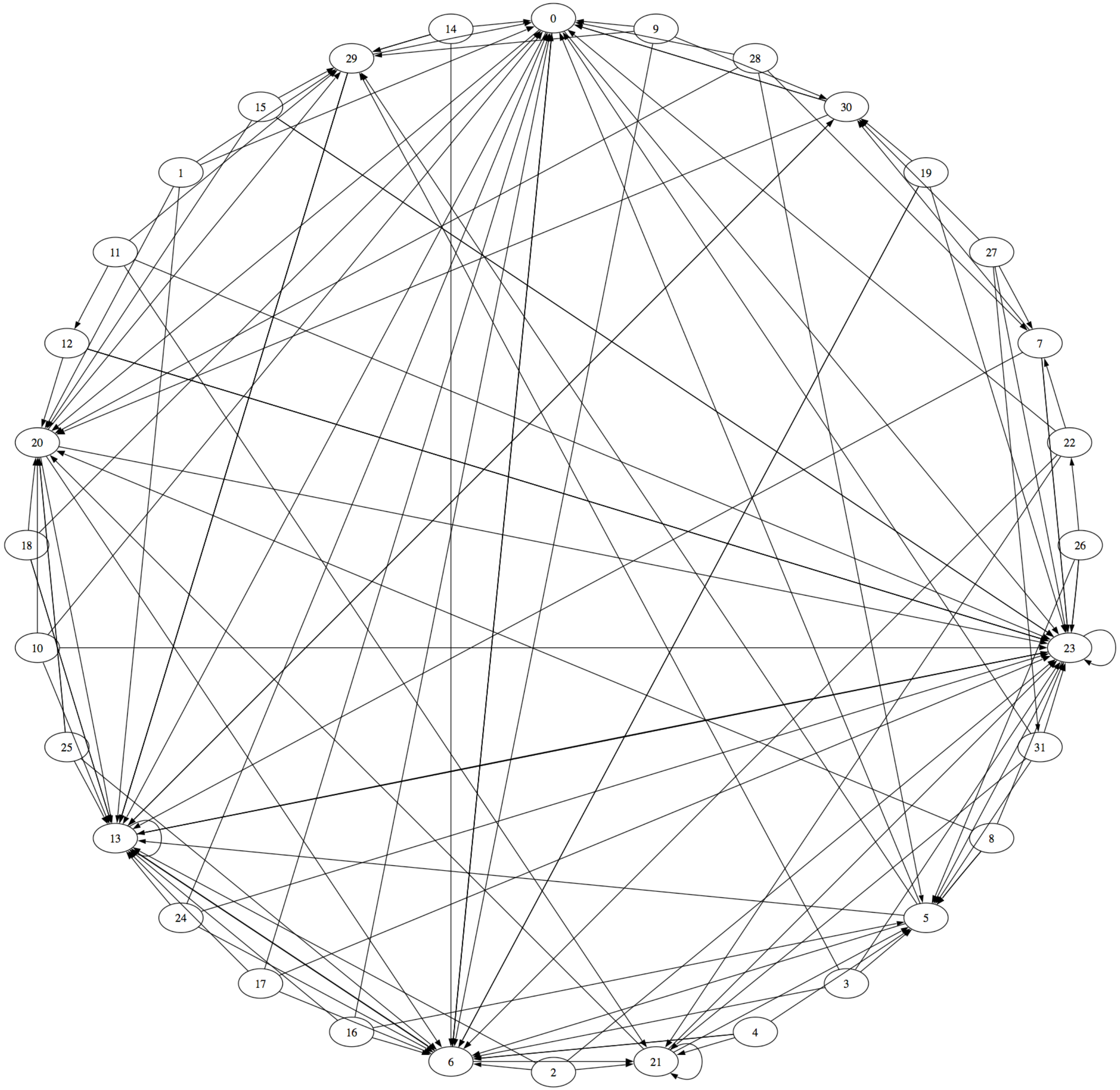
1 Iteration Push ...



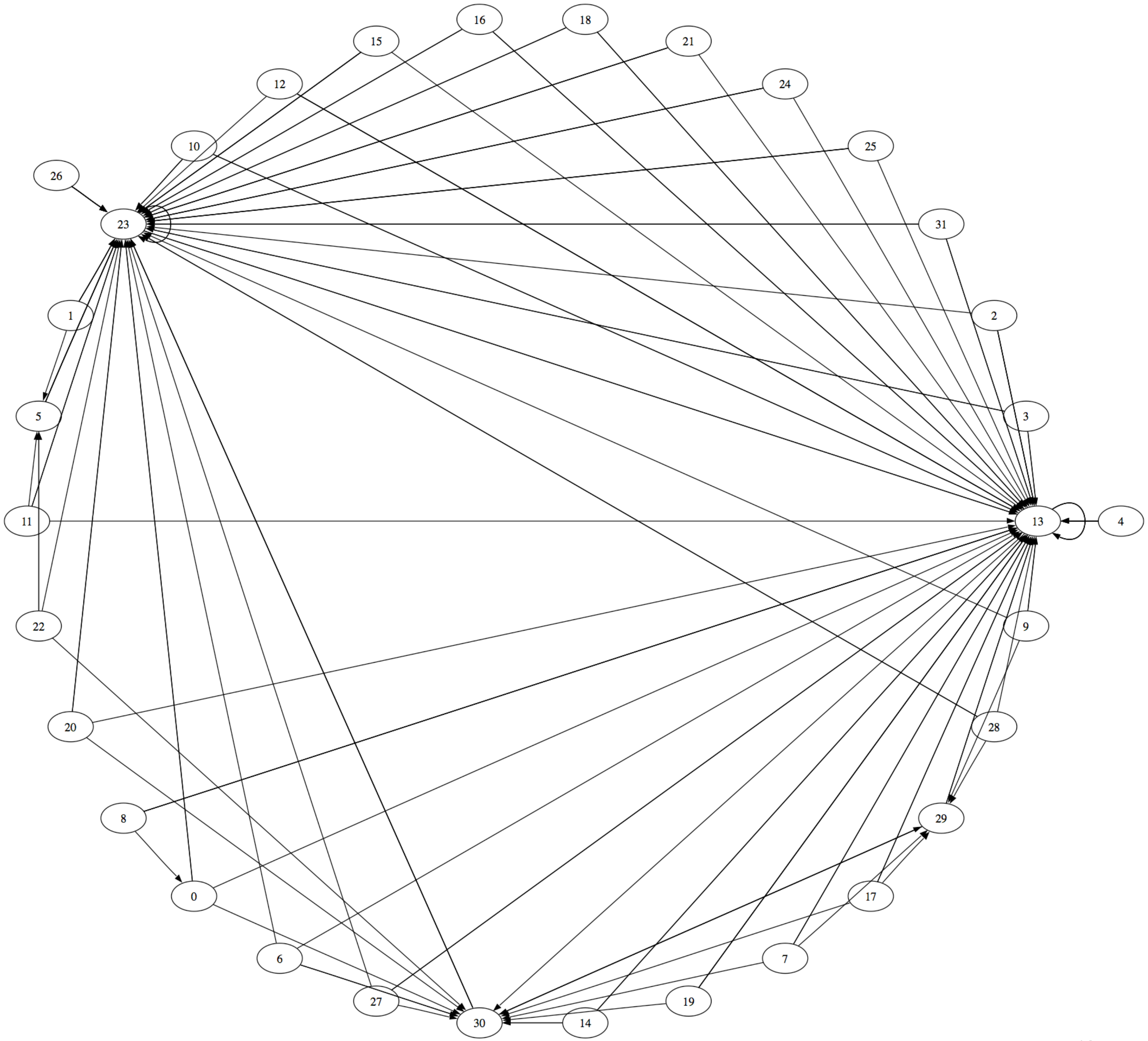
10 Iterations Push ...



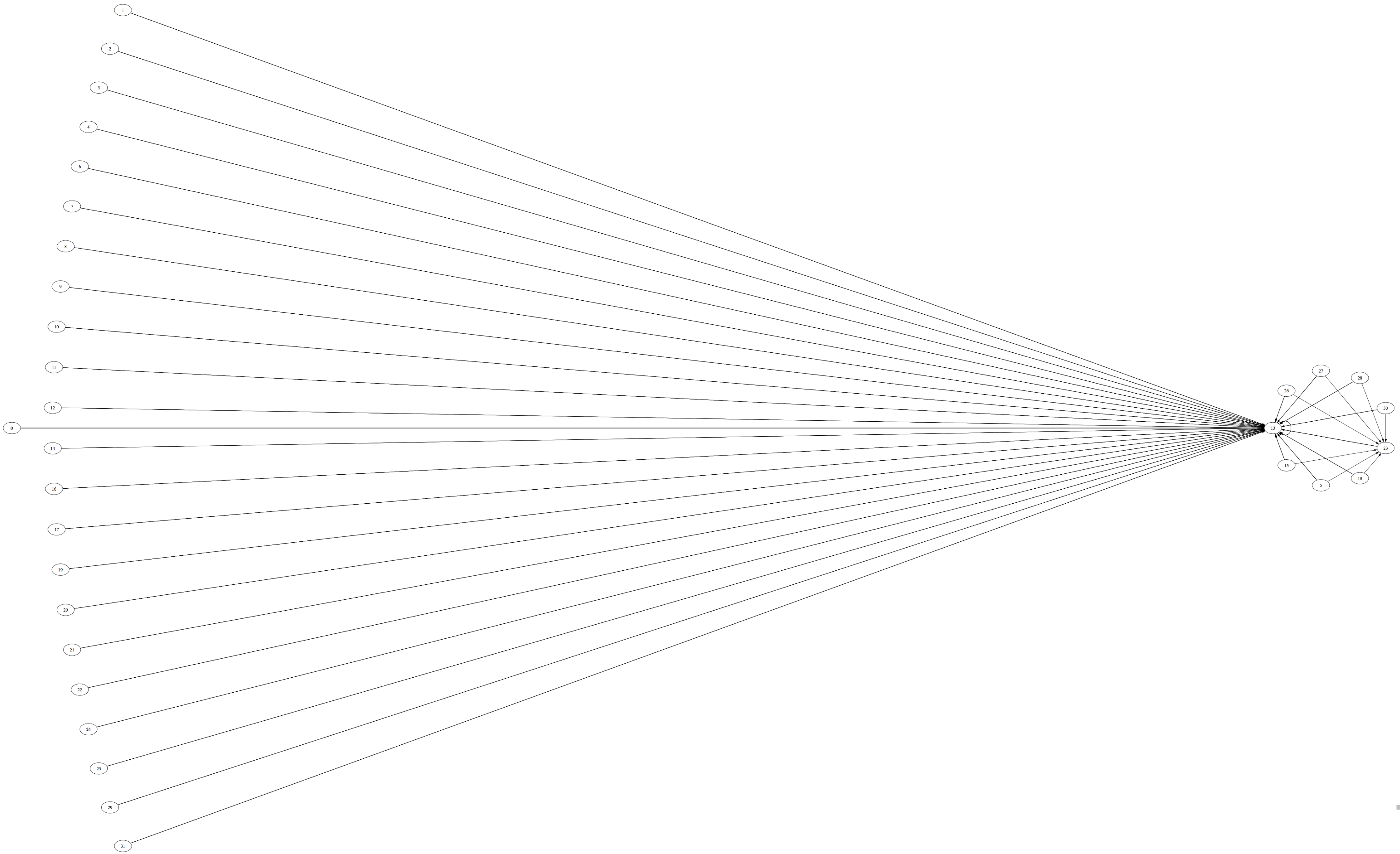
20 Iterations von Push ...



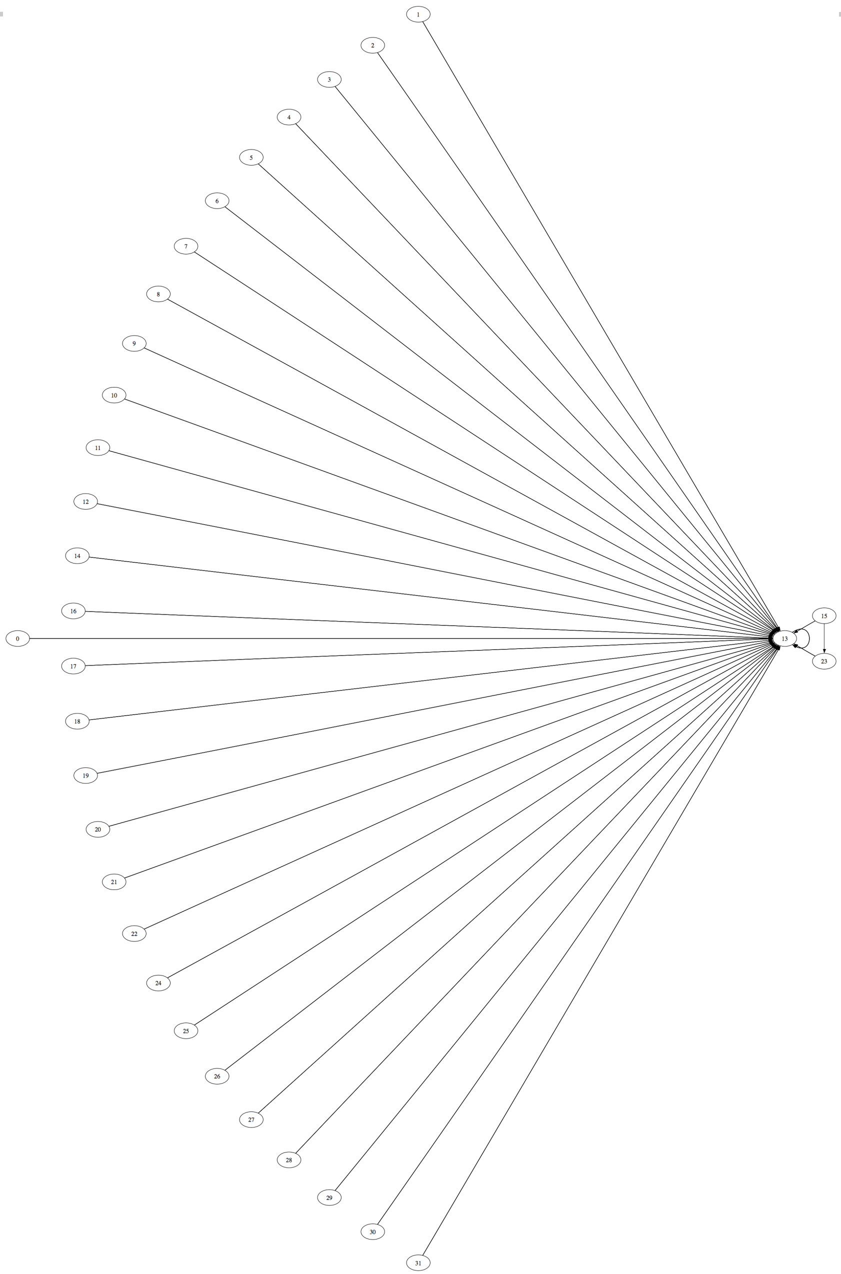
30 Iterations Push ...



