Peer-to-Peer Networks

09 Random Graphs for Peer-to-Peer-Networks

Christian Ortolf
Technical Faculty
Computer-Networks and Telematics
University of Freiburg

Peer-to-Peer Networking Facts

- Hostile environment
  - Legal situation
  - Egoistic users
  - Networking
    - ISP filter Peer-to-Peer Networking traffic
    - User arrive and leave
    - Several kinds of attacks
    - Local system administrators fight peer-to-peer networks
- Implication
  - Use stable robust network structure as a backbone
  - Napster: star
  - CAN: lattice
  - Chord, Pastry, Tapestry: ring + pointers for lookup
  - Gnutella, FastTrack: chaotic “social” network
- Idea: Use a Random d-regular Network

Why Random Networks ?

- Random Graphs ...
  - Robustness
  - Simplicity
  - Connectivity
  - Diameter
  - Graph expander
  - Security
- Random Graphs in Peer-to-Peer networks:
  - Gnutella
  - JXTApose

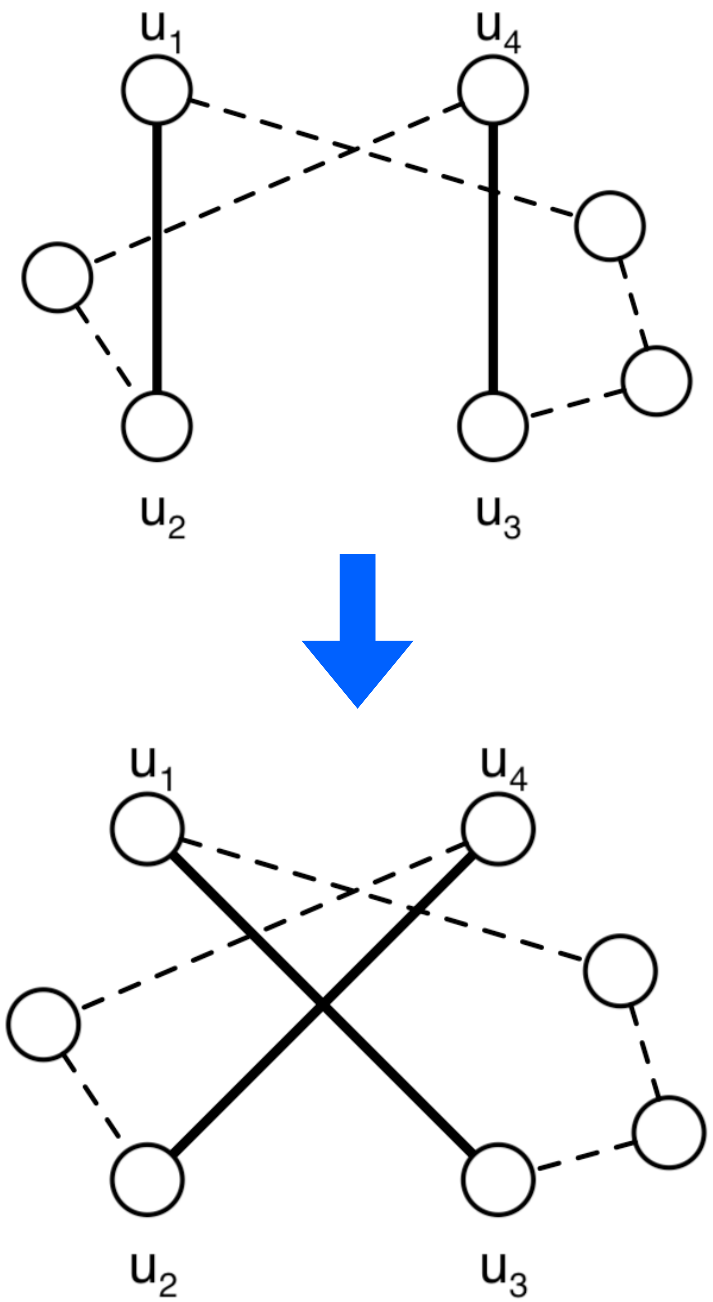
Dynamic Random Networks ...

- Peer-to-Peer networks are highly dynamic ...
  - maintenance operations are needed to preserve properties of random graphs
  - which operation can maintain (repair) a random digraph?

Desired properties:	
Soundness	Operation remains in domain (preserves connectivity and out-degree)
Generality	every graph of the domain is reachable does not converge to specific small graph set
Feasibility	can be implemented in a P2P-network
Convergence Rate	probability distribution converges quickly

Simple Switching

- Simple Switching
  - choose two random edges
    - $\{u_1,u_2\} \in E$, $\{u_3,u_4\} \in E$
  - such that $\{u_1,u_3\}$, $\{u_2,u_4\} \notin E$
    - add edges $\{u_1,u_3\}$, $\{u_2,u_4\}$ to E
    - remove $\{u_1,u_2\}$ and $\{u_3,u_4\}$ from E
- McKay, Wormald, 1990
  - Simple Switching converges to uniform probability distribution of random network
  - Convergence speed:
    - $O(nd^3)$ for $d \in O(n^{1/3})$
- Simple Switching cannot be used in Peer-to-Peer networks
  - Simple Switching disconnects the graph with positive probability
  - No network operation can re-connect disconnected graphs



Necessities of Graph Transformation

	Simple-Switching
Graphs	Undirected Graphs
Soundness	?
Generality	☇
Feasibility	✓
Convergence	✓

- Problem: Simple Switching does not preserve connectivity
- Soundness
  - Graph transformation remains in domain
  - Map connected d-regular graphs to connected d-regular graphs
- Generality
  - Works for the complete domain and can lead to any possible graph
- Feasibility
  - Can be implemented in P2P network
- Convergence Rate
  - The probability distribution converges quickly

Directed Random Graphs

- Peter Mahlmann, Christian Schindelhauer
  - Distributed Random Digraph Transformations for Peer-to-Peer Networks, 18th ACM Symposium on Parallelism in Algorithms and Architectures, Cambridge, MA, USA. July 30 - August 2, 2006

Directed Graphs

Push Operation:

1.Choose random node *u*

2.Set *v* to *u*

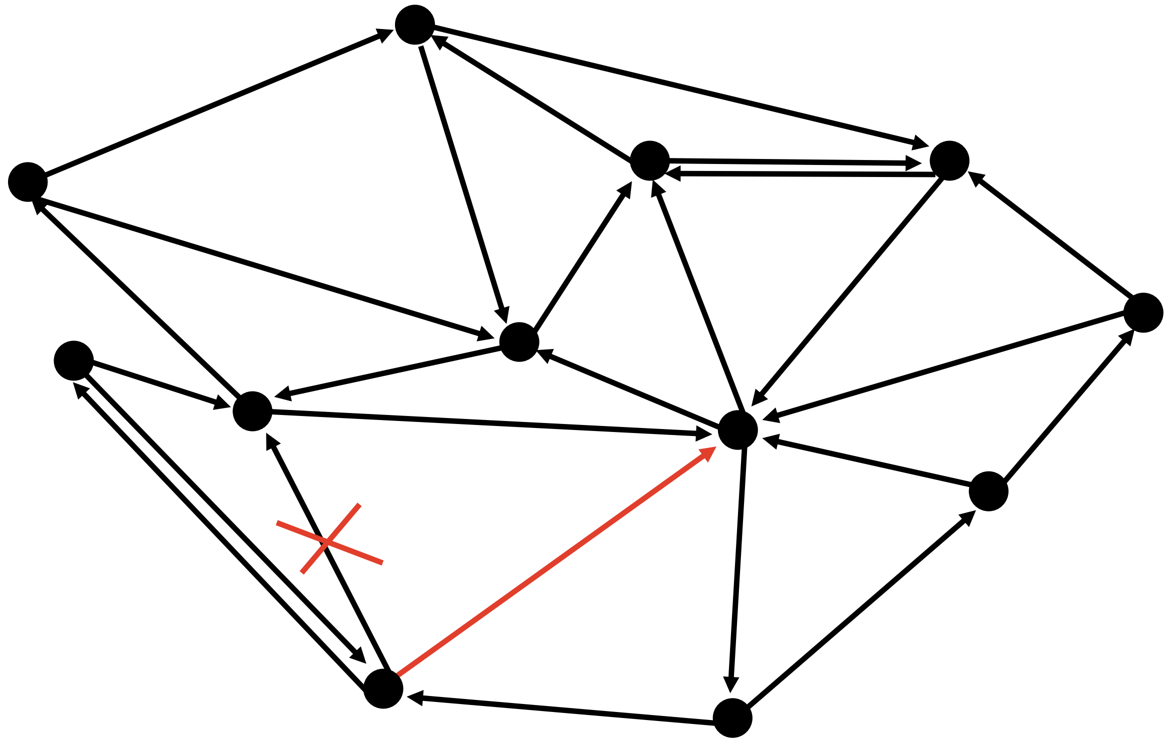
3.While a random event with p= 1/*h* appears

a) Choose random edge starting at *v* and ending at *v*‘

b) Set *v* to *v*‘

3.Insert edge (*u*,*v*)

4.Remove random edge starting at *v*



Pull Operation:

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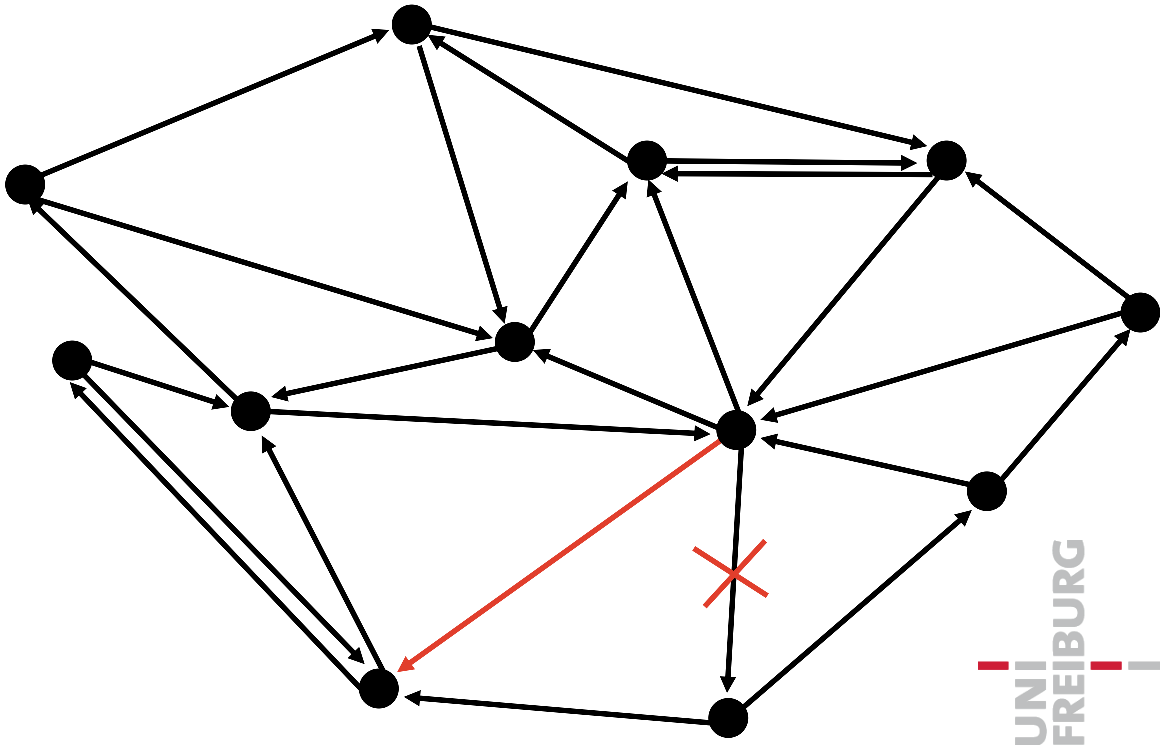
3.While a random event with p= 1/*h* appears

a)Choose random edge starting at *v* and ending at *v*‘

b)Set *v* to *v*‘

3.Insert edge (*v*,*u*)