40 Iterations Push ...

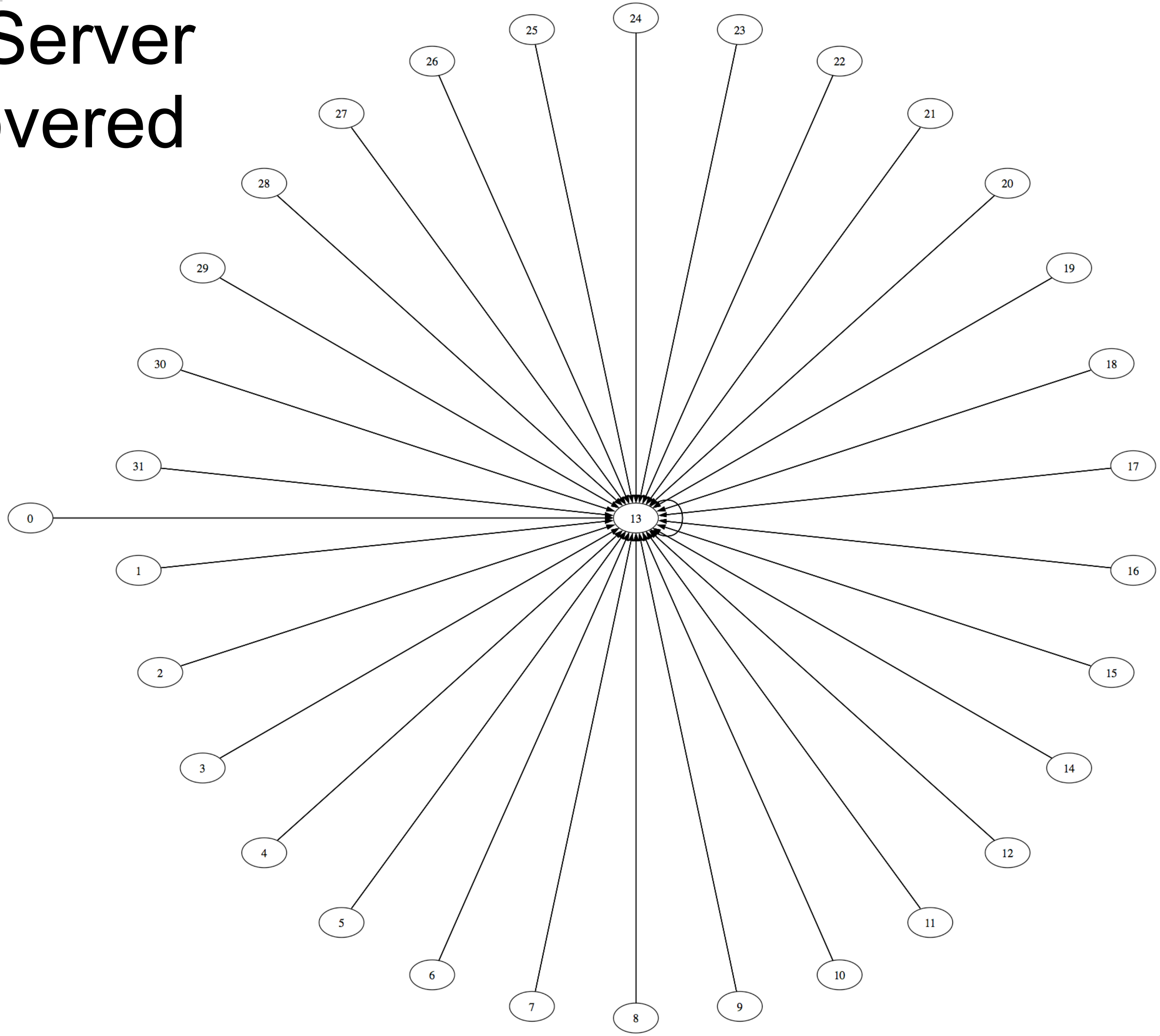


50 Iterations Push ...



70 Iterations Push ...

Client-Server rediscovered



Simulation of Pull-Operation ...

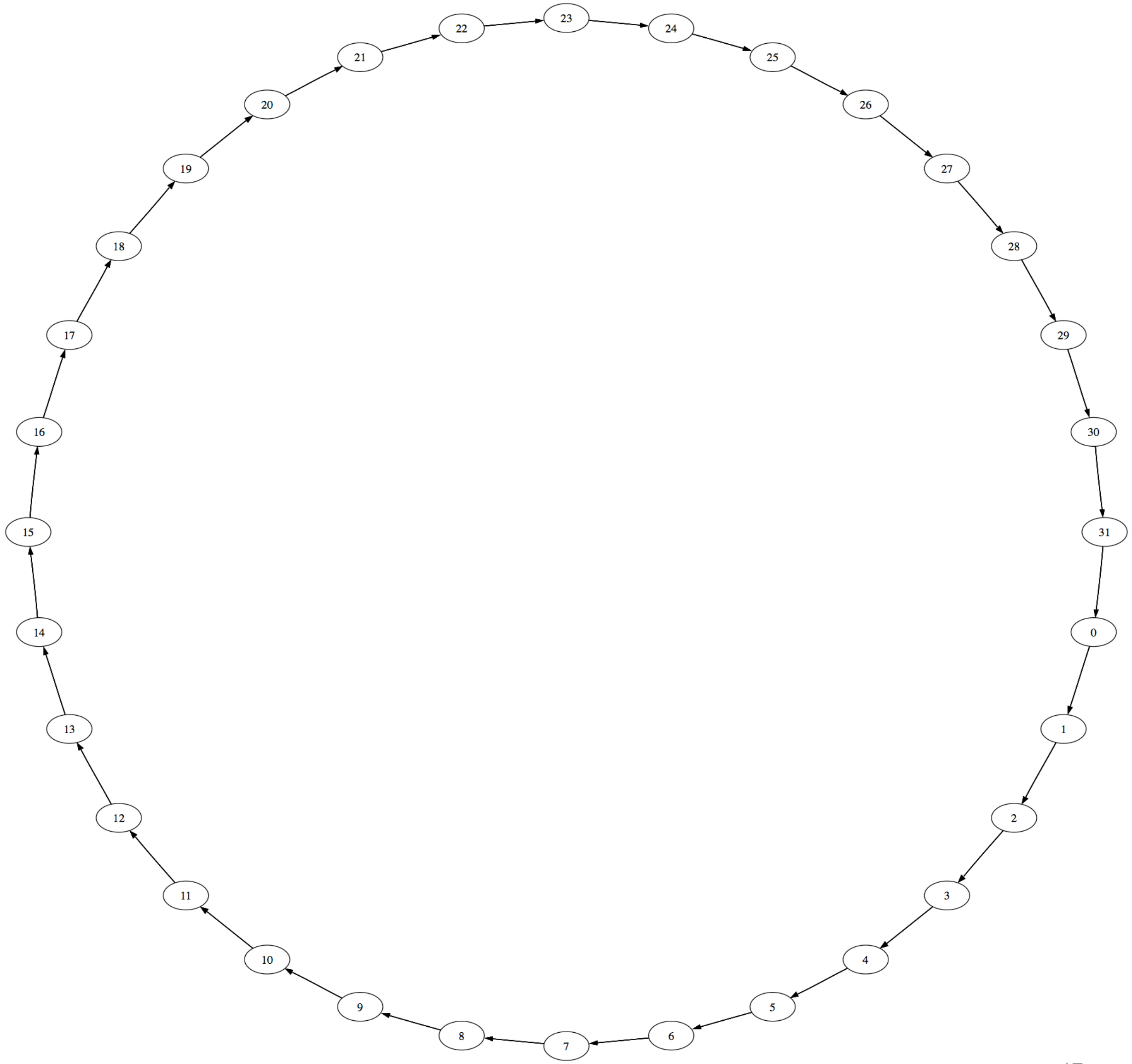
Start situation

Parameter:

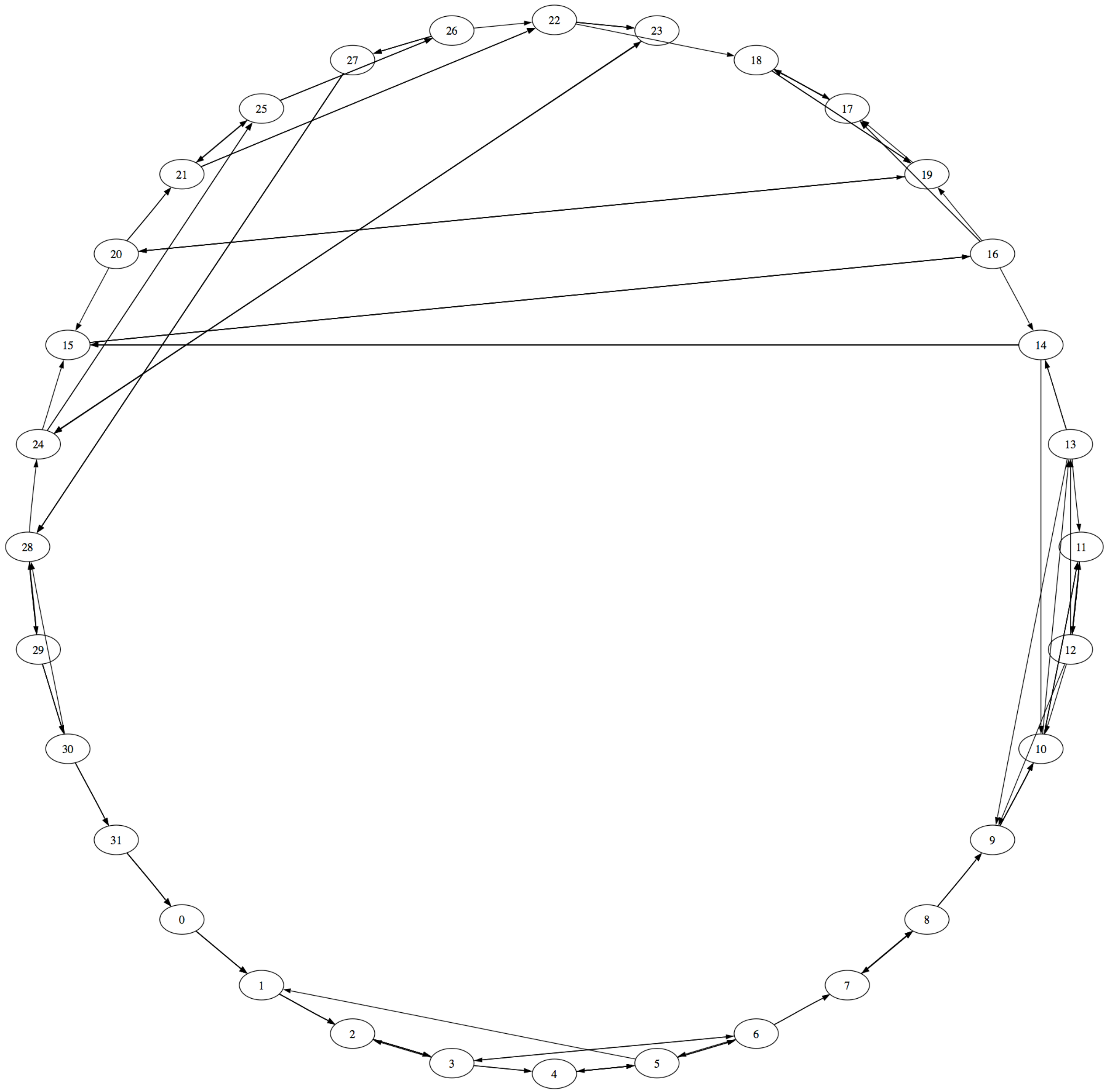
n = 32 nodes

outdegree d = 4

hop distance h = 3



1 Iteration Pull ...



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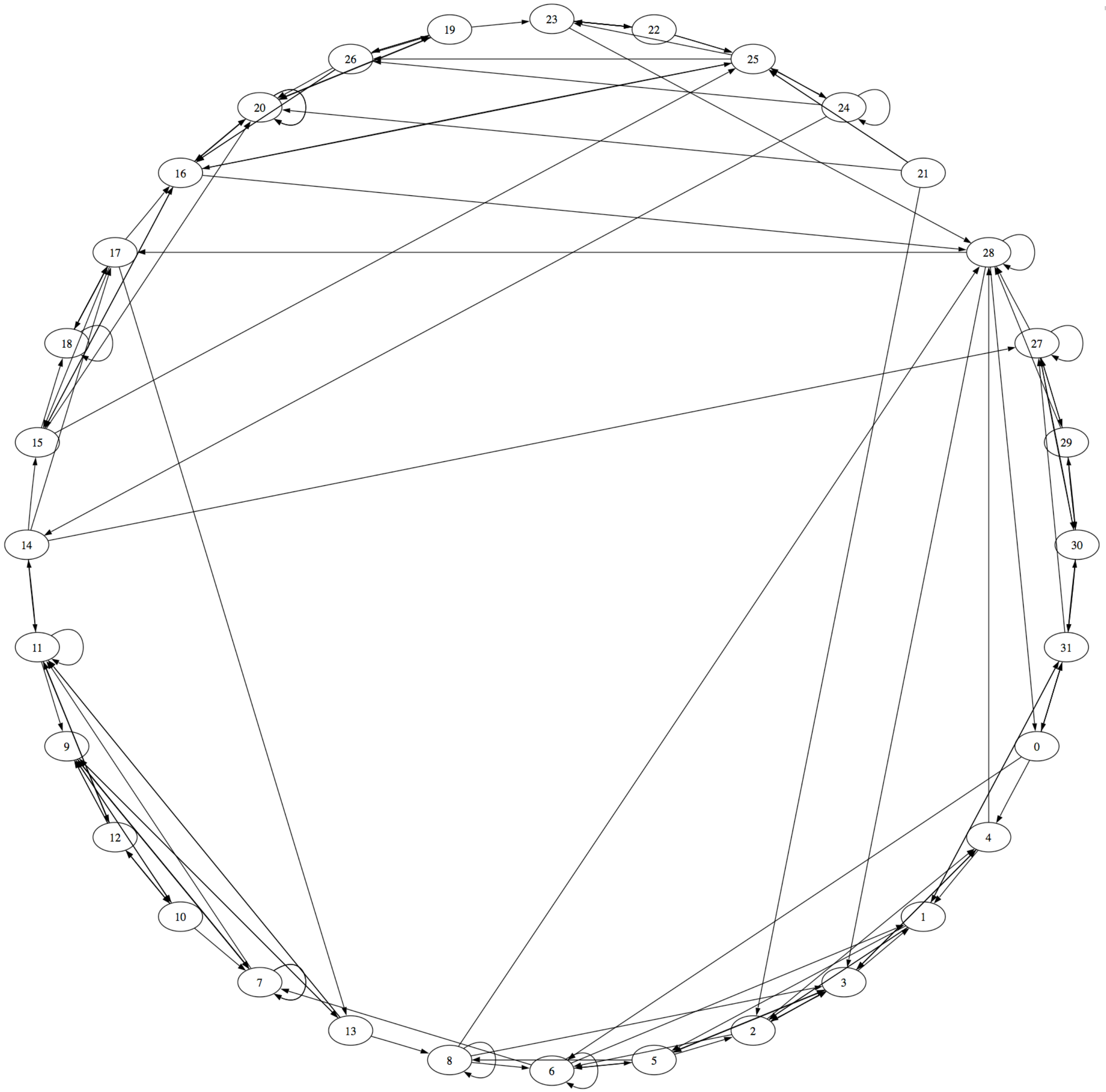
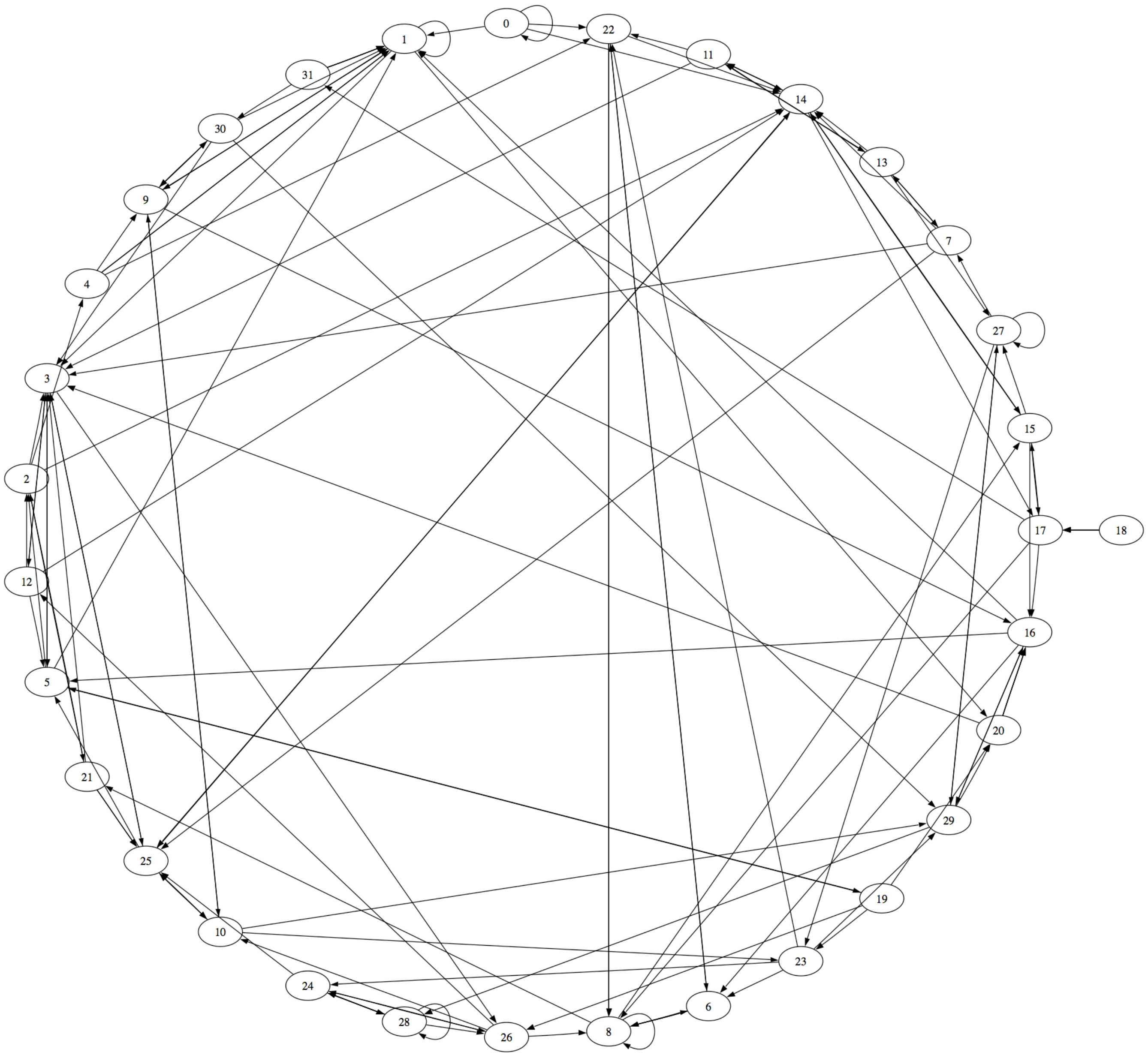
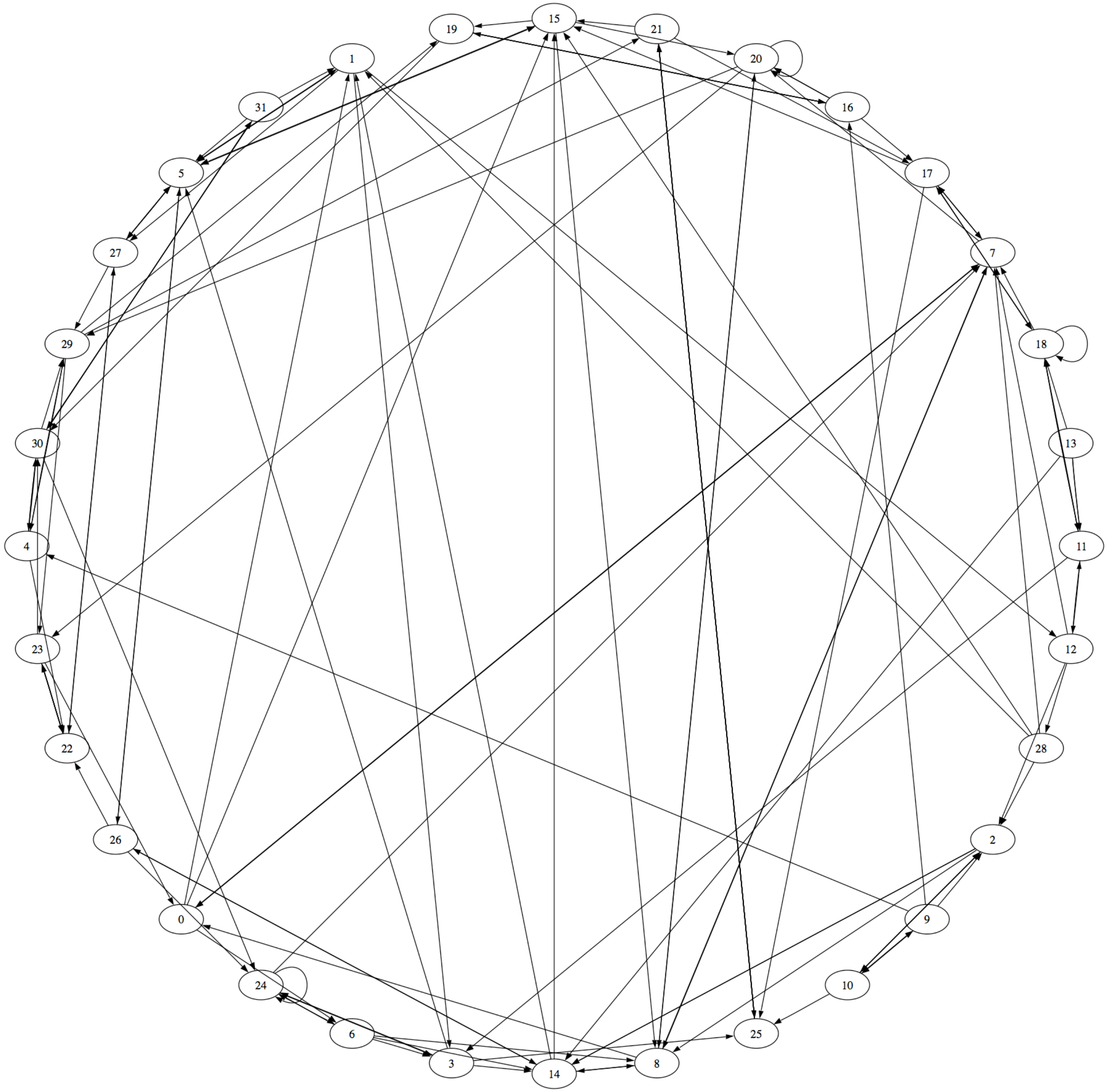


20 Iterations Pull ...



30 Iterations Pull ...



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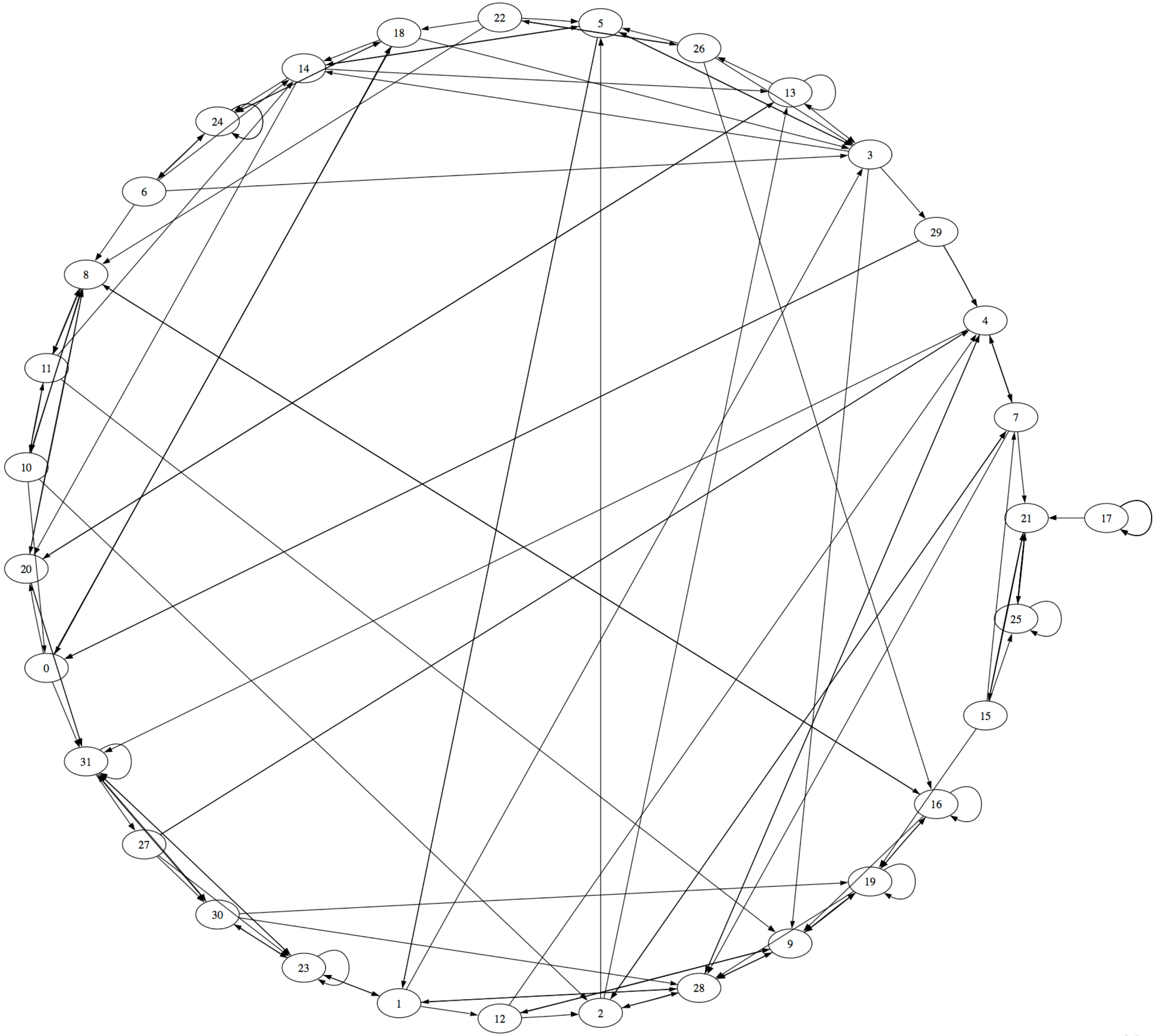
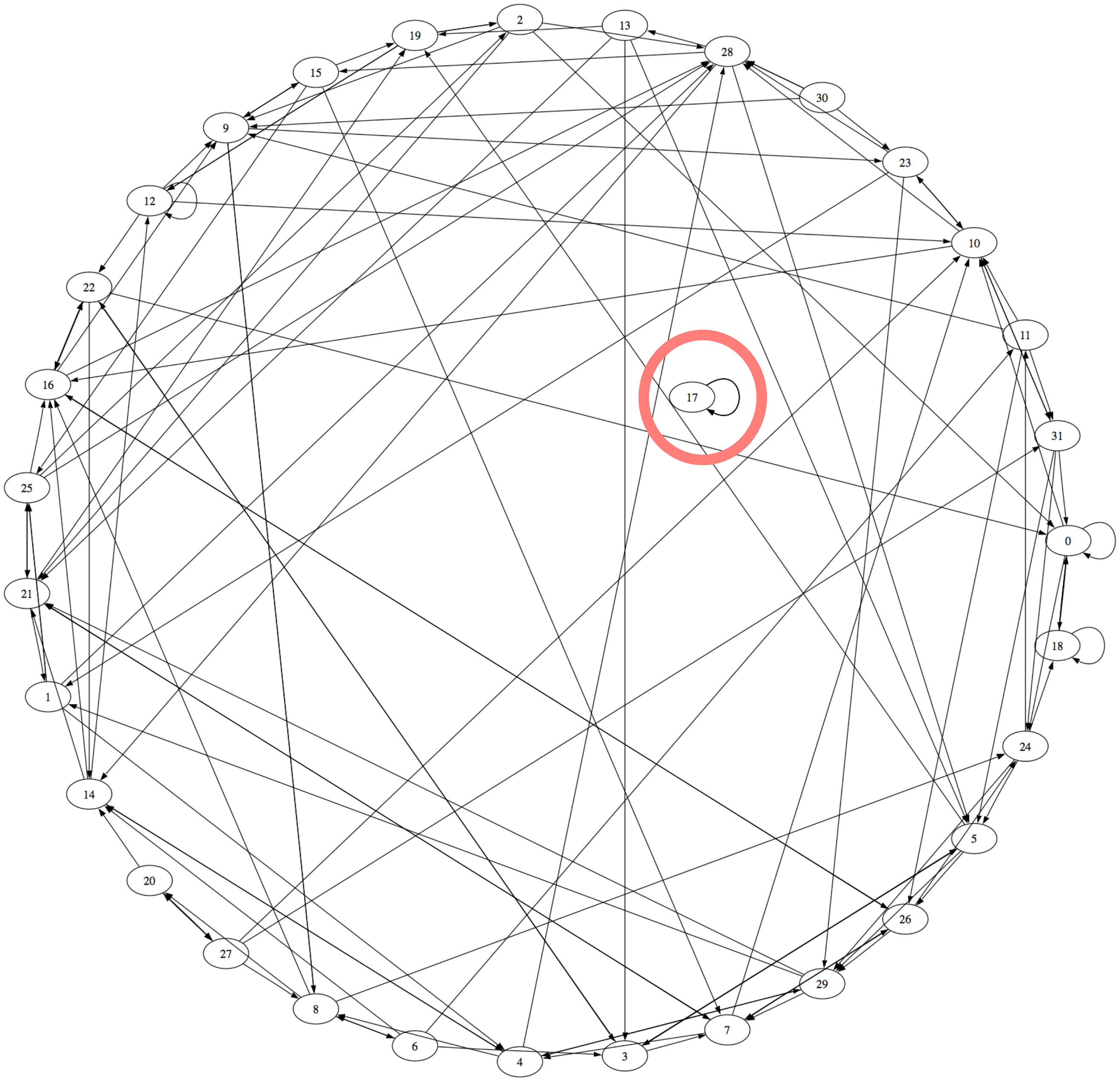
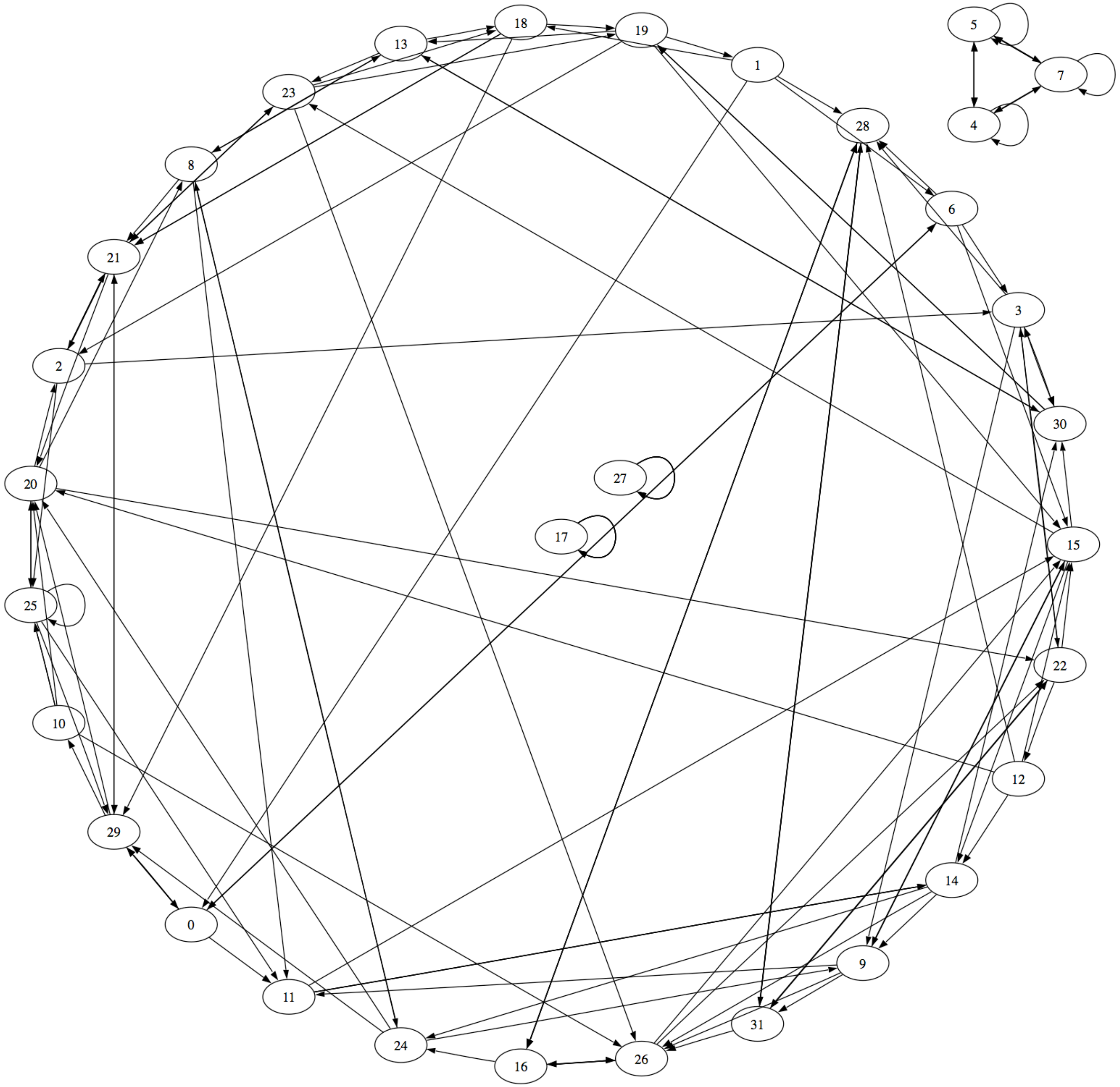


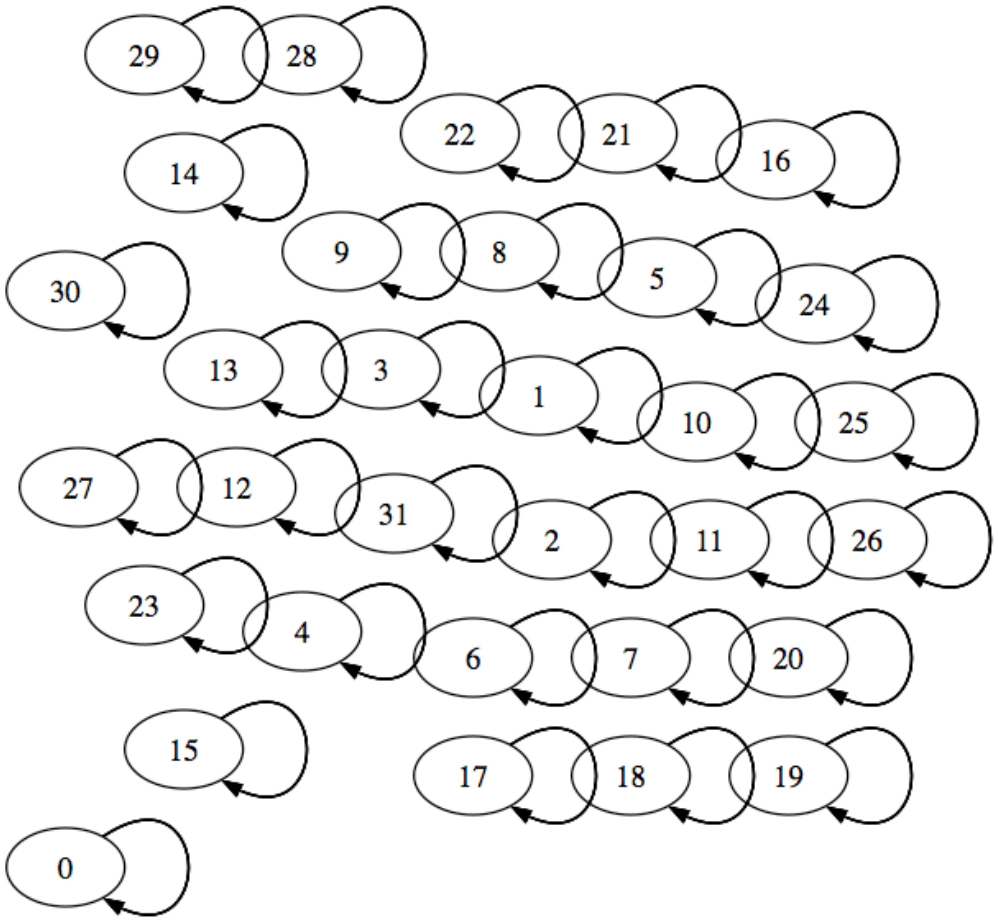
50 Iterations Pull ...



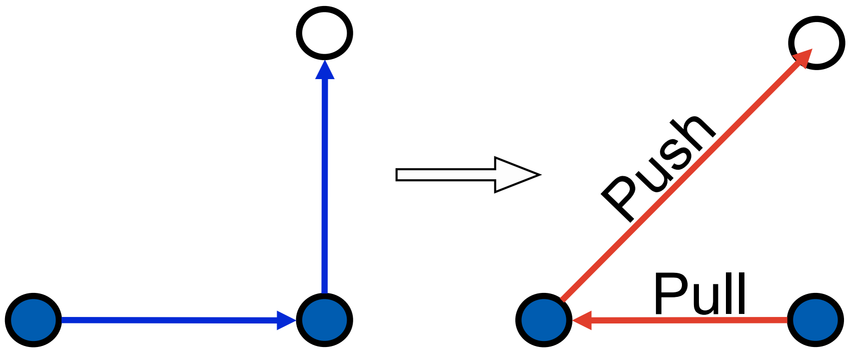
500 Iterations Pull ...



5000 Iterations Pull ...



Combination of Push and Pull

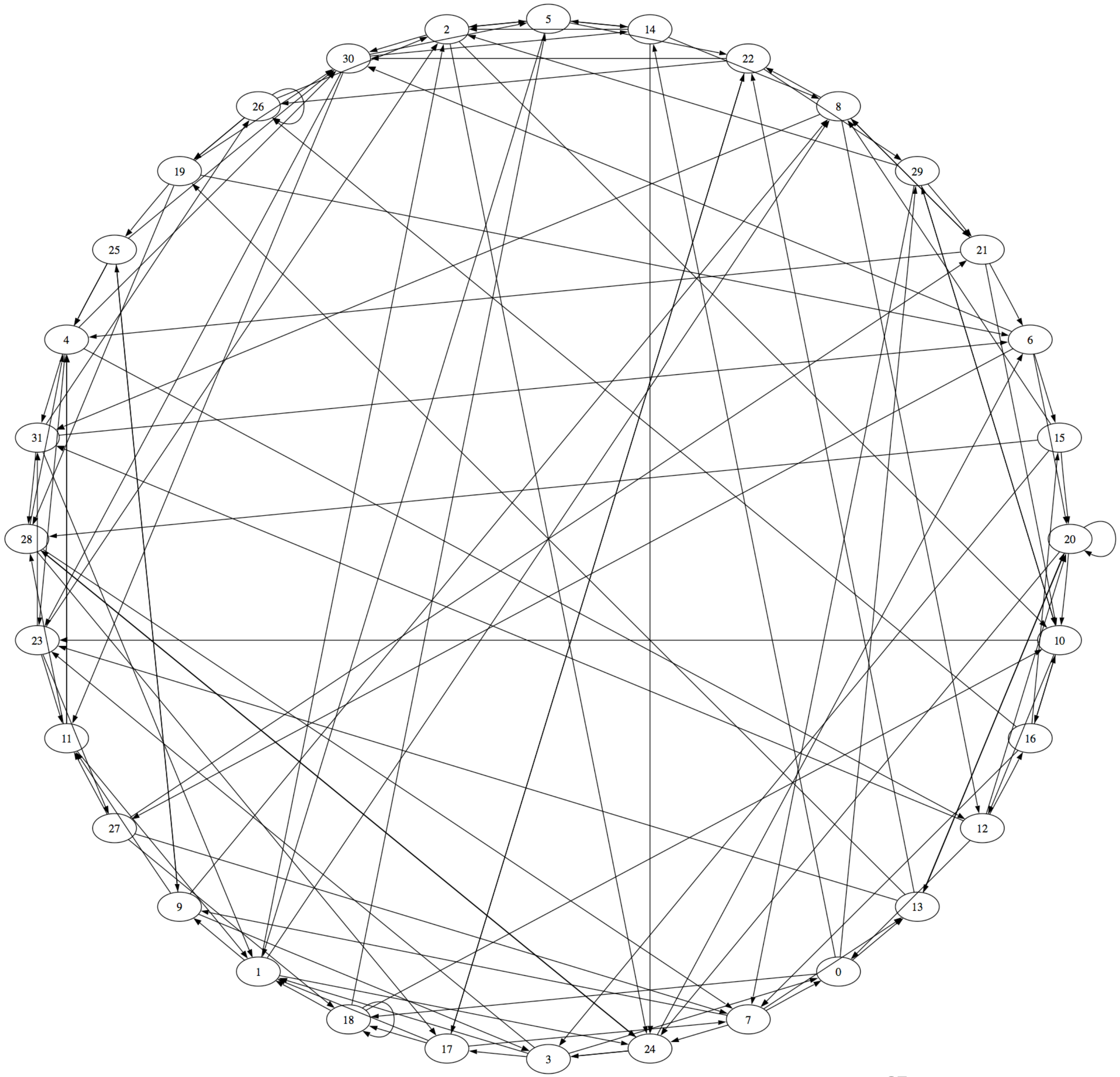


Simulation of Push&Pull-Operations ...

Same start situation

Parameters
n = 32 nodes
degree d = 4
hop-distance h = 3

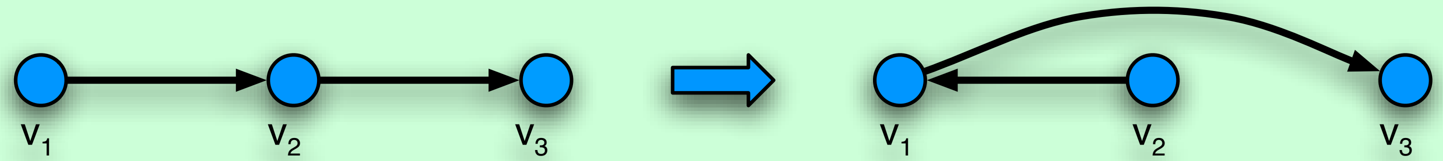
but
1.000.000 iterations



Pointer-Push&Pull for Multi-Digraphs

Pointer-Push&Pull:

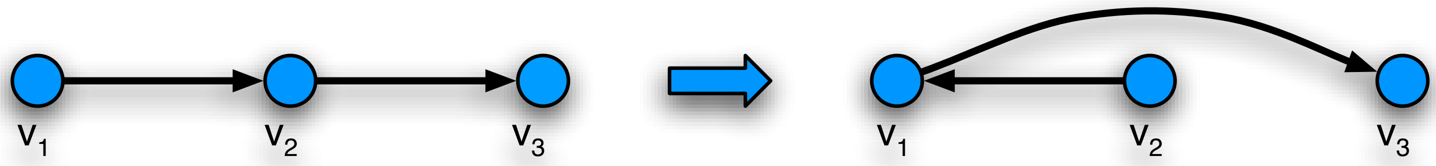
- choose random node $v_1 \in V$
- do random walk v_1, v_2, v_3
- delete edges (v_1,v_2) and (v_2,v_3)
- add edges (v_2,v_1) and (v_1,v_3)