4.Remove random edge starting at *v*‘



Simulation of Push-Operations

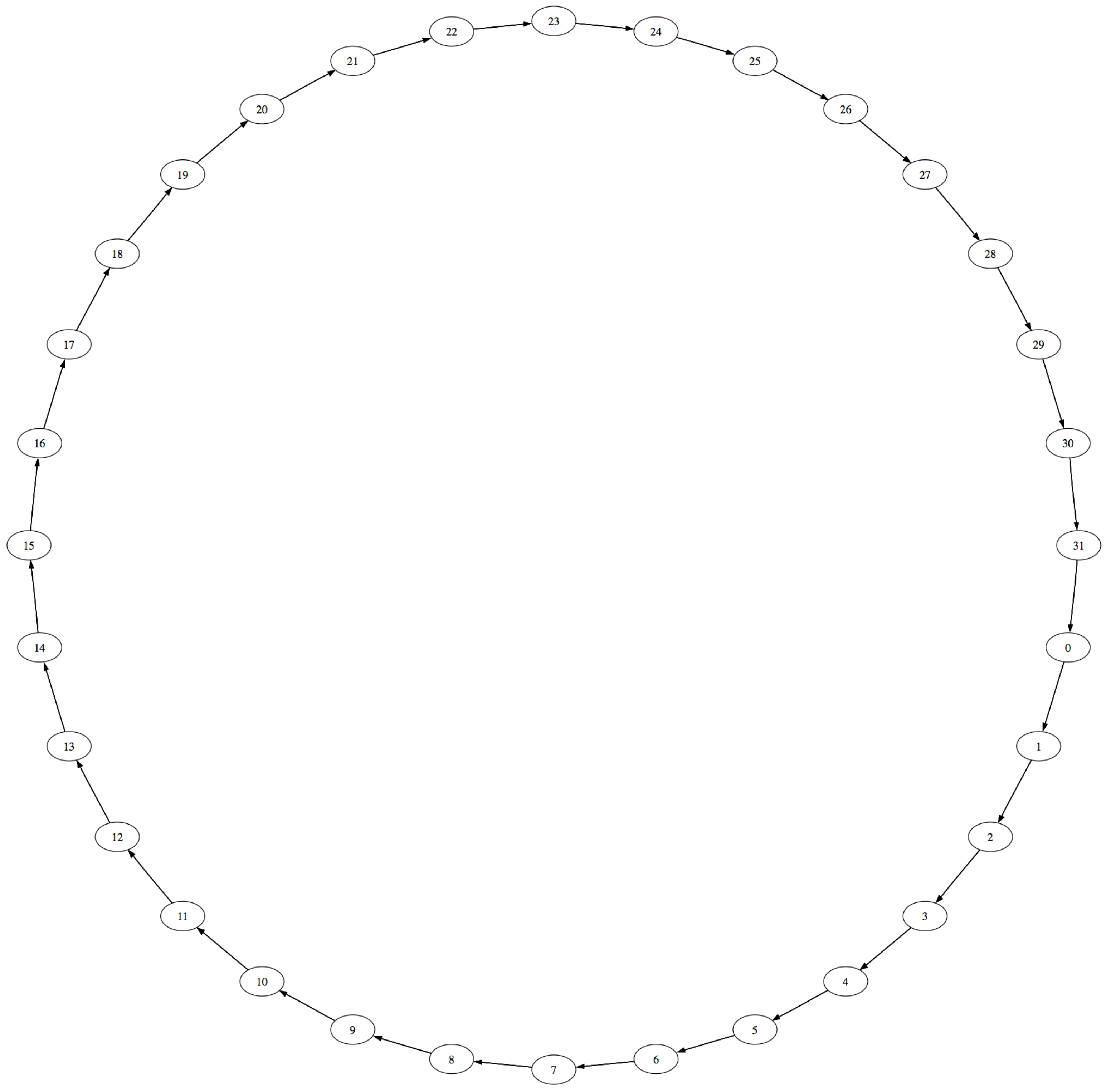
Start situation

Parameter:

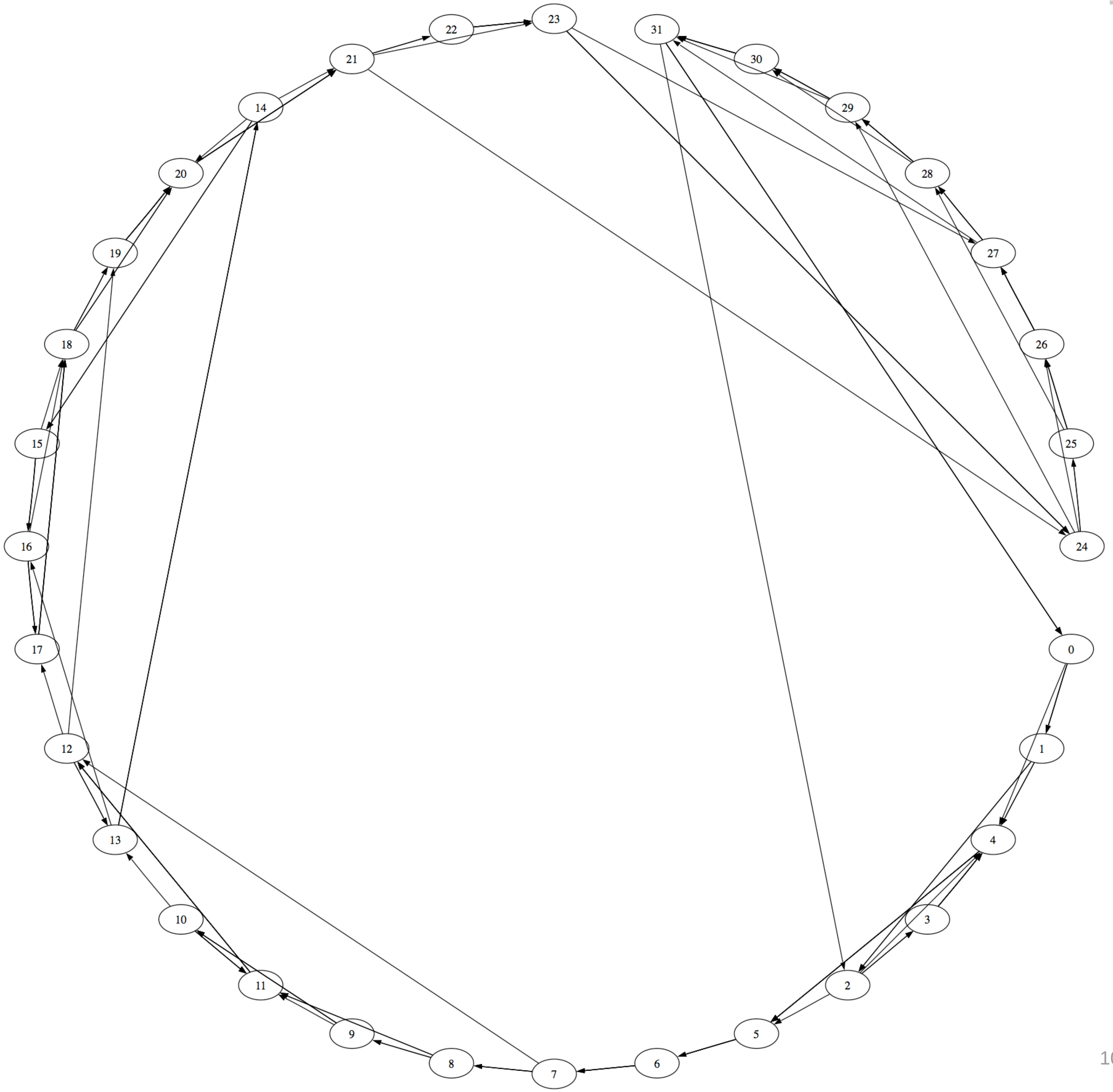
n = 32 nodes

out-degree d = 4

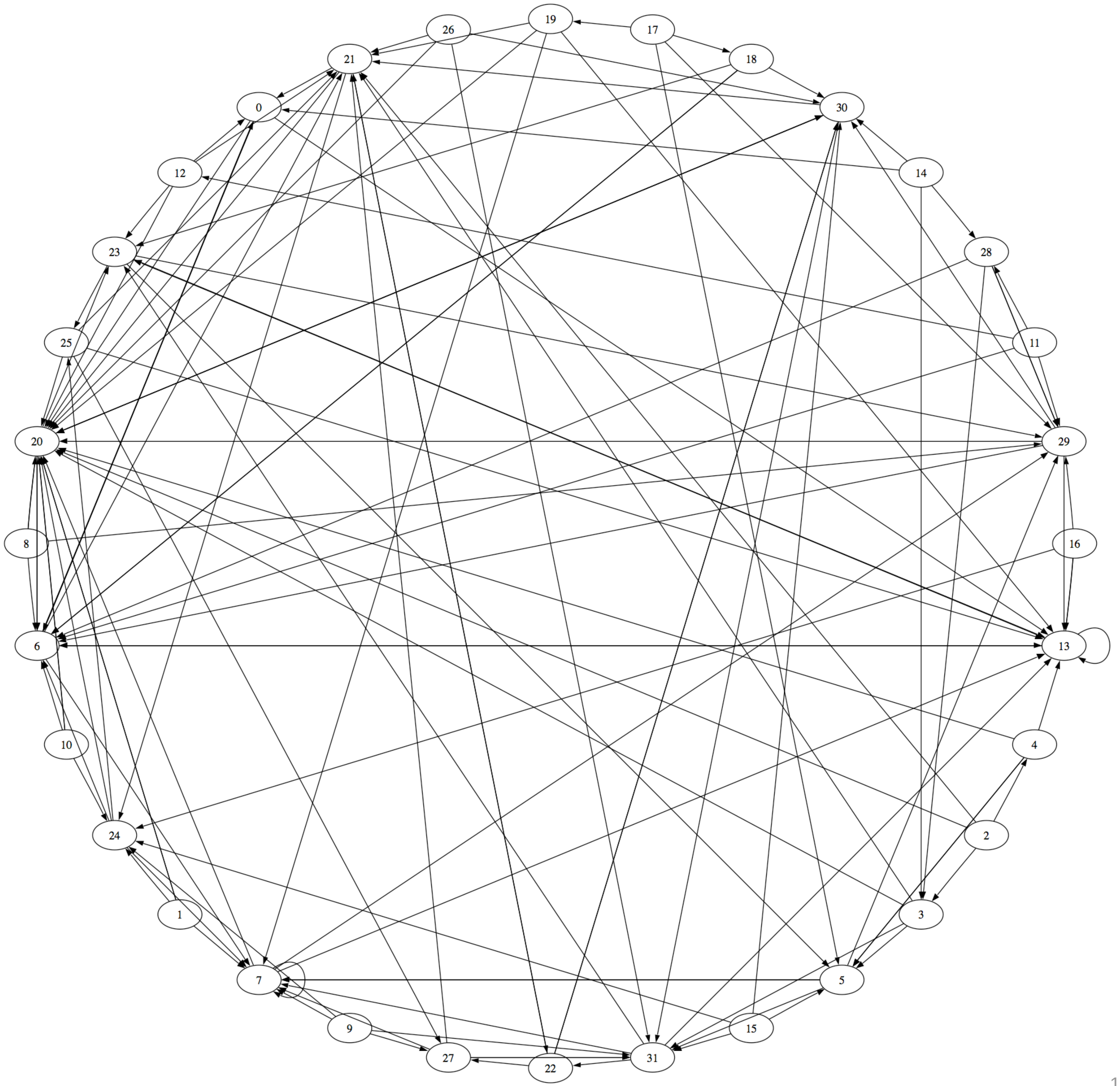
Hop-distance h = 3



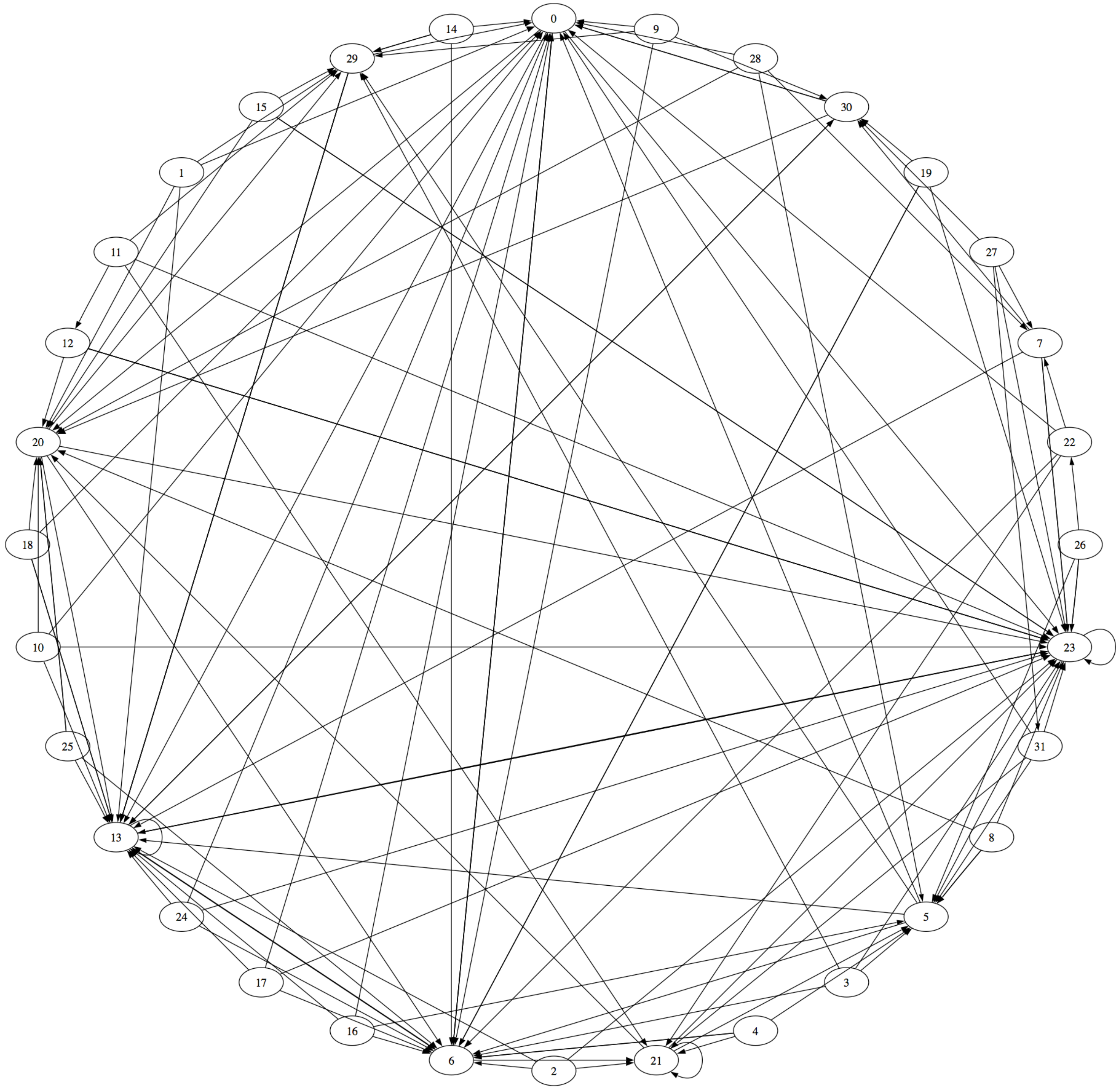
1 Iteration Push ...



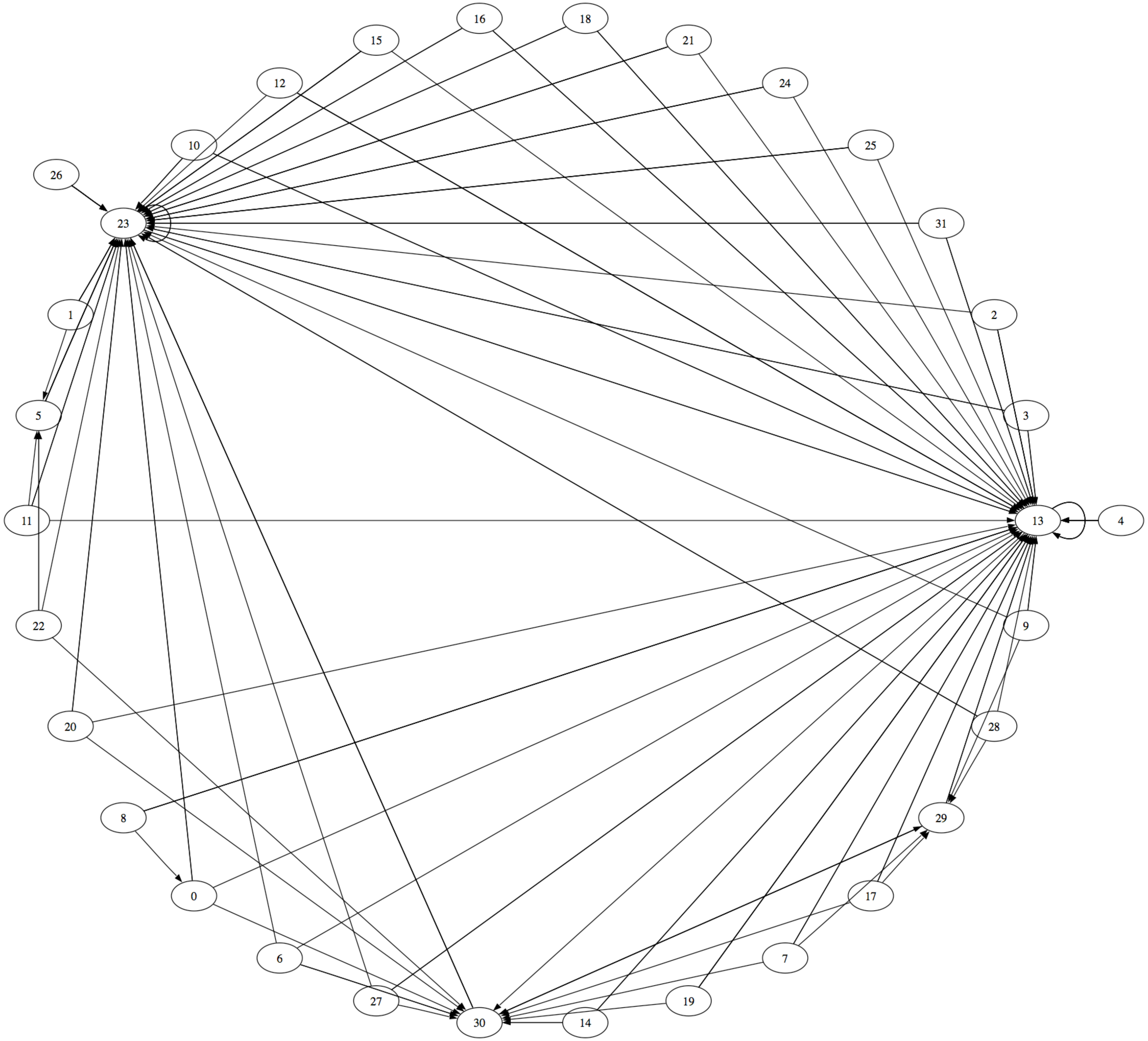
10 Iterations Push ...



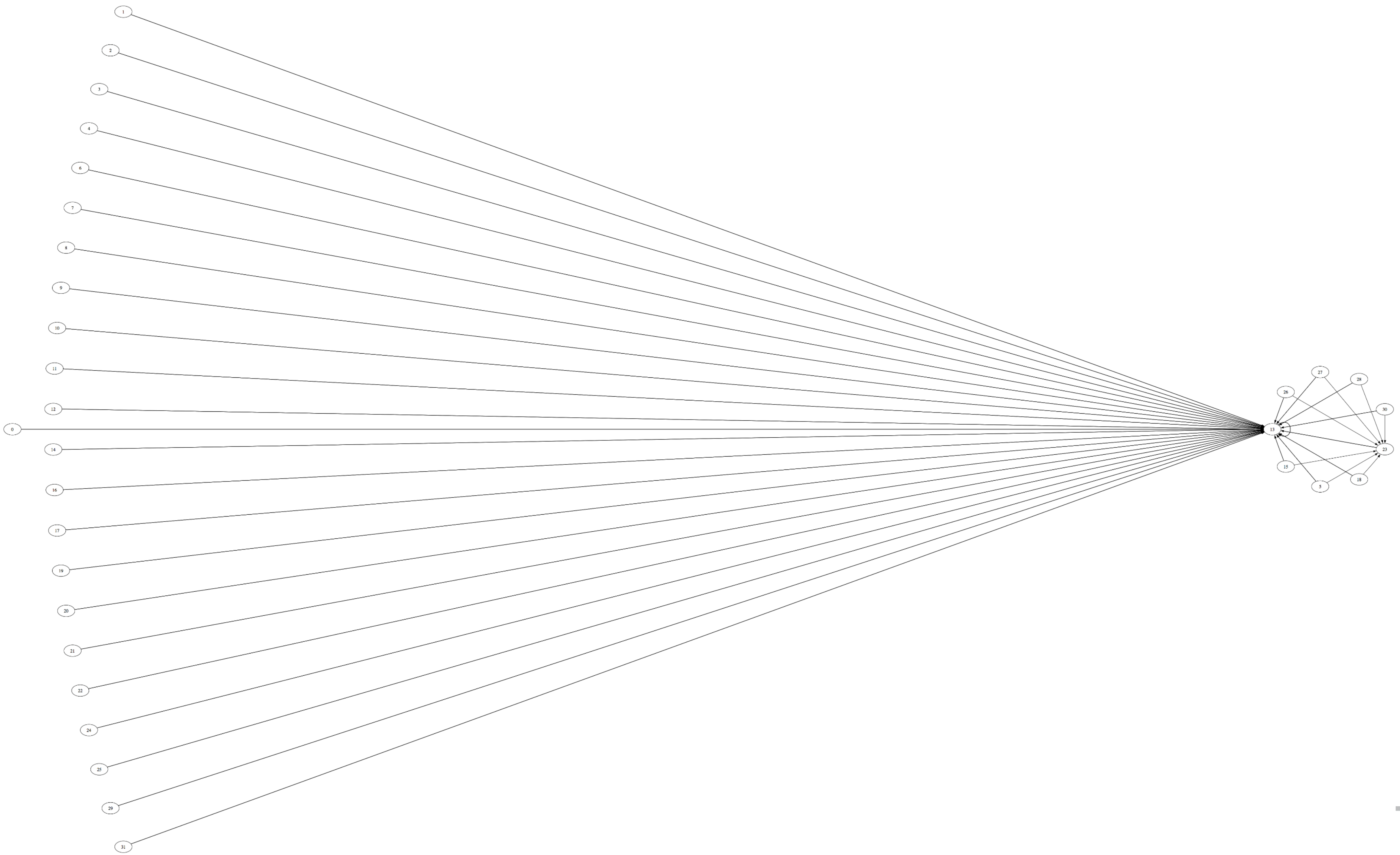
20 Iterations von Push ...



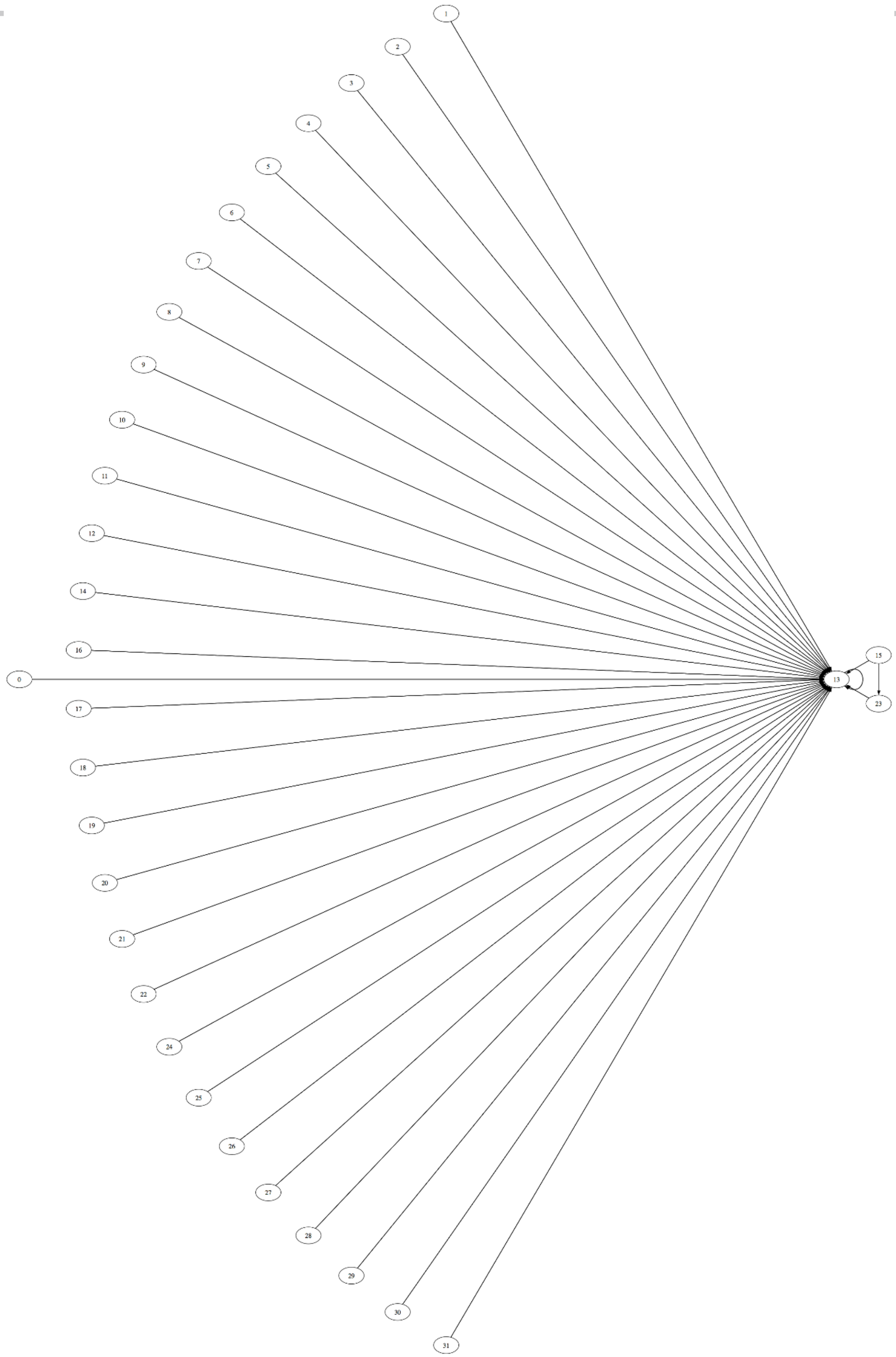
30 Iterations Push ...