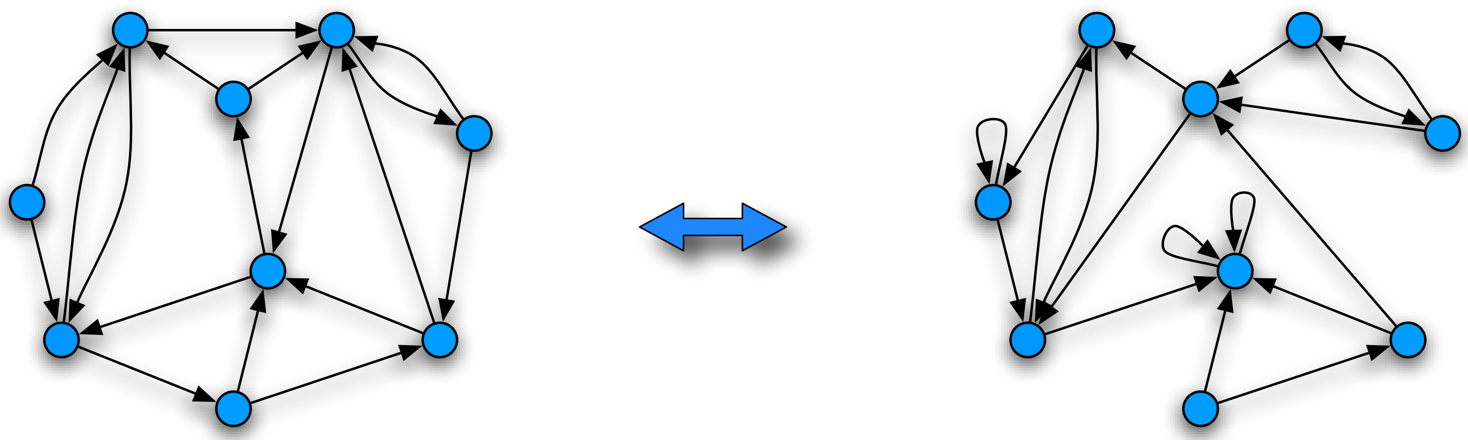
- obviously:
  - preserves connectivity of G
  - does not change out-degrees

➡ Pointer-Push&Pull is **sound** for the domain of out-regular connected multi-digraphs

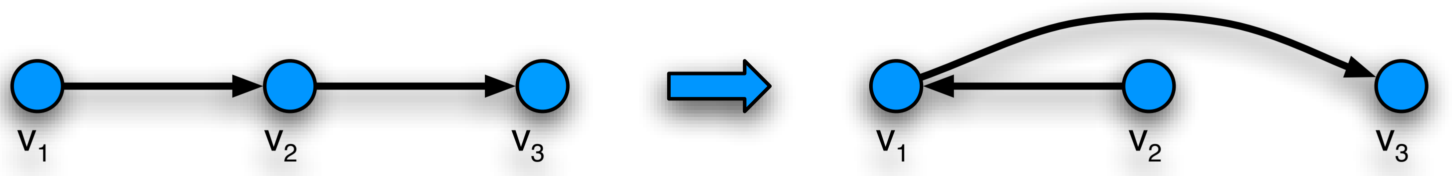
Pointer-Push&Pull: Reachability



Lemma *A series of random Pointer-Push&Pull operations can transform an arbitrary connected out-regular multi-digraph, to every other graph within this domain*



Pointer-Push&Pull: Uniformity



What is the stationary prob. distribution generated by Pointer-Push&Pull?

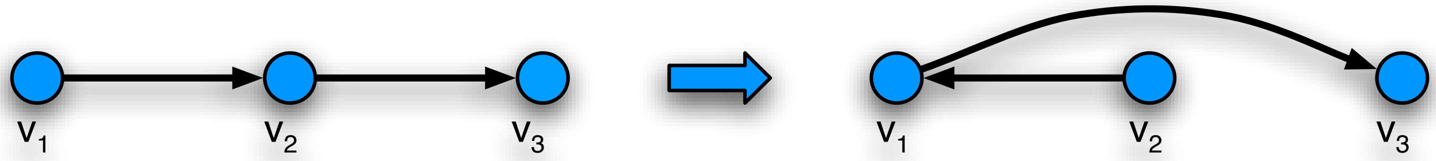
- depends on random walk

example: ***node oriented random walk***

- choose random neighboring node with $p=1/d$ respectively
- due to multi-edges possibly less than d neighbors
- if no node was chosen operation is canceled

$$P[G \xrightarrow{\mathcal{PP}} G'] = P[G' \xrightarrow{\mathcal{PP}} G]$$

Uniform Generality

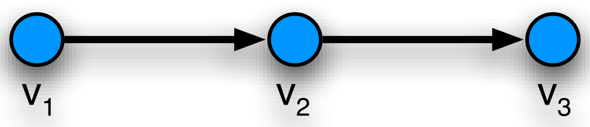


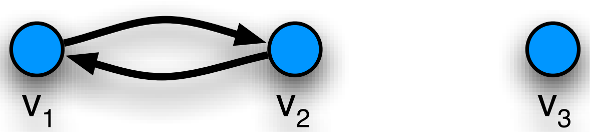
Theorem: *Let G' be a d-out-regular connected multi-digraph with n nodes. Applying Pointer-Push&Pull operations repeatedly will construct every d-out-regular connected multi-digraph with the same probability in the limit, i.e.*

$$\lim_{t\to\infty} P[G' \xrightarrow{t} G] = \frac{1}{|\mathcal{MDG}_{n,d}|}$$

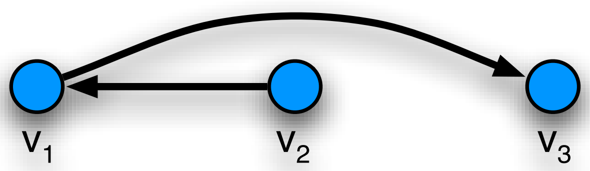
Feasibility ...

A Pointer-Push&Pull operation in the network ...





(2) v_2 replaces (v_2,v_3) by (v_2,v_1) and sends ID of v_3 to v_1



- only 2 messages between two nodes, carrying the information of one edge only
- verification of neighborhood is mandatory in dynamic networks

  $\Rightarrow$ **combine neighbor-check with Pointer-Push&Pull**

Properties of Pointer-Push&Pull

	Pointer-Push&Pull
Graphs	Directed Multigraphs
Soundness	✔
Generality	✔
Feasibility	✔
Convergence	?

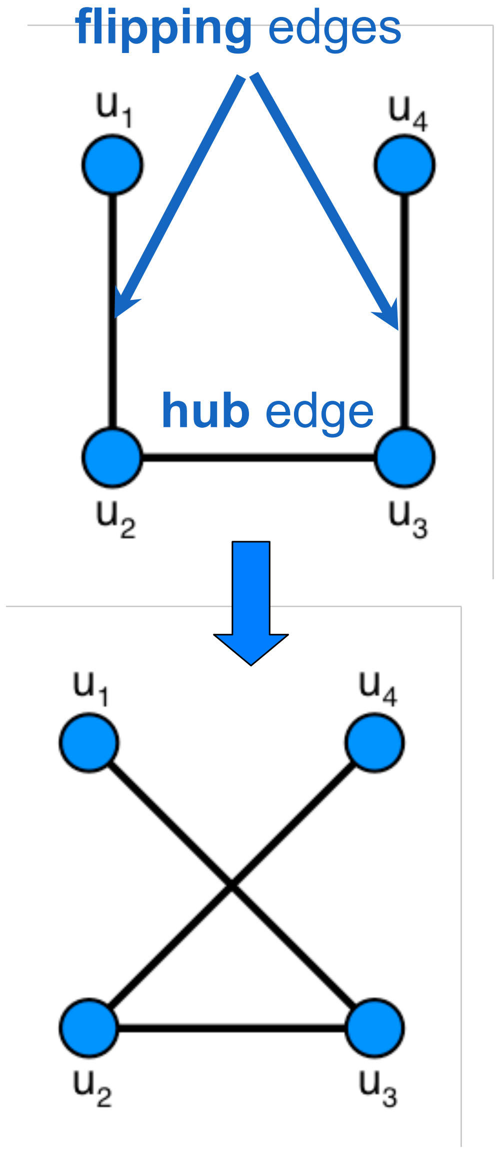
- strength of Pointer-Push&Pull is its **simplicity**
- generates truly random digraphs
- the price you have to pay: multi-edges

Open Problems:

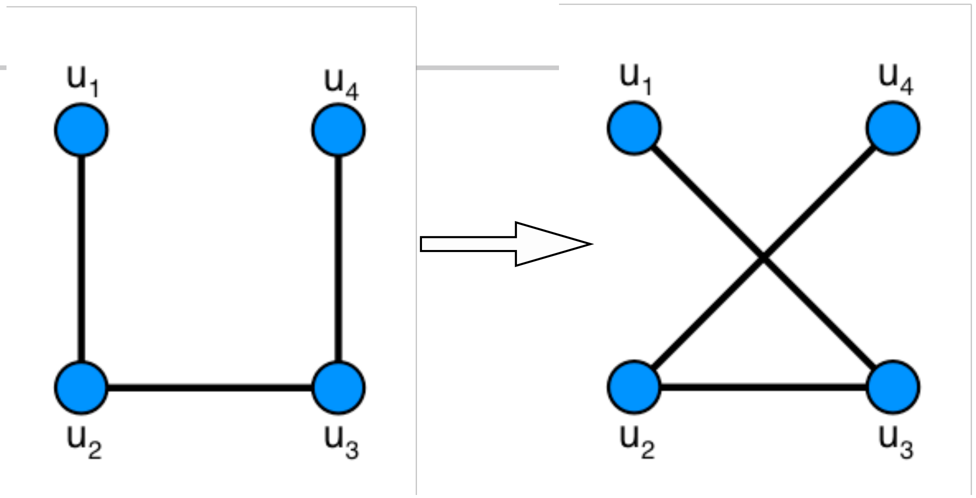
- convergence rate is unknown, conjecture $O(dn \log n)$
- is there a similar operation for simple digraphs?

The 1-Flipper (F1)

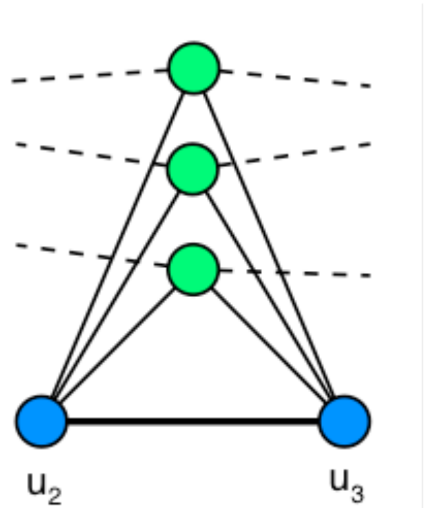
- The operation
  - choose random edge $\{u_2,u_3\} \in E$,
    - hub edge
  - choose random node $u_1 \in N(u_2)$
    - 1st flipping edge
  - choose random node $u_4 \in N(u_3)$
    - 2nd flipping edge
  - if $\{u_1,u_3\}, \{u_2,u_4\} \notin E$
    - flip edges, i.e.
    - add edges $\{u_1,u_3\}, \{u_2,u_4\}$ to E
    - remove $\{u_1,u_2\}$ and $\{u_3,u_4\}$ from E



1-Flipper is sound



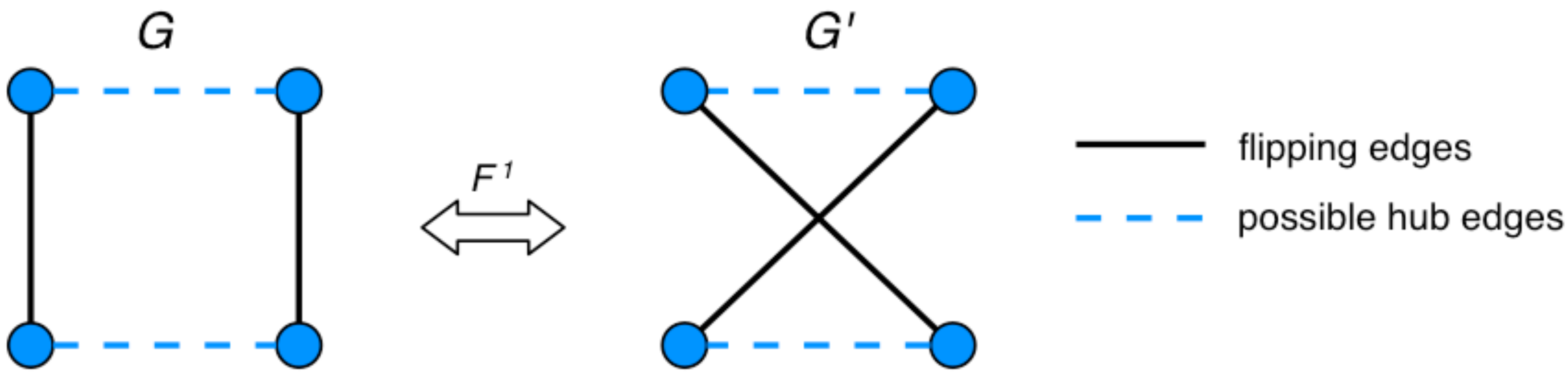
- Soundness:
  - 1-Flipper preserves d-regularity
    - follows from the definition
  - 1-Flipper preserves connectivity
    - because of the hub edge
- Observation:
  - For all d > 2 there is a connected d-regular graph G such that $P[G \xrightarrow{F^1} G] \neq 0$
  - For all d ≥ 2 and for all d-regular connected graphs at least one 1-Flipper-operation changes the graph with positive probability
    - This does not imply generality



1-Flipper is symmetric

- Lemma (symmetry):
  - For all undirected regular graphs G,G':

$$P[G \xrightarrow{F^1} G'] = P[G' \xrightarrow{F^1} G]$$



1-Flipper provides generality

- Lemma (reachability):
  - For all pairs G,G' of connected d-regular graphs there exists a sequence of 1-Flipper operations transforming G into G'.

1-Flipper properties: uniformity

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- Theorem (uniformity):
  - Let G_0 be a d-regular connected graph with n nodes and d > 2. Then in the limit the 1-Flipper operation constructs all connected d-regular graphs with the same probability:

$$\lim_{t \to \infty} P[G_0 \xrightarrow{t} G] = \frac{1}{|\mathcal{C}_{n,d}|}$$

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1-Flipper properties: Expansion

- Definition (edge boundary):
  - The edge boundary δS of a set $S \subset V$ is the set of edges with exactly one endpoint in S.

- Definition (expansion):
  A graph $G=(V,E)$ has expansion $\beta > 0$
  - if for all node sets S with $|S| \leq |V|/2$:
  - $|\delta S| \geq \beta\, |S|$

- Since for $d \in \omega(1)$ a random connected d-regular graph is a $\theta(d)$ expander asymptotically almost surely (a.a.s: in the limit with probability 1), we have
- Theorem:
  - For $d > 2$ consider any d-regular connected Graph G0. Then in the limit the 1-Flipper operation establishes an expander graph after a sufficiently large number of applications a.a.s.