40 Iterations Push ...

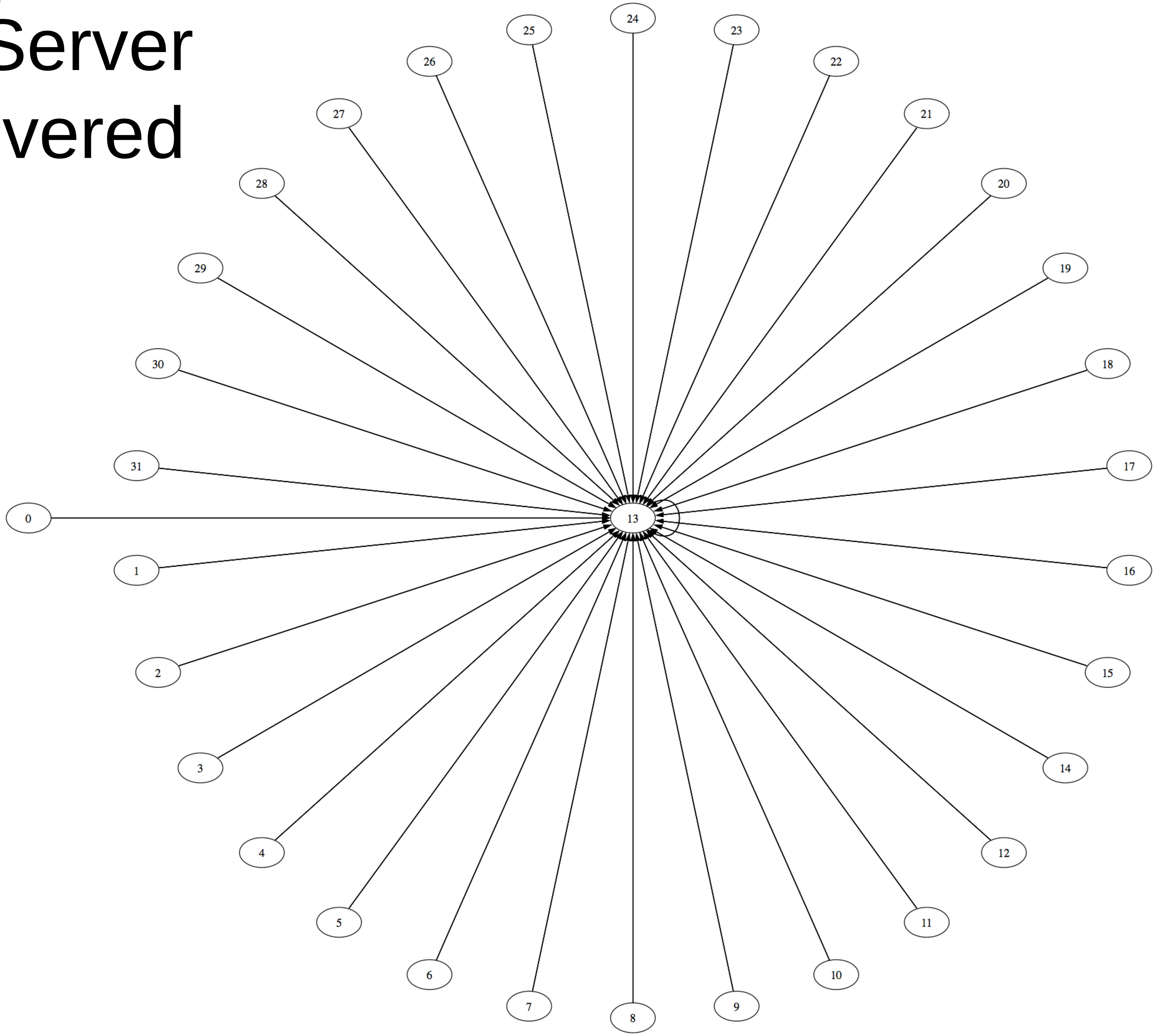


50 Iterations Push ...



70 Iterations Push ...

Client-Server rediscovered



Simulation of Pull-Operation ...

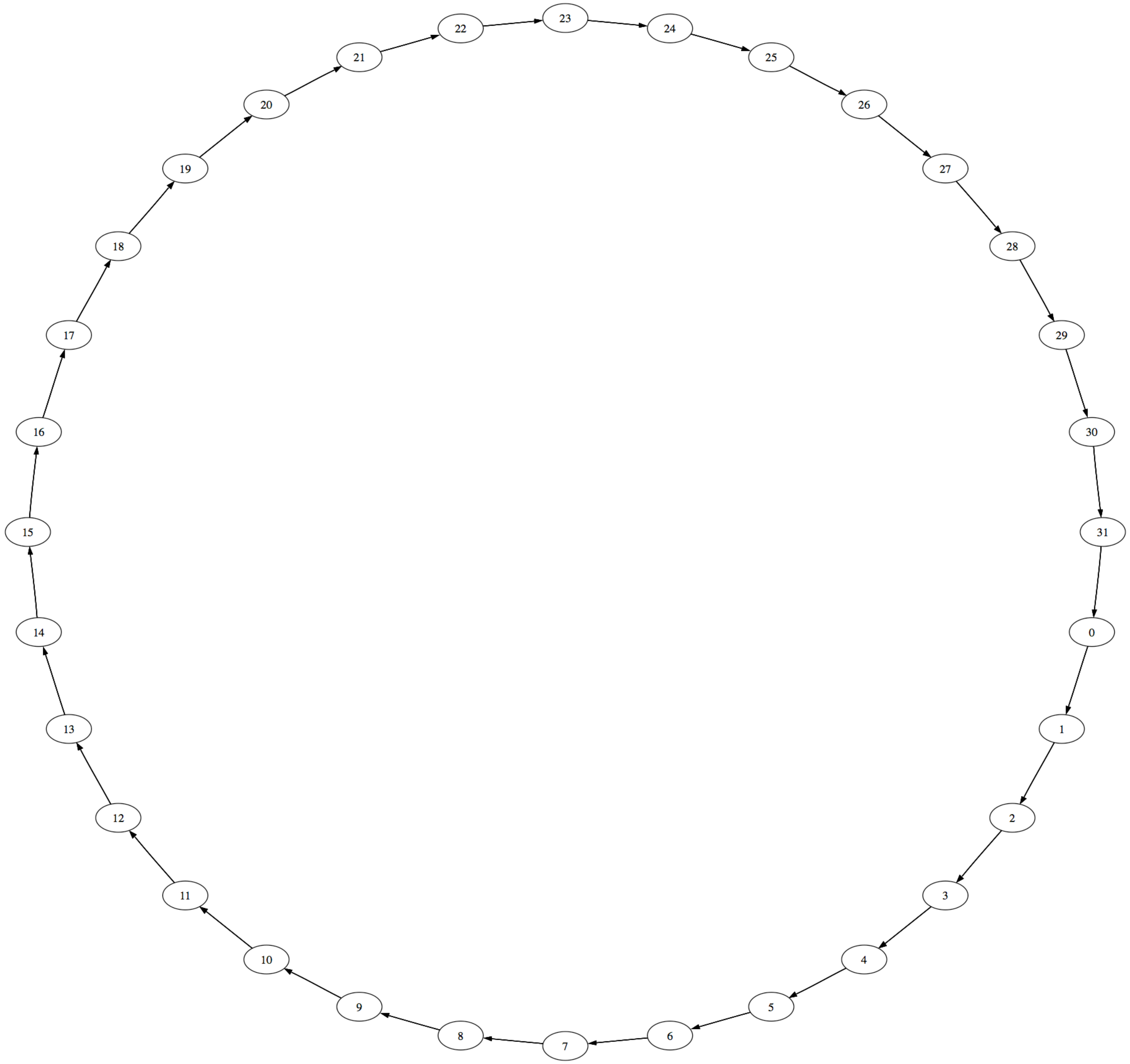
Start situation

Parameter:

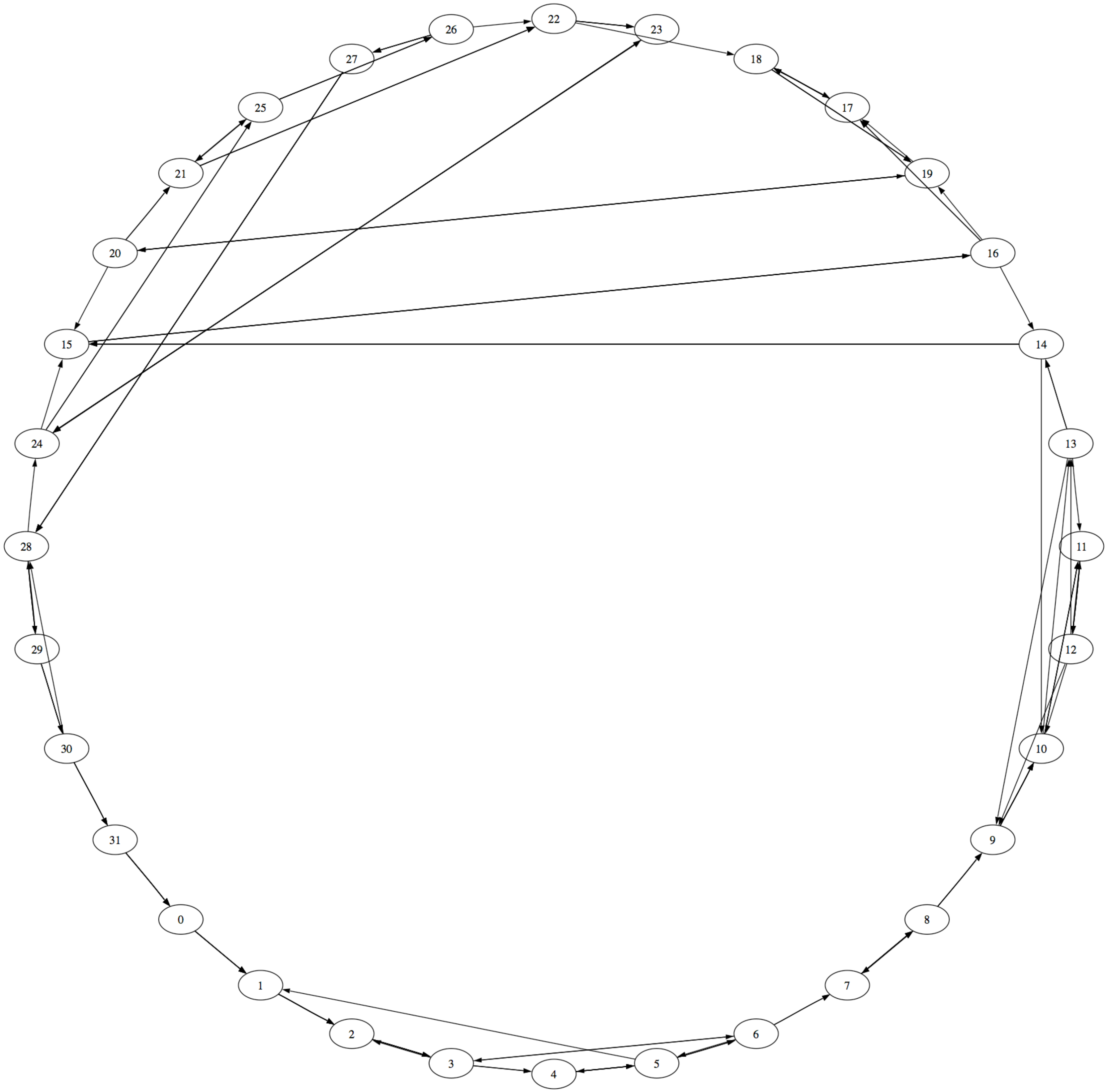
n = 32 nodes

outdegree d = 4

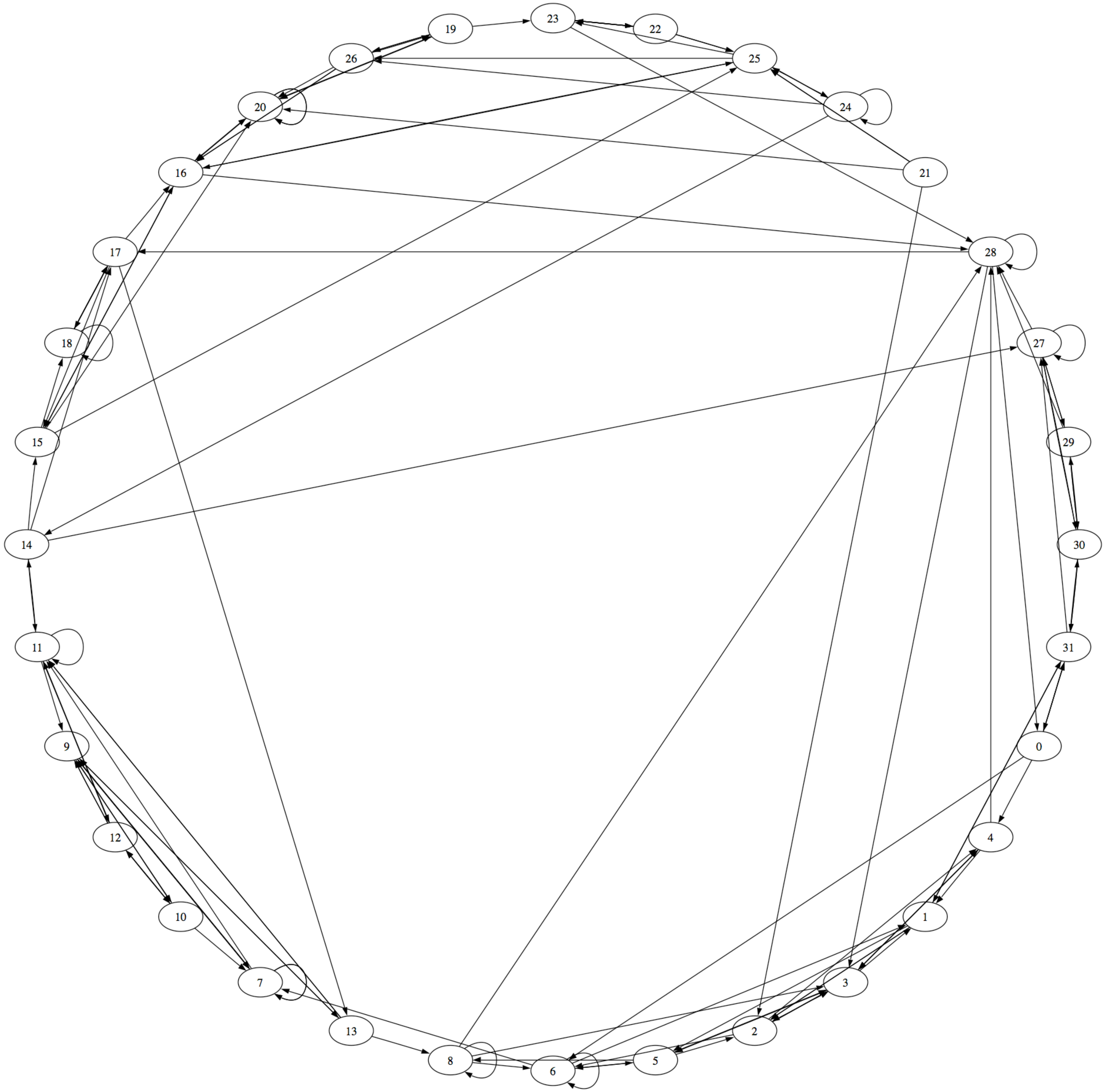
hop distance h = 3



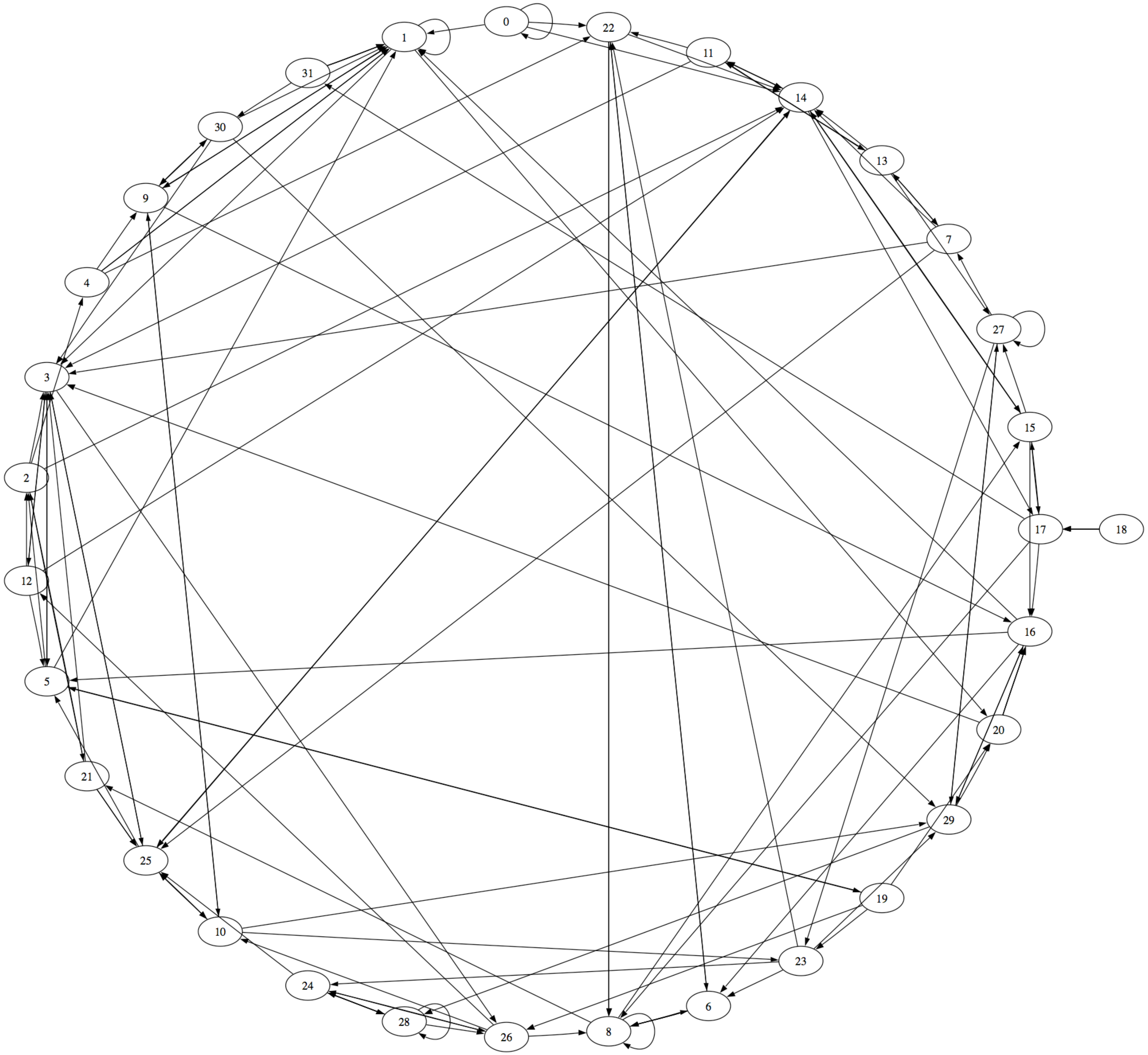
1 Iteration Pull ...



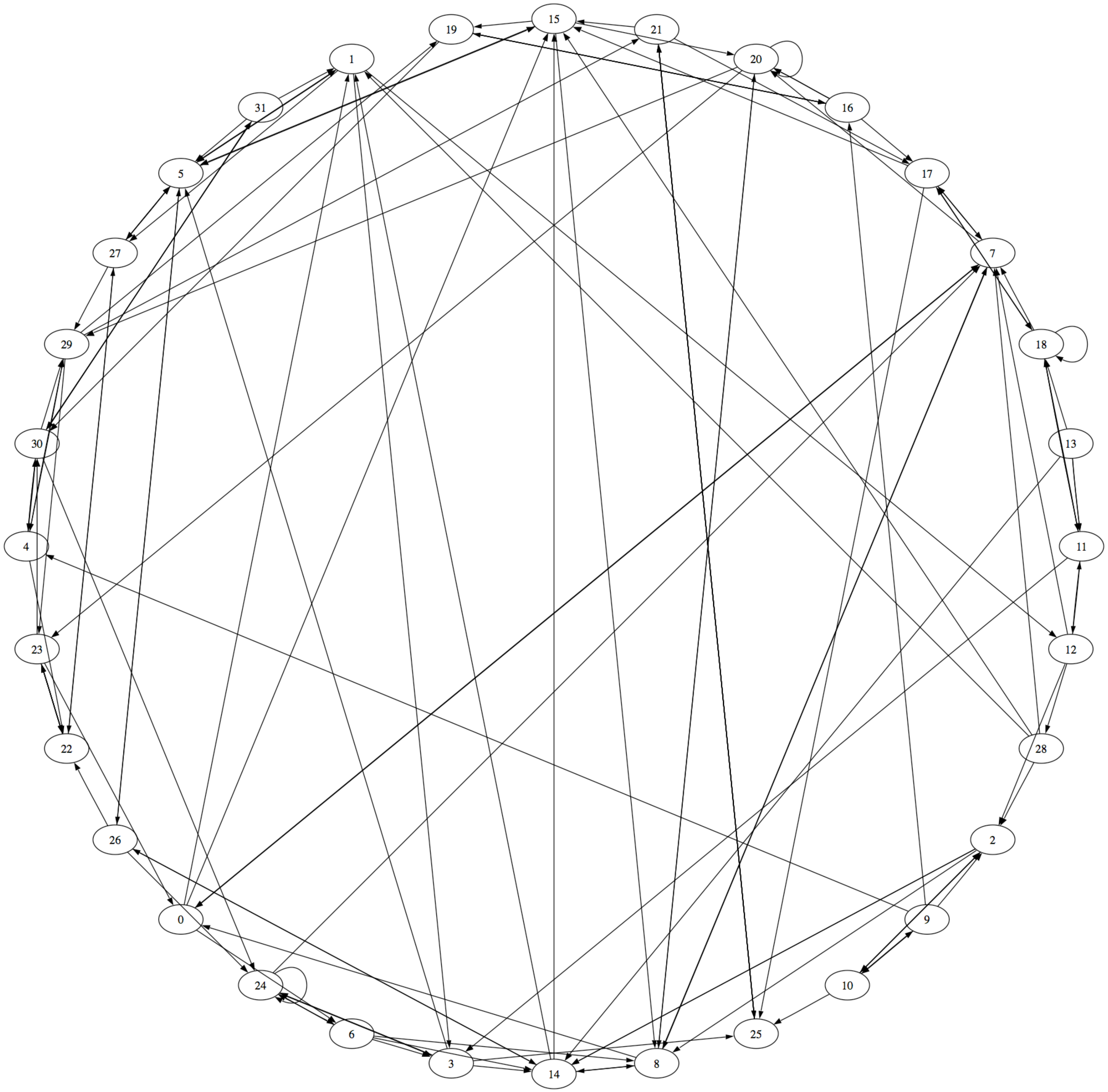
10 Iterations Pull ...



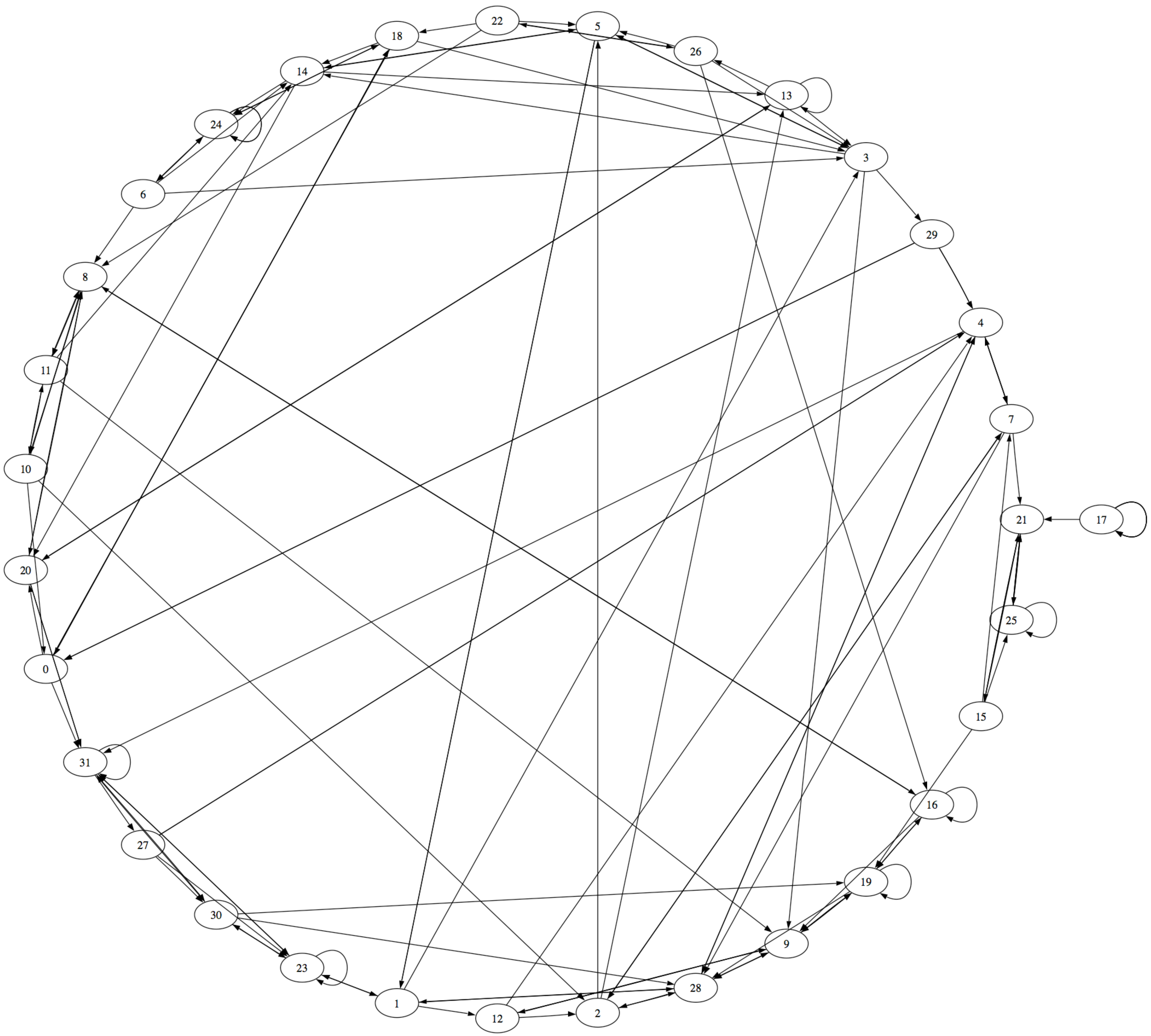
20 Iterations Pull ...



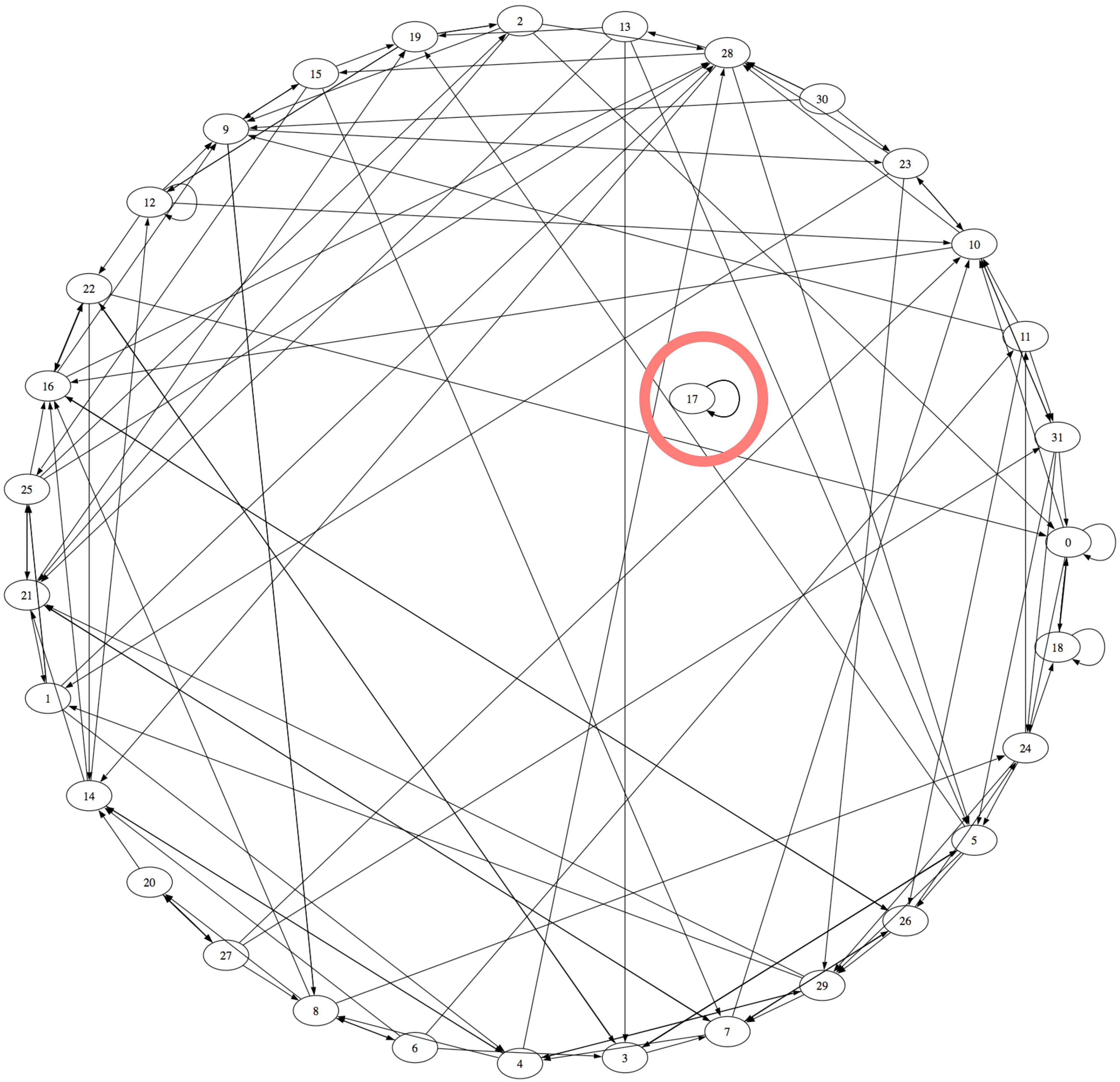
30 Iterations Pull ...



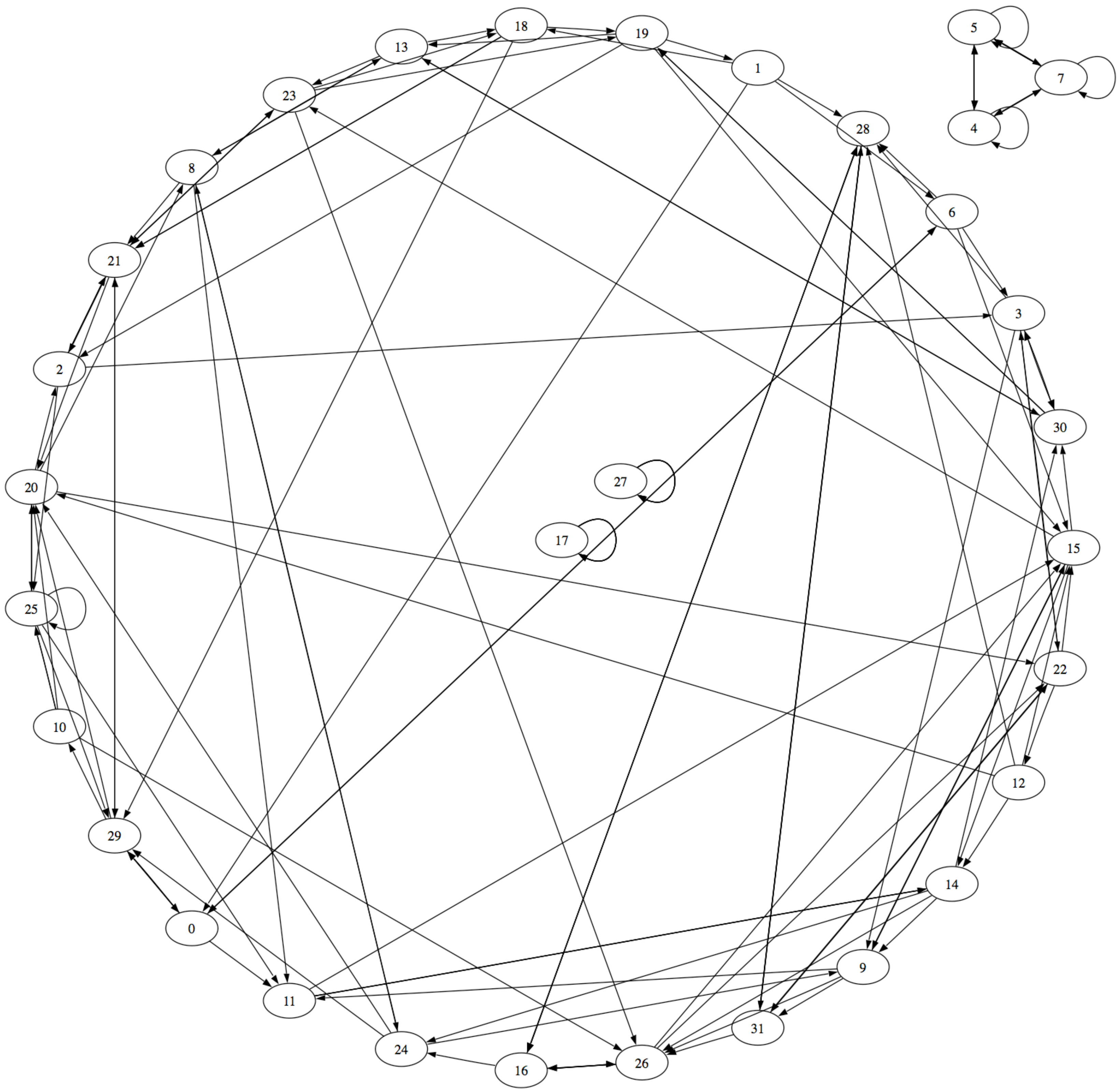
40 Iterationen Pull ...



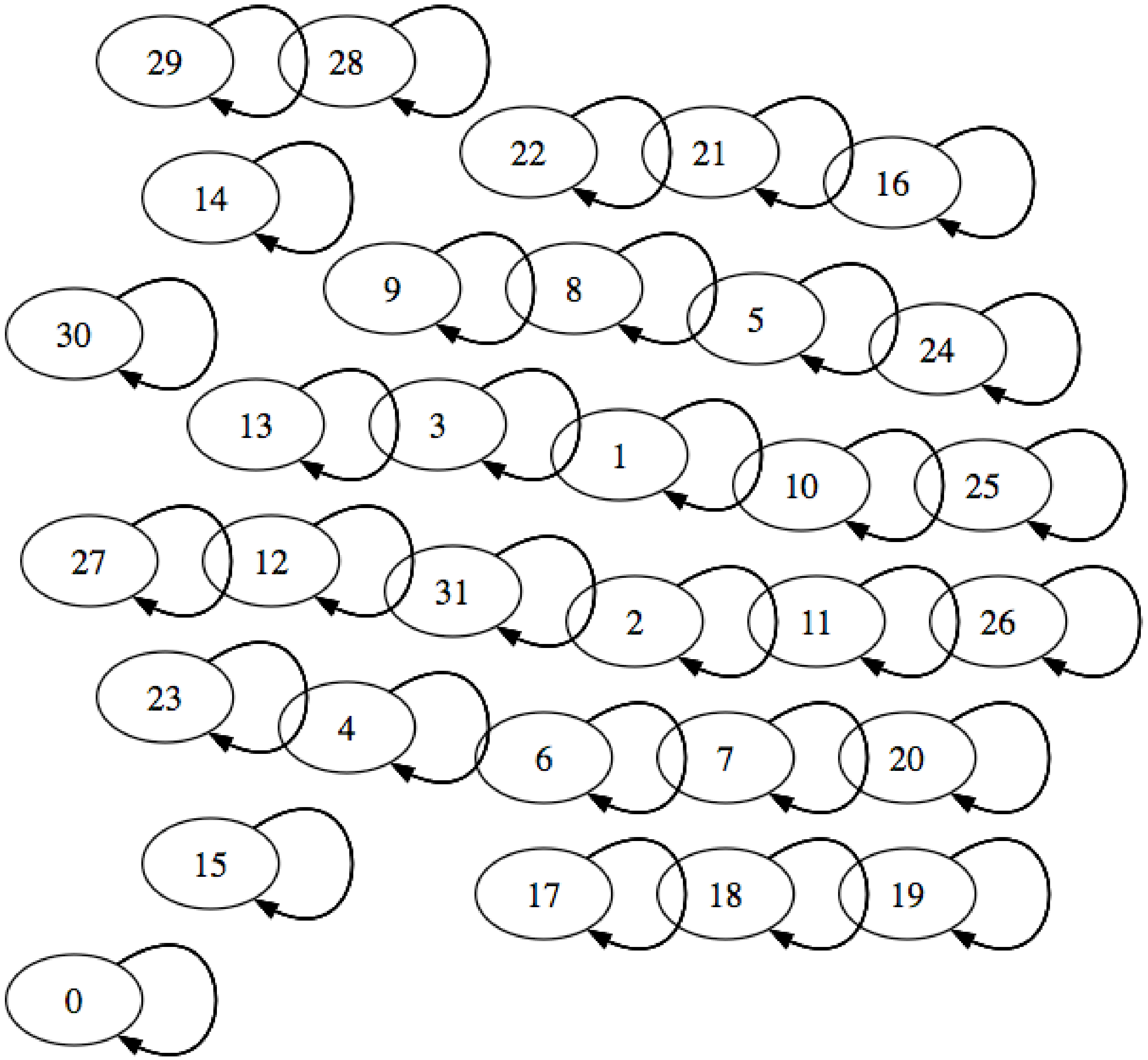
50 Iterations Pull ...