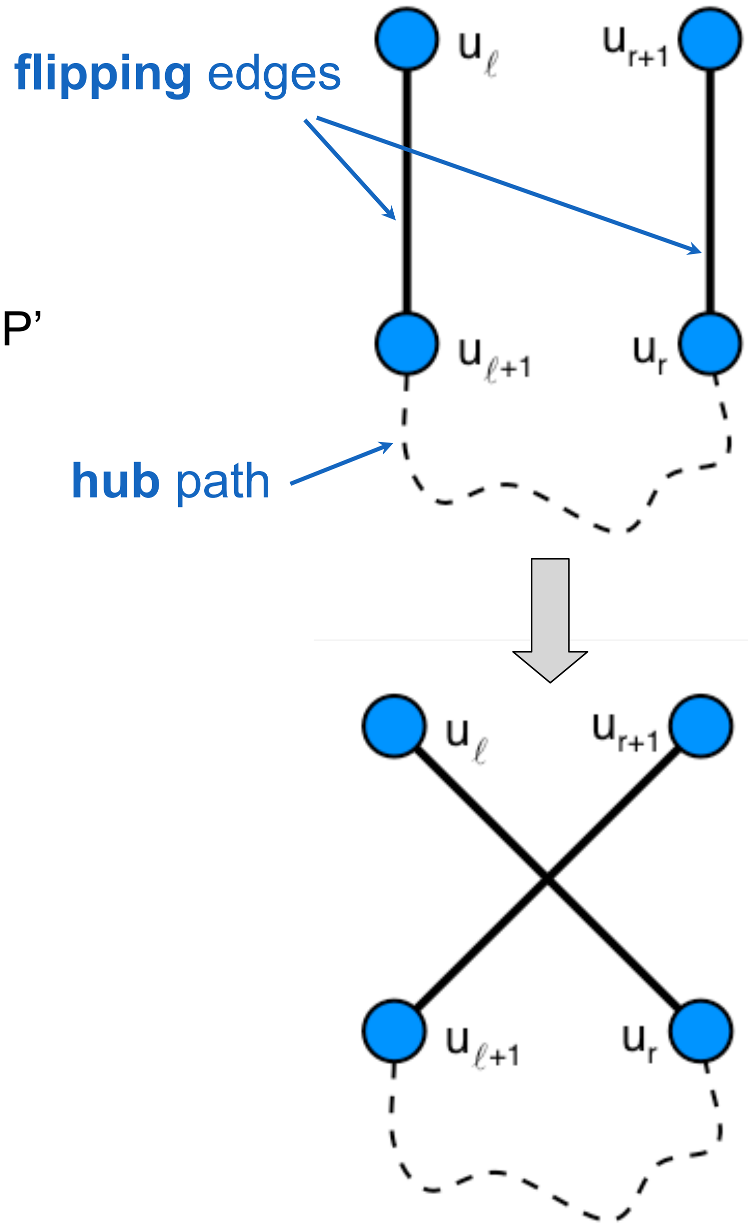
Flipper

	Flipper
Graphs	Undirected Graphs
Soundness	✔
Generality	✔
Feasibility	✔
Convergence	?

- **Flipper involves 4 nodes**
- **Generates truly random graphs**
- **Open Problems:**
  - convergence rate is polynomial
  - conjecture: $O(dn \log n)$

The k-Flipper (Fk)

- The operation
  - choose random node
  - random walk P' in G
  - choose hub path with nodes
    - $\{u_l, u_r\}$, $\{u_{l+1}, u_{r+1}\}$ occur only once in P'
  - if $\{u_l, u_r\}, \{u_{l+1}, u_{r+1}\} \notin E$
    - add edges $\{u_l, u_r\}$, $\{u_{l+1}, u_{r+1}\}$ to E
    - remove $\{u_l, u_{l+1}\}$ and $\{u_r, u_{r+1}\}$ from E

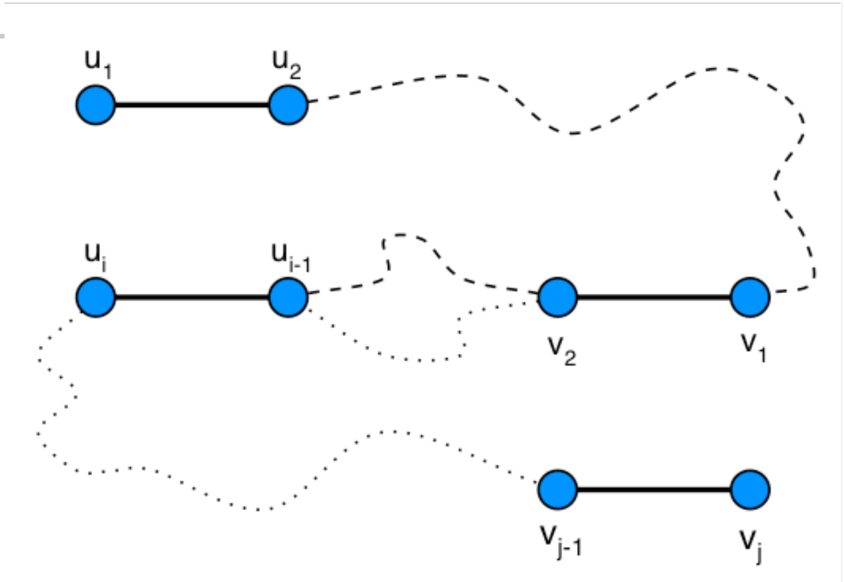


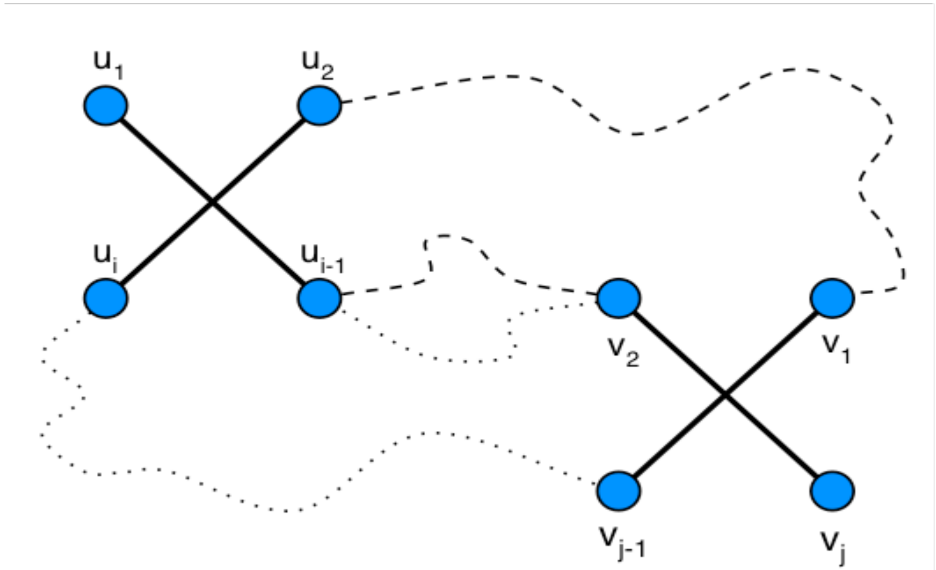
k-Flipper: Properties ...

- k-Flipper preserves connectivity and d-regularity
  - proof analogously to the 1-Flipper
- k-Flipper provides reachable,
  - since the 1-Flipper provides reachability
  - k-Flipper can emulate 1-Flipper
- But: k-Flipper is not symmetric:
  - a new proof for expansion property is needed

Concurrency ...

- In a P2P-network there are concurrent Flipper operations
  - No central coordination
  - Concurrent Flipper operations can speed up the convergence process
  - However concurrent Flipper operations can disconnect the network





k-Flipper

	k-Flipper large k	k-Flipper small k
Graphs	Undirected Graphs	Undirected Graphs
Soundness	✔	✔
Generality	✔	✔
Feasibility	⸢	✔
Convergence	✔	?

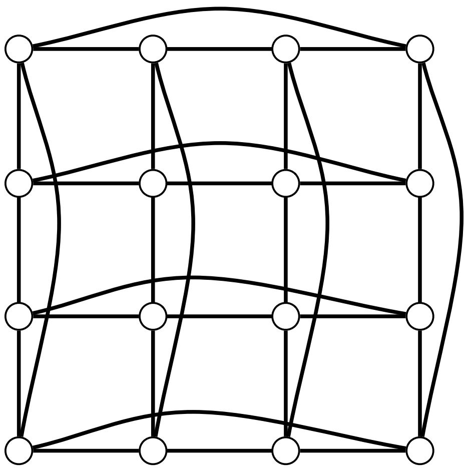
- Convergence only proven for too long paths
  - Operation is not feasible then.
  - Does k-Flipper quickly converge for small k?
- Open problem:
  - Which k is optimal?

All Graph Transformation

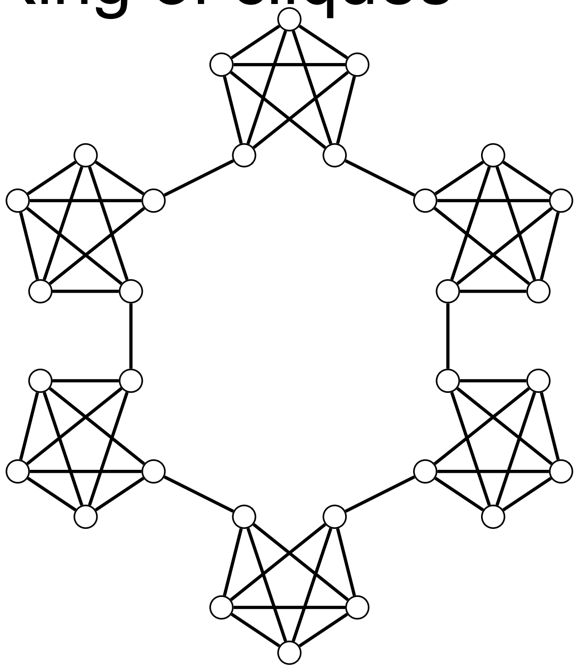
	Simple-Switching	Flipper	Pointer-Push&Pull	k-Flipper small k	k-Flipper large k
Graphs	Undirected Graphs	Undirected Graphs	Directed Multigraphs	Undirected Graphs	Undirected Graphs
Soundness	?	✔	✔	✔	✔
Generality	<	✔	✔	✔	✔
Feasibility	✔	✔	✔	✔	<
Convergence	✔	?	?	?	✔

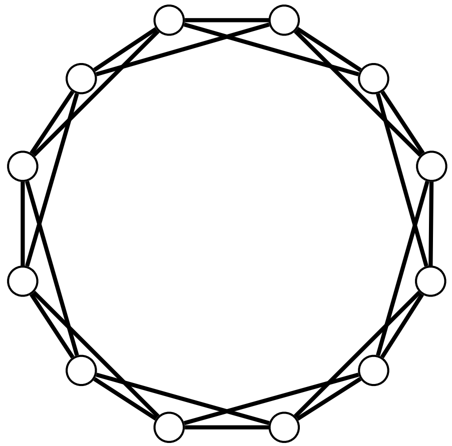
- **Open Problems**
  - Conjecture: Flipper converges in after O(dn log n) operations to a truly random graph
  - Conjecture: k-Flipper converges faster, but involves more nodes and flags
  - Conjecture: k-Flipper does not pay out
- **Empirical Simulations**
  - Estimate expansion by eingenvalue gap
  - Estimate eigenvalue gap by iterated multiplication of a start vector