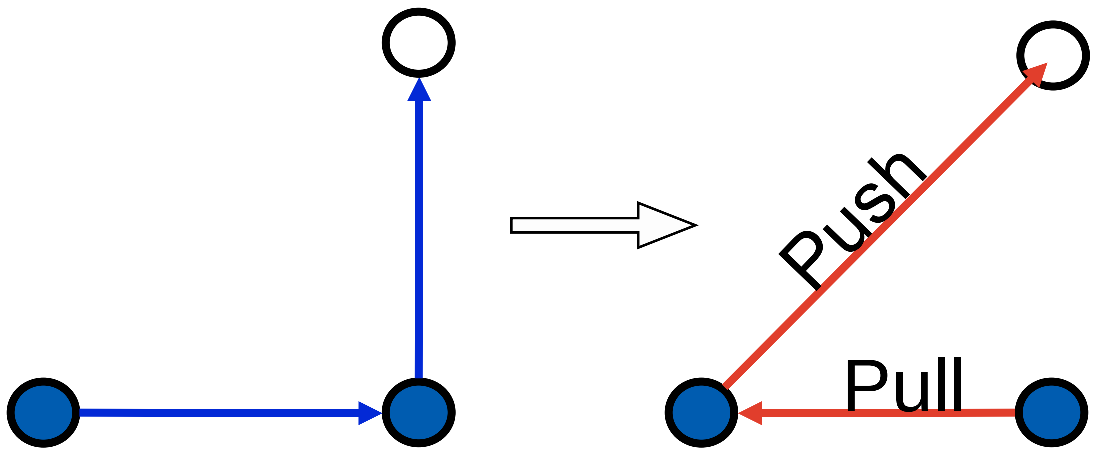
500 Iterations Pull ...



5000 Iterations Pull ...



Combination of Push and Pull

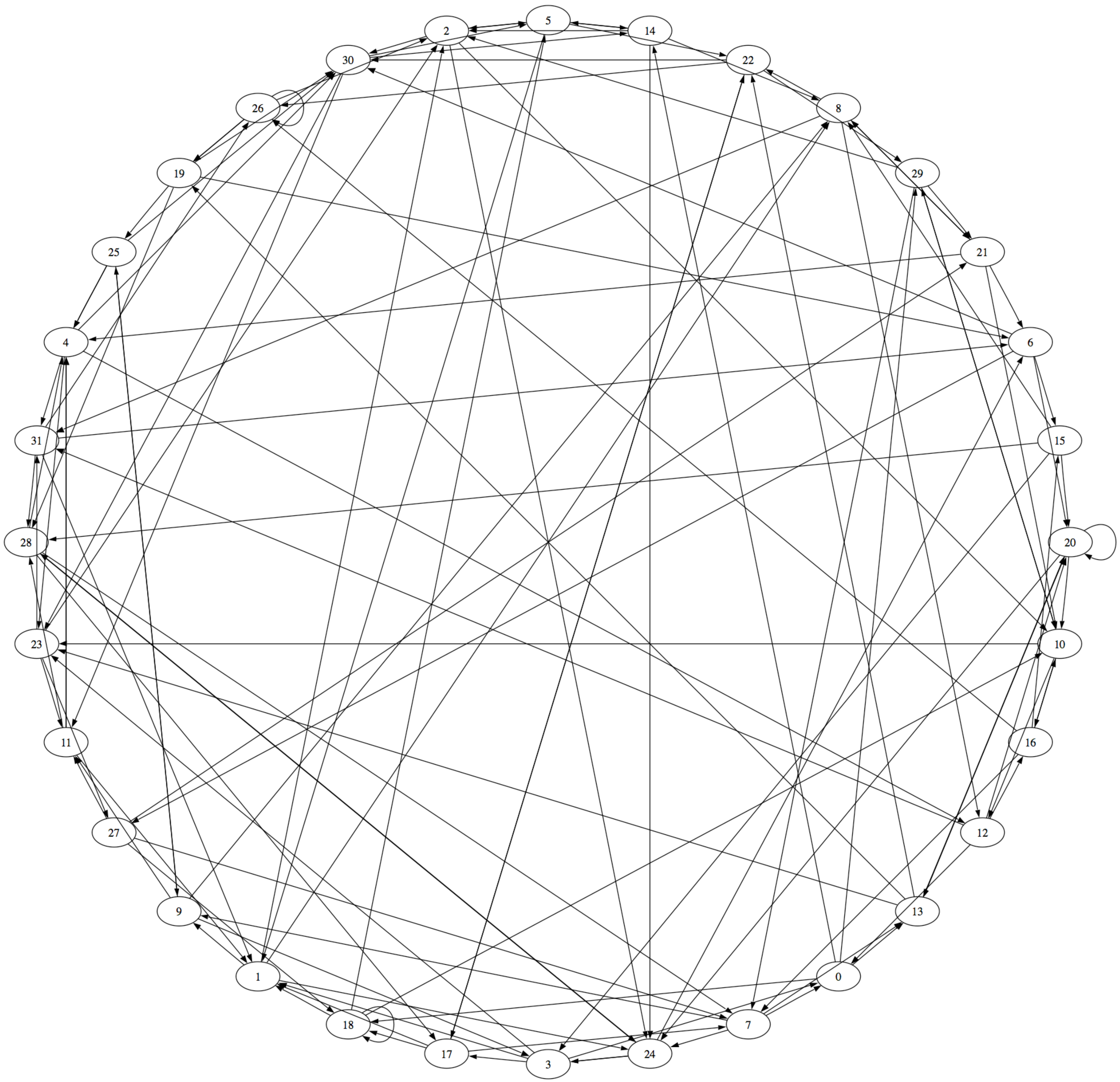


Simulation of Push&Pull-Operations ...

Same start situation

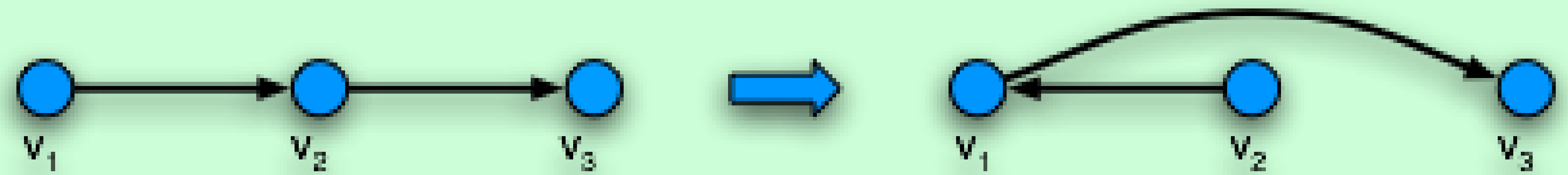
Parameters
n = 32 nodes
degree d = 4
hop-distance h = 3

but
1.000.000 iterations



Pointer-Push&Pull for Multi-Digraphs

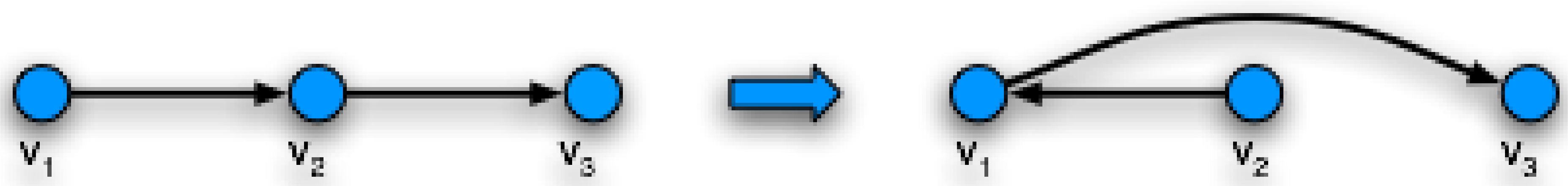
Pointer-Push&Pull:
- choose random node $v_1 \in V$
- do random walk v_1, v_2, v_3
- delete edges (v_1,v_2) and (v_2,v_3)
- add edges (v_2,v_1) and (v_1,v_3)



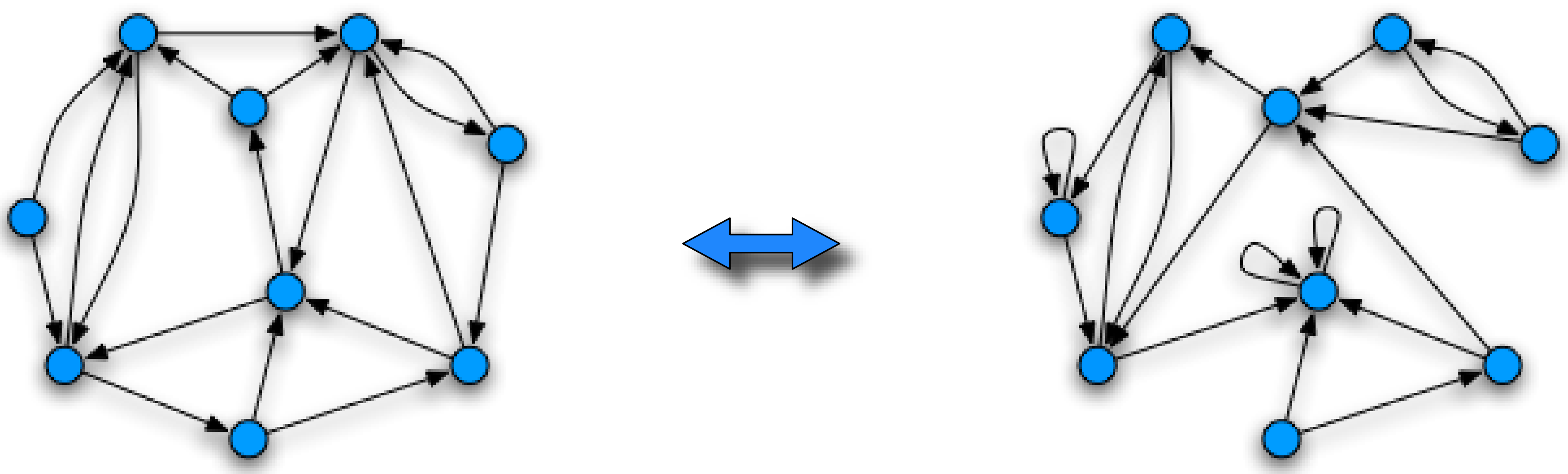
- obviously:
  - preserves connectivity of G
  - does not change out-degrees

➞ Pointer-Push&Pull is **sound** for the domain of out-regular connected multi-digraphs

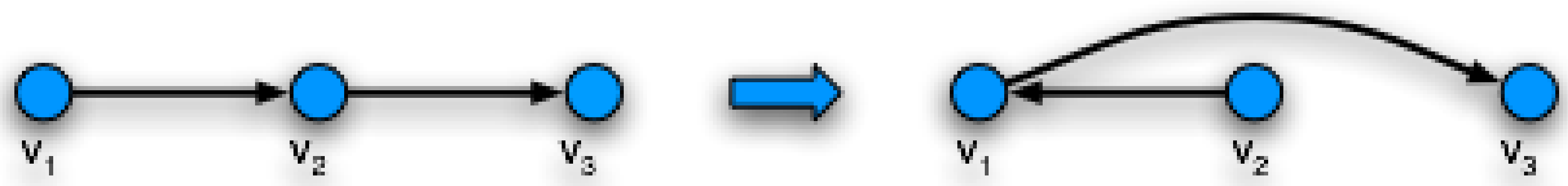
Pointer-Push&Pull: Reachability



Lemma *A series of random Pointer-Push&Pull operations can transform an arbitrary connected out-regular multi-digraph, to every other graph within this domain*



Pointer-Push&Pull: Uniformity



What is the stationary prob. distribution generated by Pointer-Push&Pull?