Start Graphs

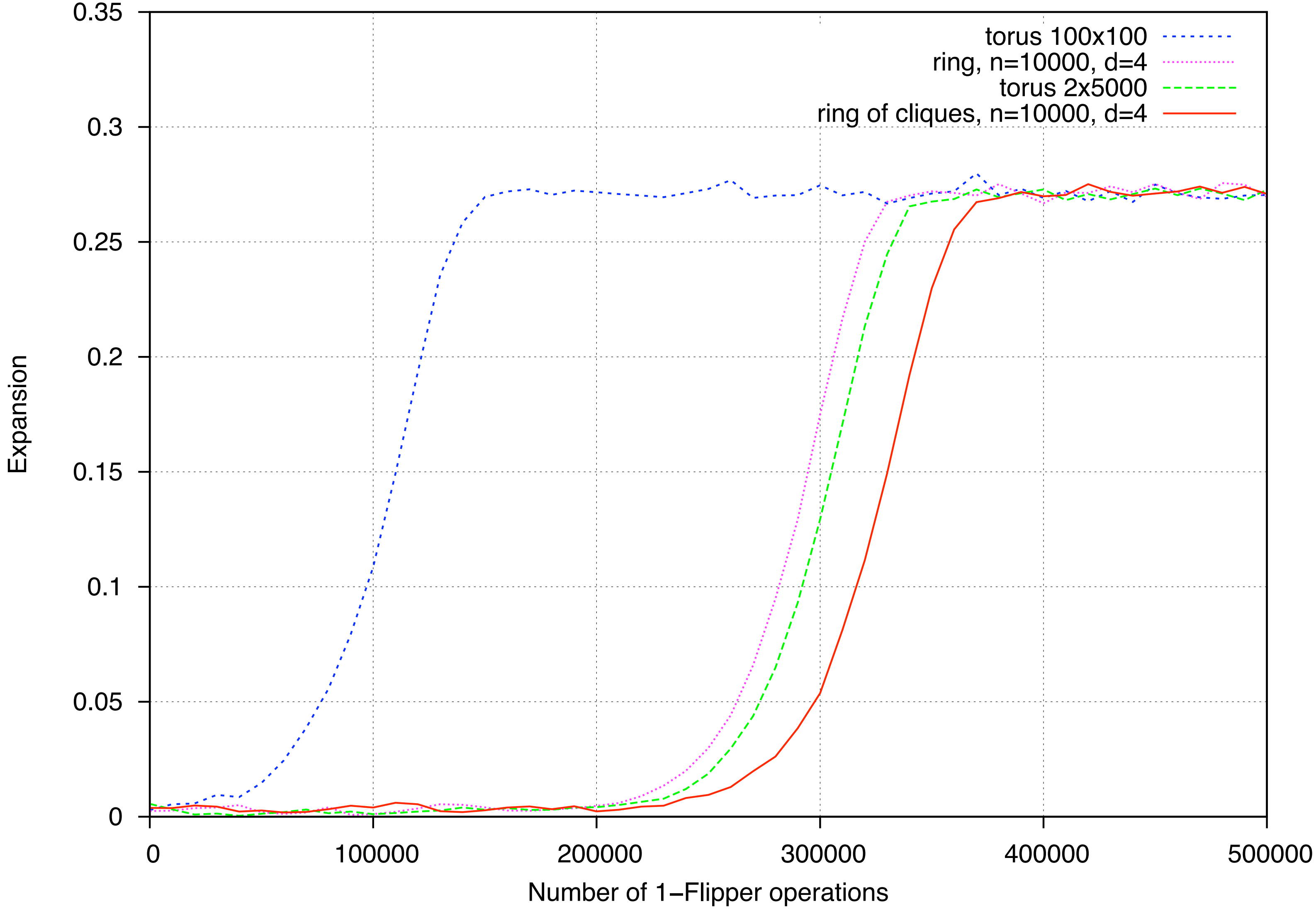


- Ring with neighbor edges
- Torus
- Ring of cliques

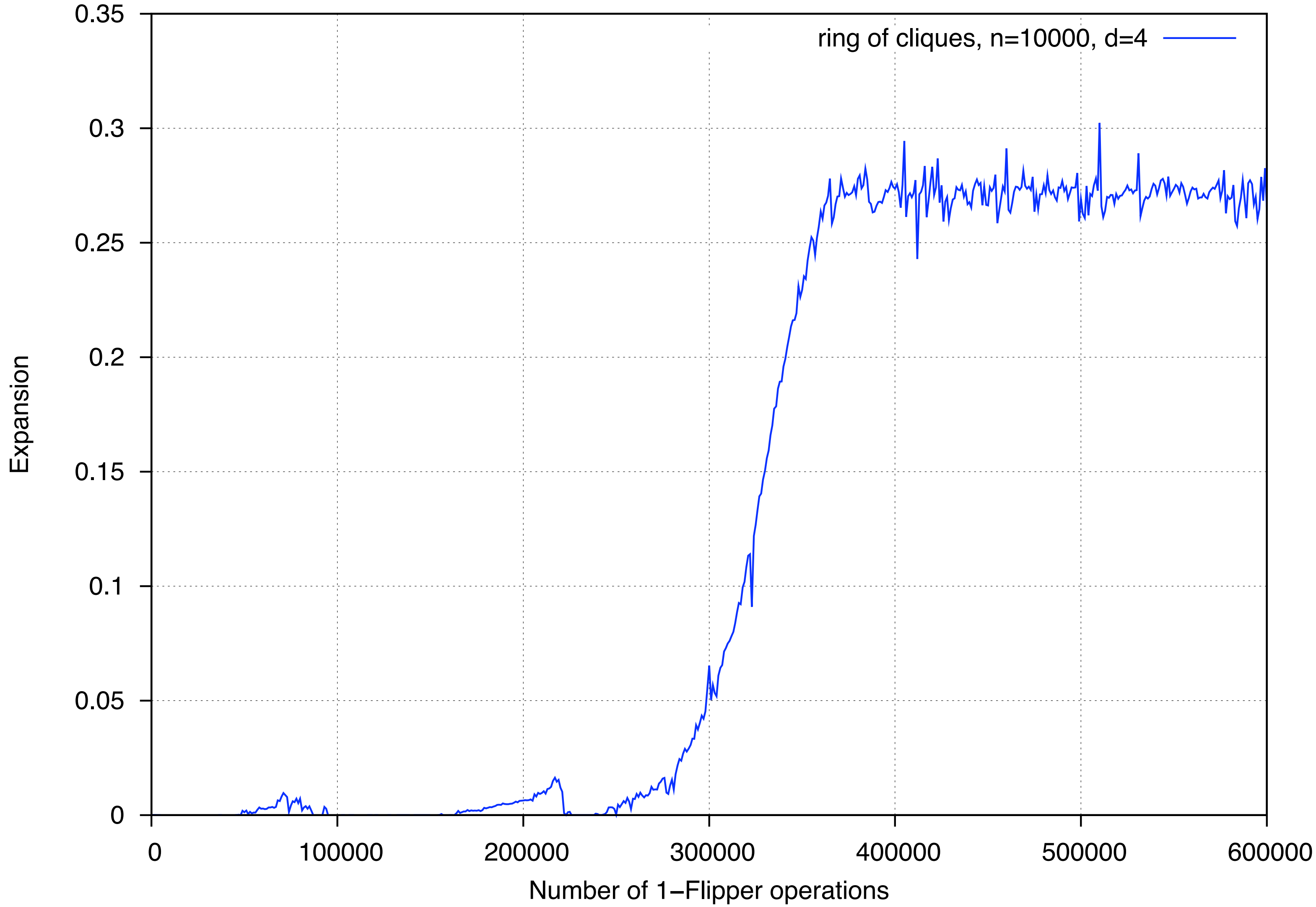




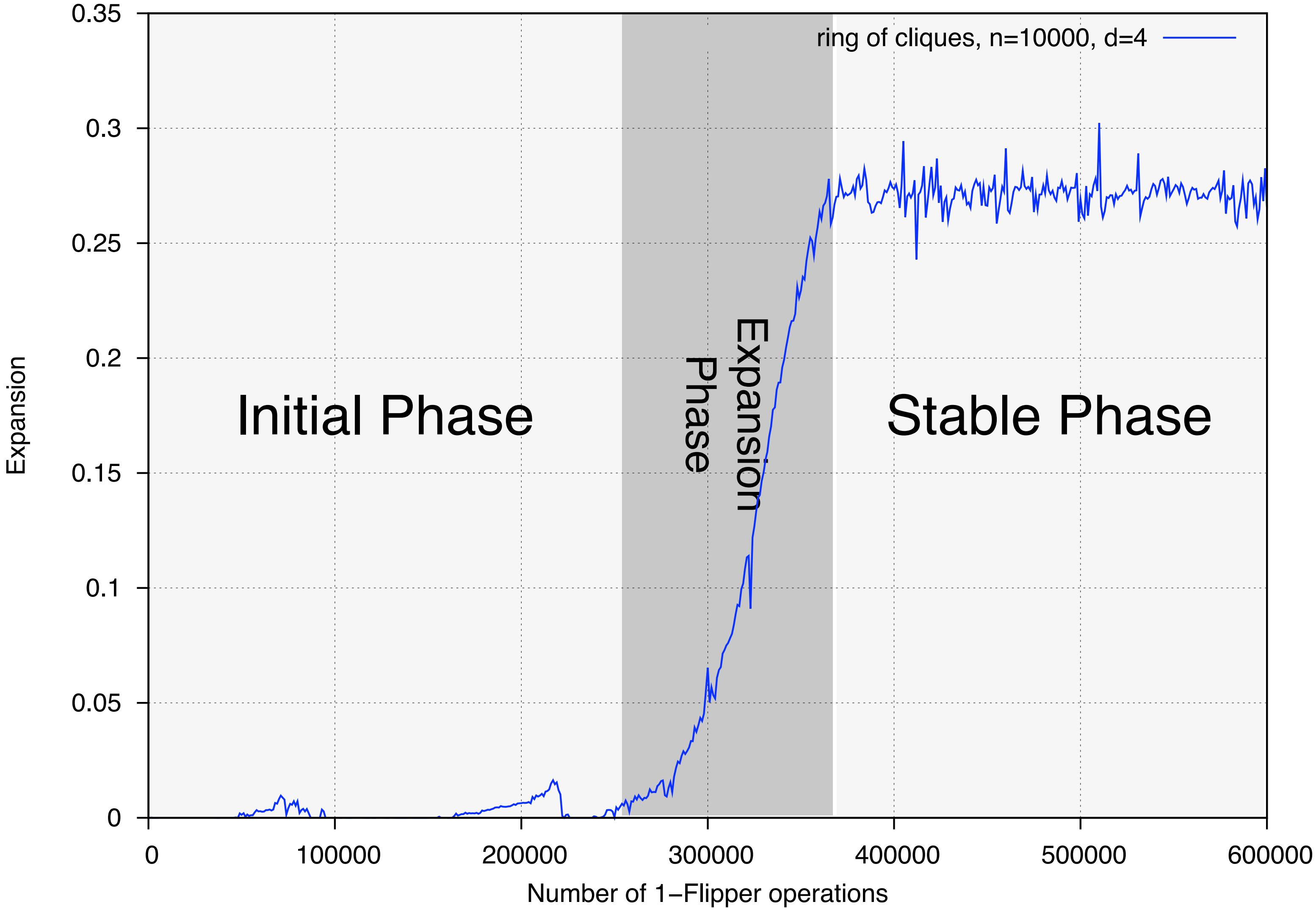
Flipper
Influence of the Start Graph



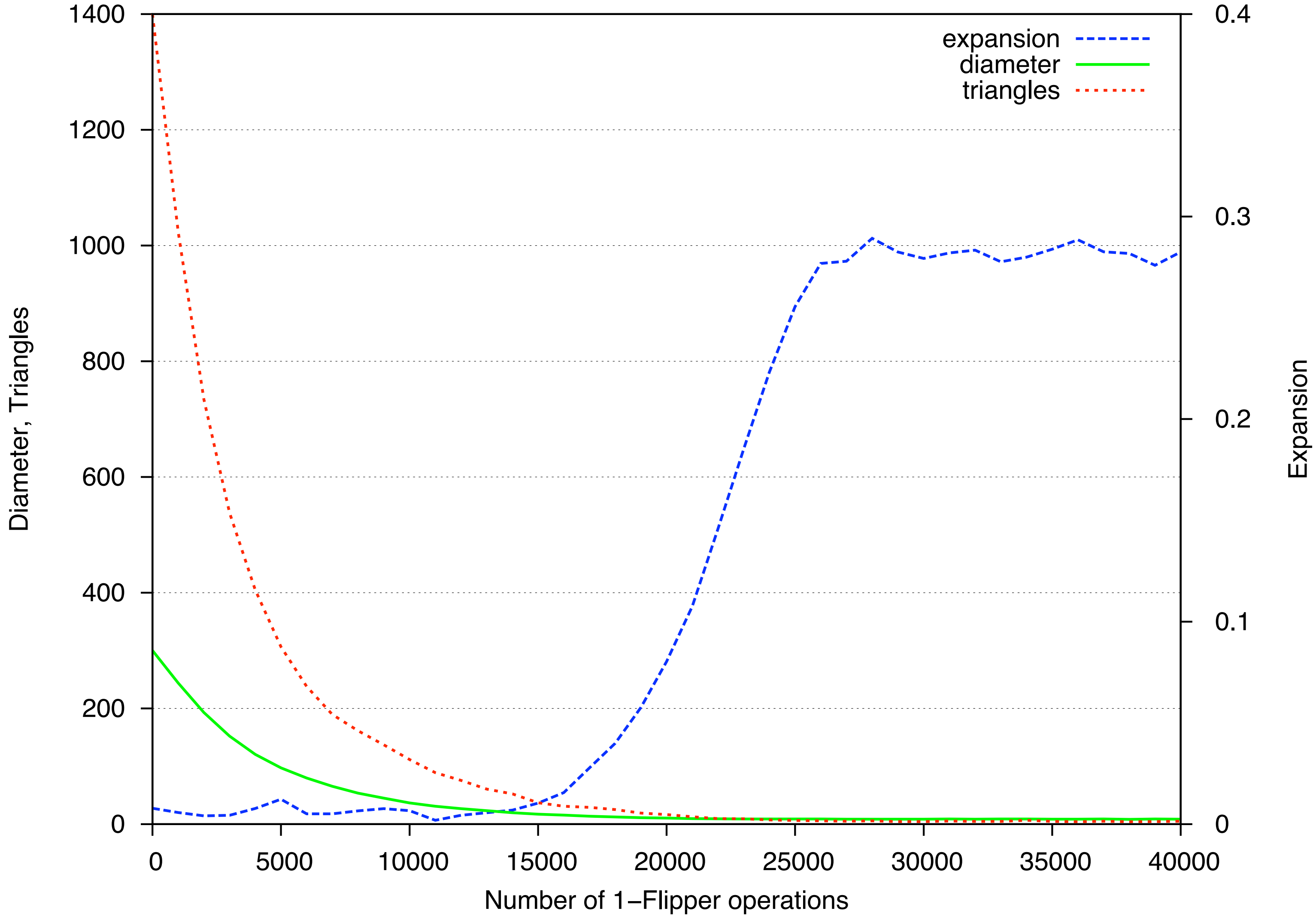
Development of Expansion



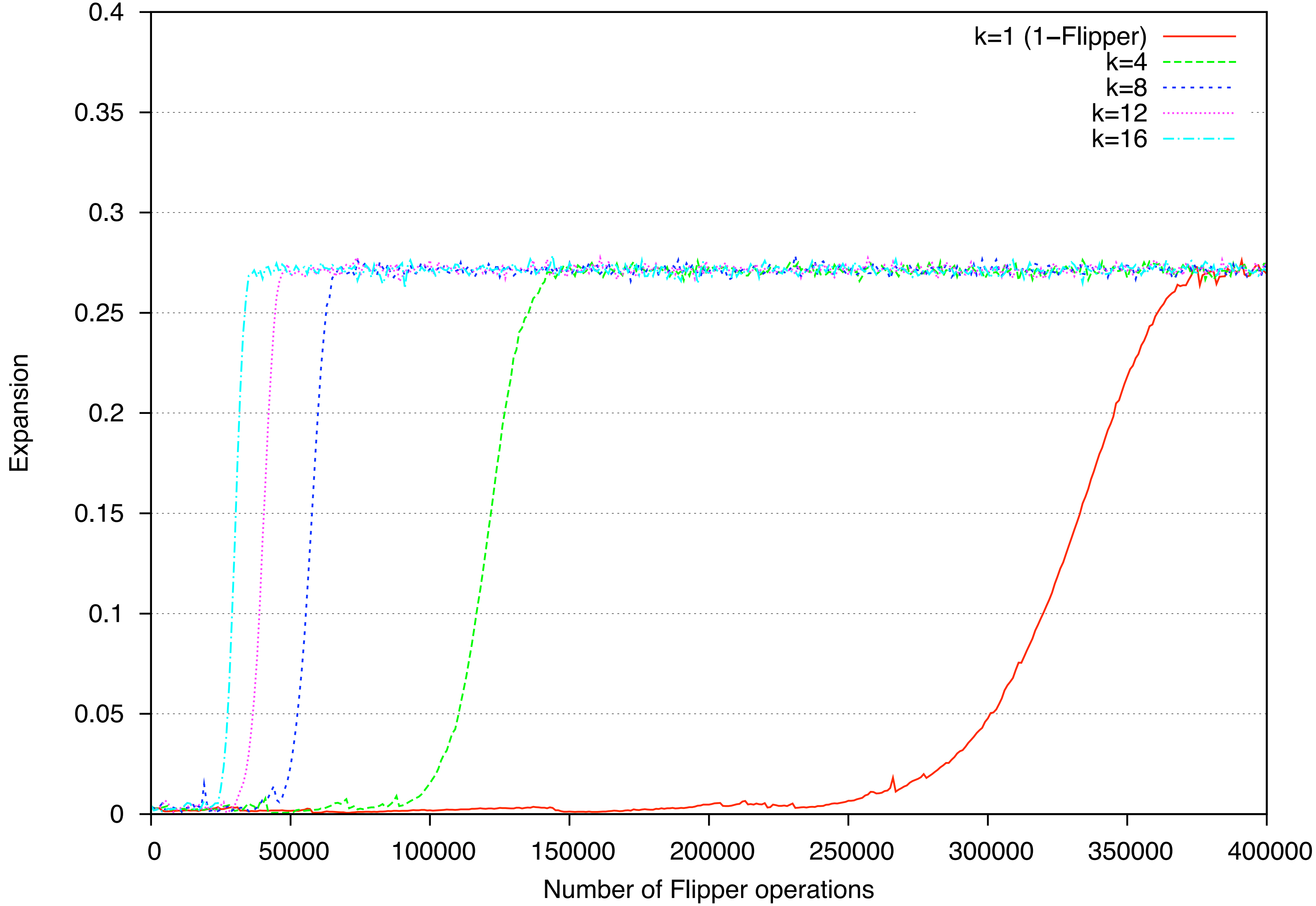
Development of Expansion



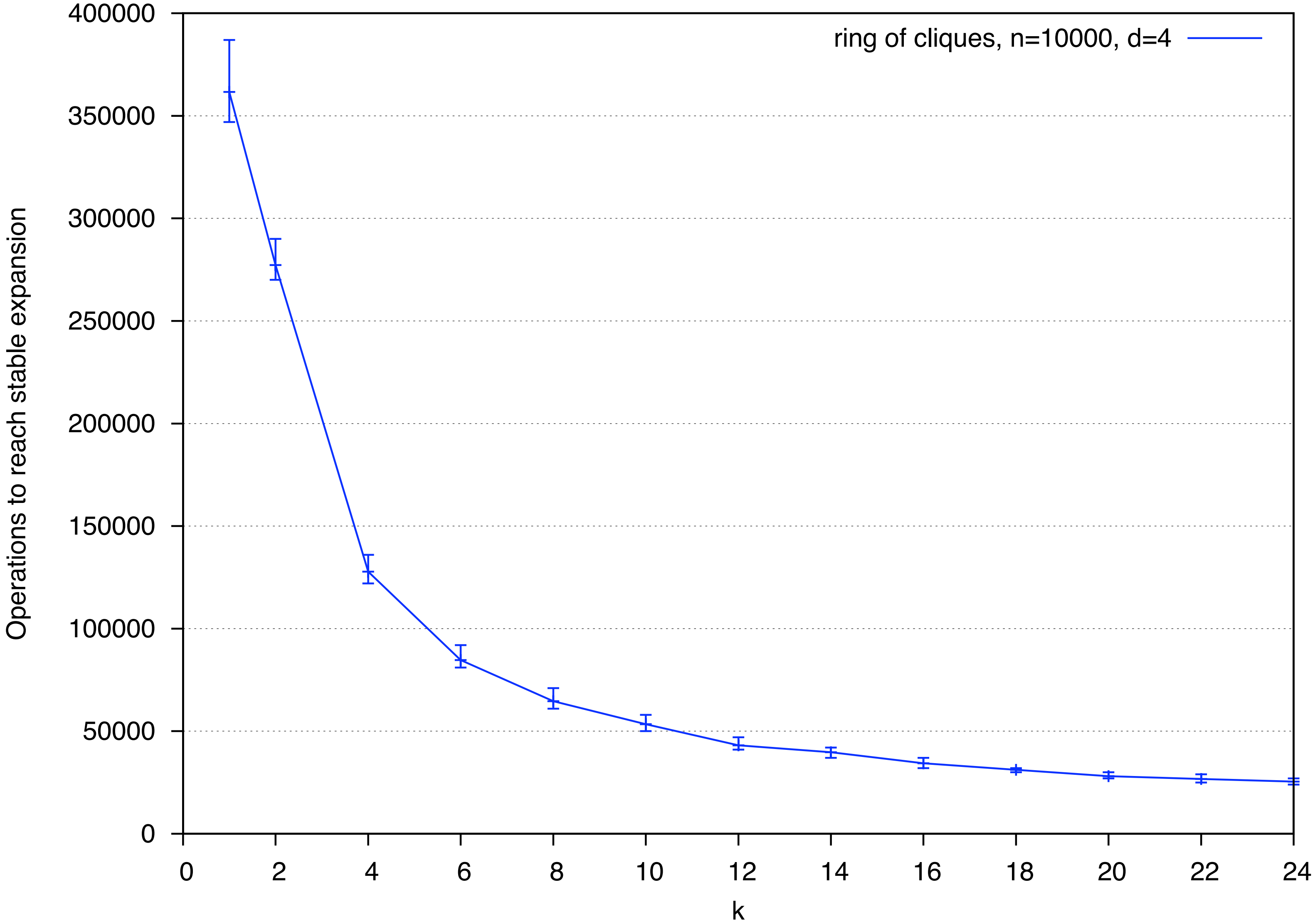
Expansion, Diameter & Triangles



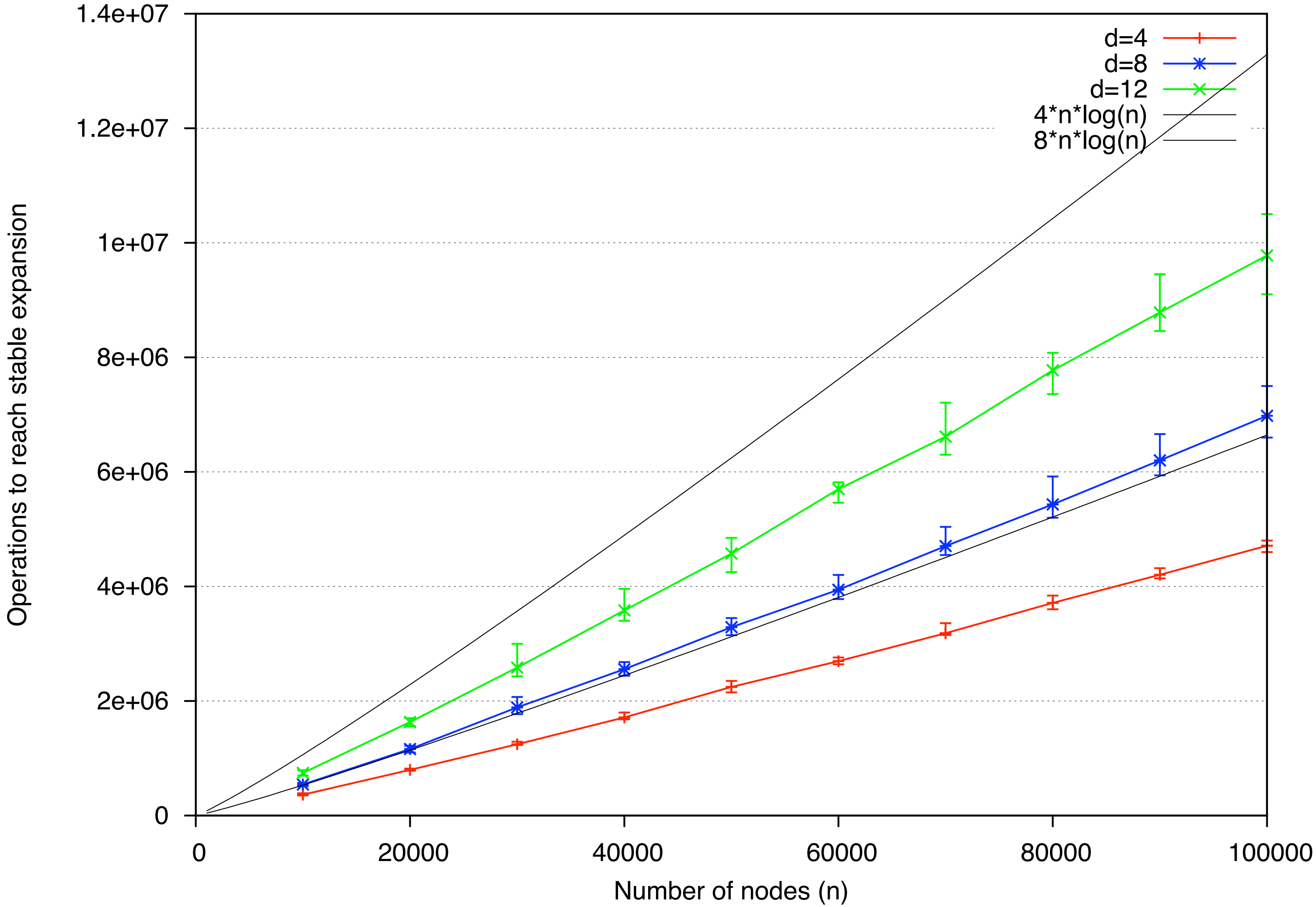
k-Flipper
Start Graph: Ring of Cliques



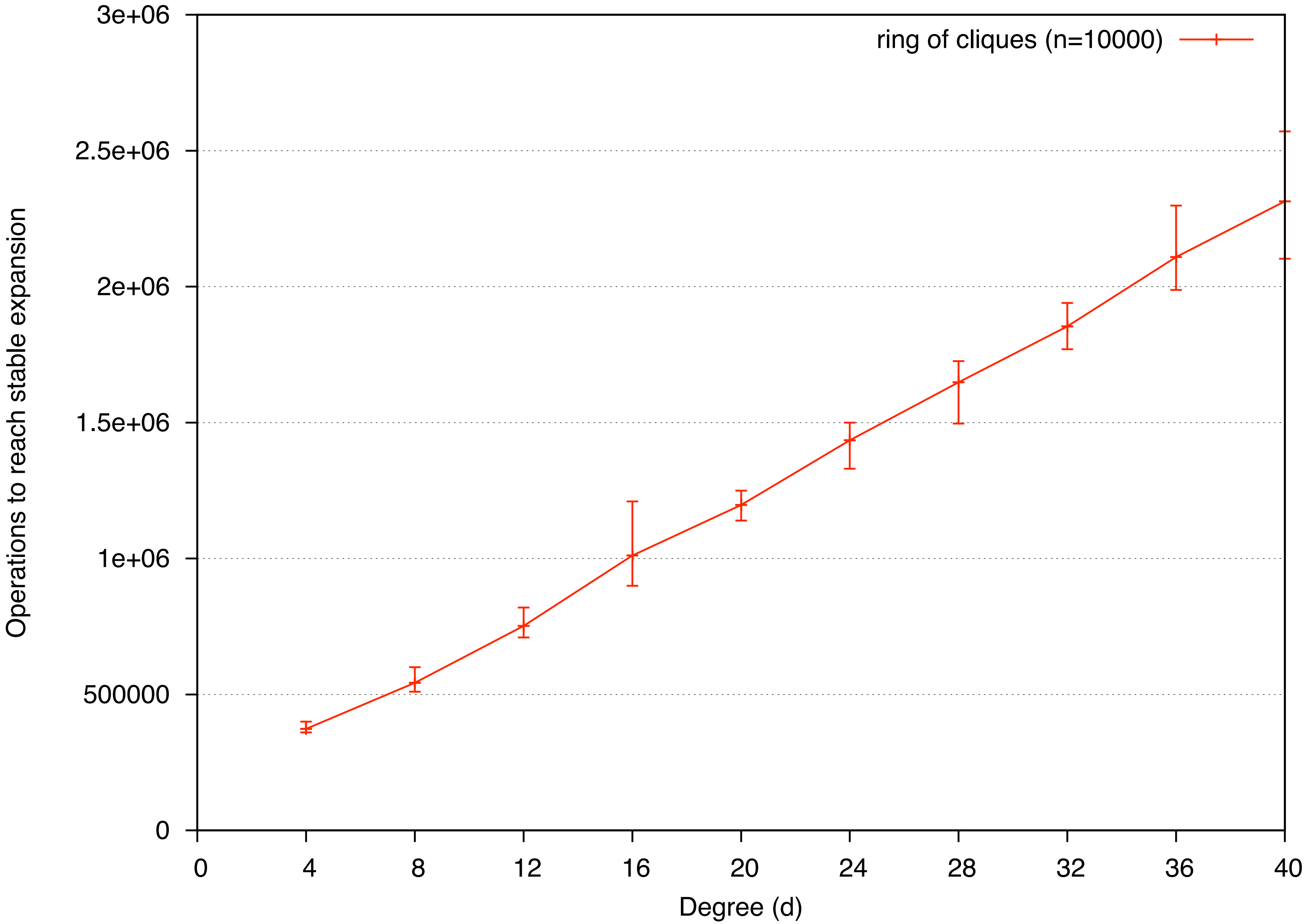
k-Flipper
Start Graph: Ring of Cliques



Convergence of Flipper



Convergence of Flipper
Varying Degree



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All Graph Transformation

	Simple-Switching	Flipper	Pointer-Push&Pull	k-Flipper small k	k-Flipper large k
Graphs	Undirected Graphs	Undirected Graphs	Directed Multigraphs	Undirected Graphs	Undirected Graphs
Soundness	?	✔	✔	✔	✔
Generality	☇	✔	✔	✔	✔
Feasibility	✔	✔	✔	✔	☇
Convergence	✔	✔	?	✔	✔

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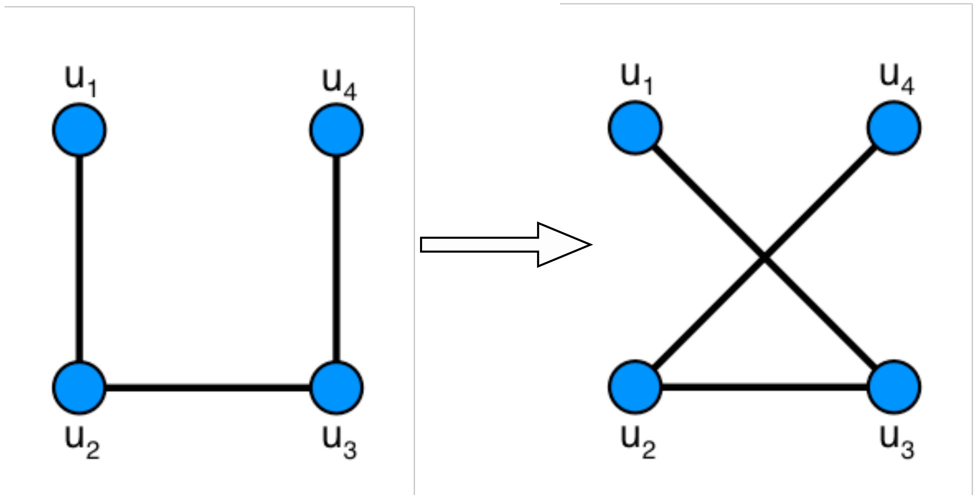
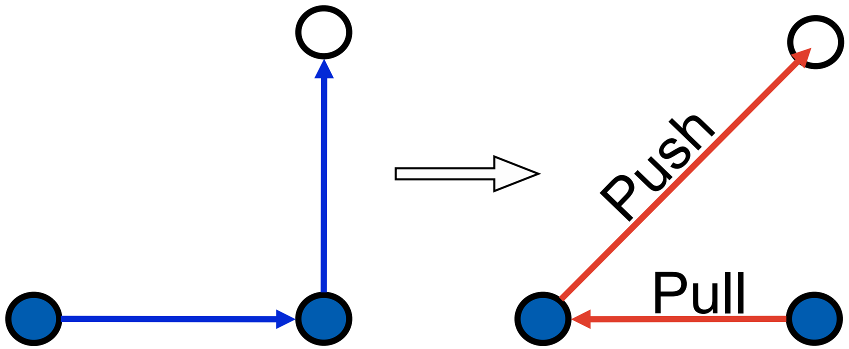
Good Peer-to-Peer-Operations





Peer-to-Peer Networks

09 Random Graphs for Peer-to-Peer-Networks

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