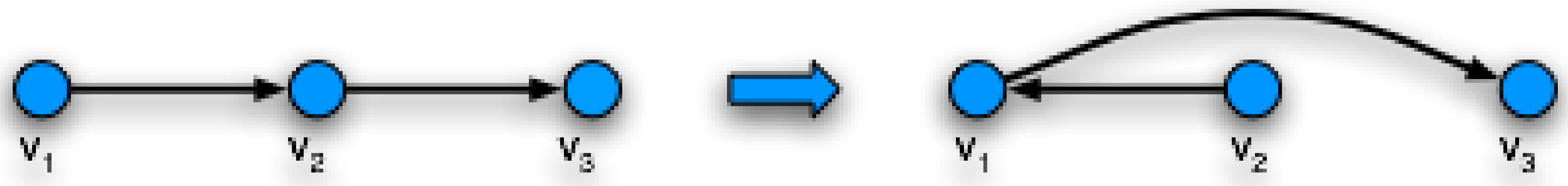
- depends on random walk

example: ***node oriented random walk***

- choose random neighboring node with $p=1/d$ respectively
- due to multi-edges possibly less than d neighbors
- if no node was chosen operation is canceled

$$P[G \xrightarrow{\mathcal{PP}} G'] = P[G' \xrightarrow{\mathcal{PP}} G]$$

Uniform Generality

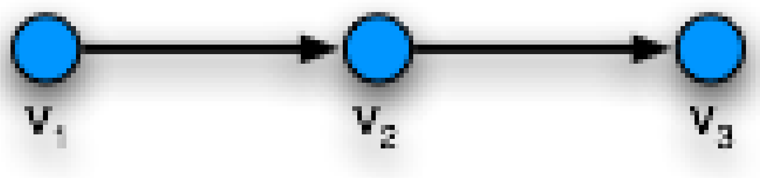


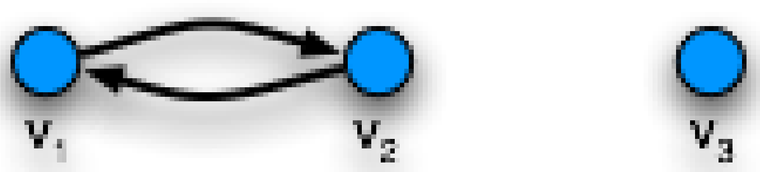
Theorem: *Let G' be a d-out-regular connected multi-digraph with n nodes. Applying Pointer-Push&Pull operations repeatedly will construct every d-out-regular connected multi-digraph with the same probability in the limit, i.e.*

$$\lim_{t \to \infty} P[G' \xrightarrow{t} G] = \frac{1}{|\mathcal{MDG}_{n,d}|}$$

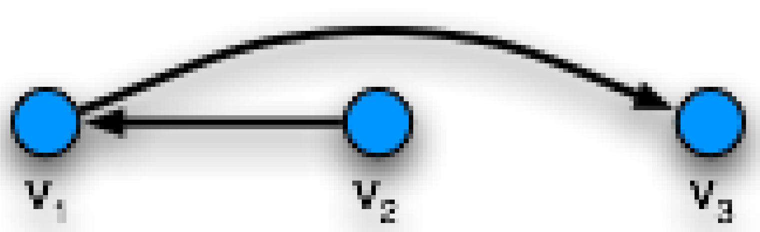
Feasibility ...

A Pointer-Push&Pull operation in the network ...





(2) v_2 replaces (v_2,v_3) by (v_2,v_1) and sends ID of v_3 to v_1



- only 2 messages between two nodes, carrying the information of one edge only
- verification of neighborhood is mandatory in dynamic networks

⇒ combine neighbor-check with Pointer-Push&Pull

Properties of Pointer-Push&Pull

	Pointer-Push&Pull
Graphs	Directed Multigraphs
Soundness	✓
Generality	✓
Feasibility	✓
Convergence	?

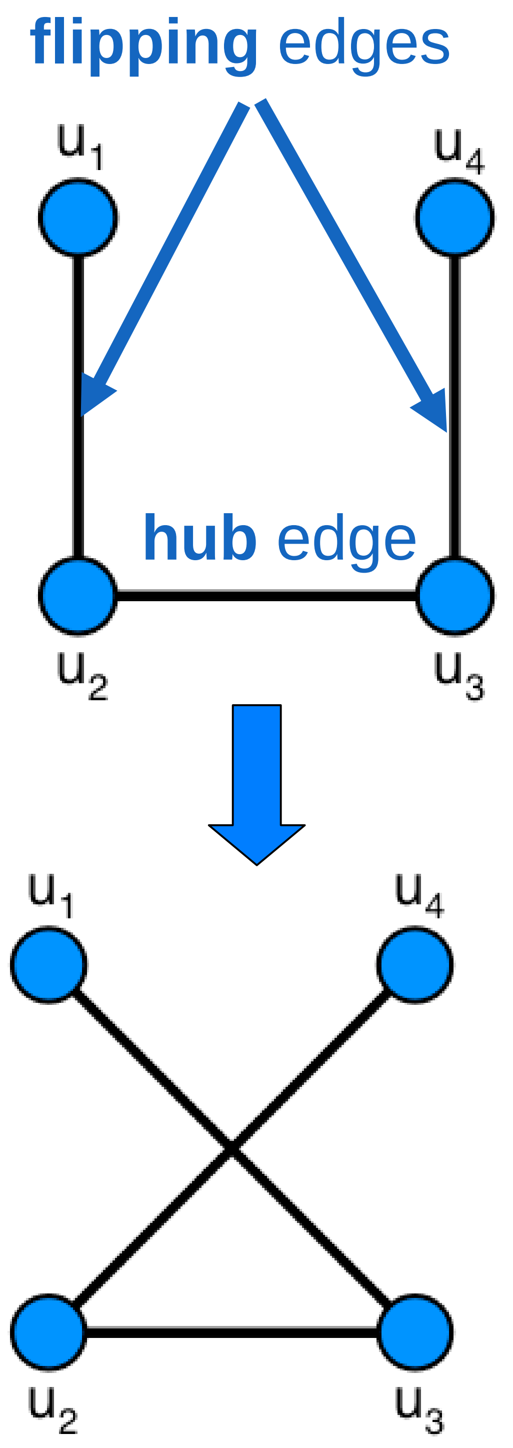
- strength of Pointer-Push&Pull is its **simplicity**
- generates truly random digraphs
- the price you have to pay: multi-edges

Open Problems:

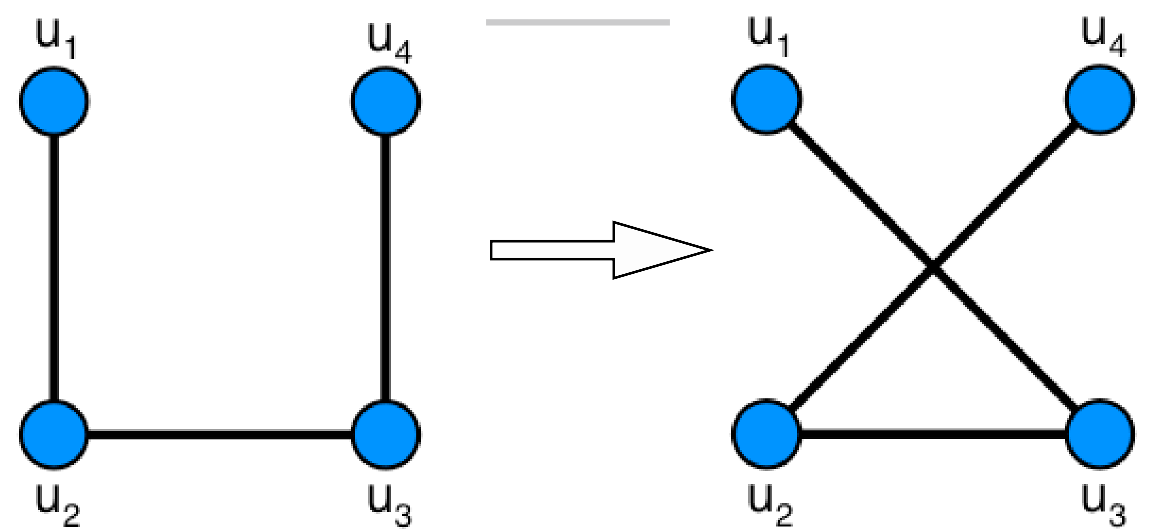
- convergence rate is unknown, conjecture $O(dn \log n)$
- is there a similar operation for simple digraphs?

The 1-Flipper (F1)

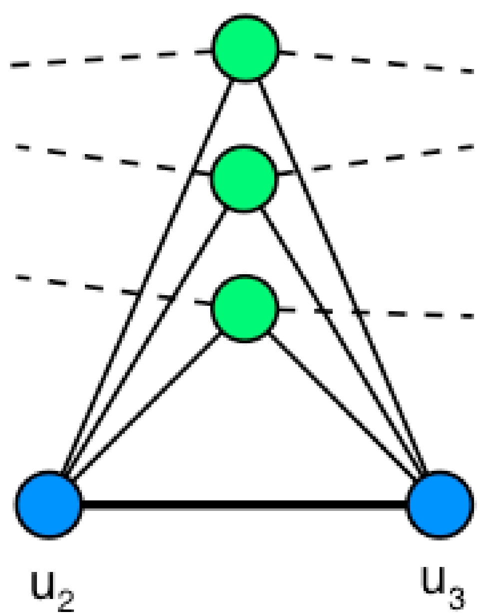
- The operation
  - choose random edge $\{u_2,u_3\} \in E$,
    - hub edge
  - choose random node $u_1 \in N(u_2)$
    - 1st flipping edge
  - choose random node $u_4 \in N(u_3)$
    - 2nd flipping edge
  - if $\{u_1,u_3\}, \{u_2,u_4\} \notin E$
    - flip edges, i.e.
    - add edges $\{u_1,u_3\}, \{u_2,u_4\}$ to E
    - remove $\{u_1,u_2\}$ and $\{u_3,u_4\}$ from E



1-Flipper is sound



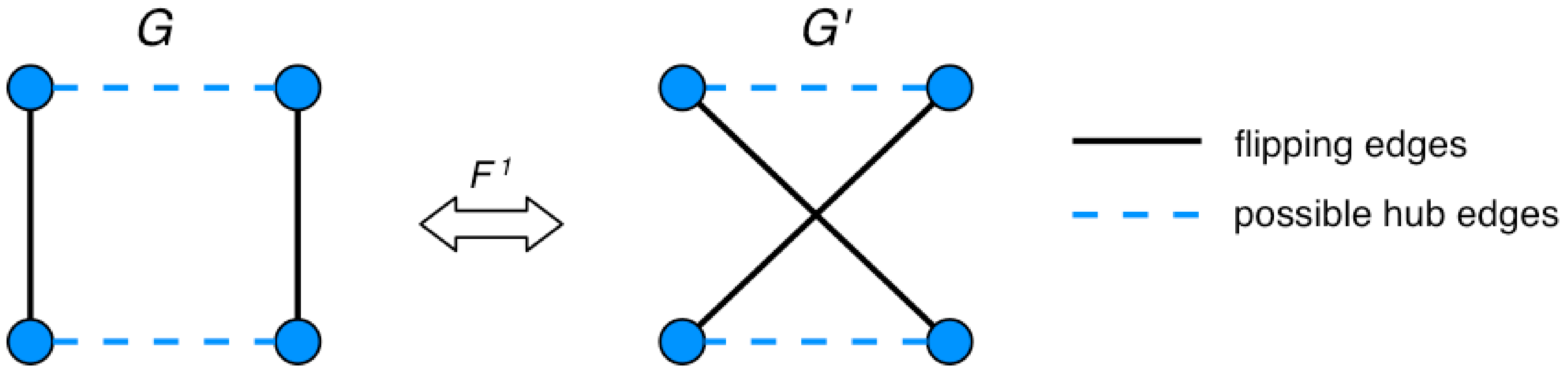
- Soundness:
  - 1-Flipper preserves d-regularity
    - follows from the definition
  - 1-Flipper preserves connectivity
    - because of the hub edge
- Observation:
  - For all d > 2 there is a connected d-regular graph G such that $P[G \xrightarrow{F^1} G] \neq 0$
  - For all d ≥ 2 and for all d-regular connected graphs at least one 1-Flipper-operation changes the graph with positive probability
    - This does not imply generality



1-Flipper is symmetric

- Lemma (symmetry):
  - For all undirected regular graphs G,G':

$$P[G \xrightarrow{F^1} G'] = P[G' \xrightarrow{F^1} G]$$



1-Flipper provides generality

- Lemma (reachability):
  - For all pairs G,G’ of connected d-regular graphs there exists a sequence of 1-Flipper operations transforming G into G’.

1-Flipper properties: uniformity

- Theorem (uniformity):
  - Let G_0 be a d-regular connected graph with n nodes and d > 2. Then in the limit the 1-Flipper operation constructs all connected d-regular graphs with the same probability:

$$\lim_{t \to \infty} P[G_0 \xrightarrow{t} G] = \frac{1}{|\mathcal{C}_{n,d}|}$$

1-Flipper properties: Expansion

- Definition (edge boundary):
  - The edge boundary δS of a set $S \subset V$ is the set of edges with exactly one endpoint in S.

- Definition (expansion):
  A graph G=(V,E) has expansion $\beta > 0$
  - if for all node sets S with $|S| \le |V|/2$:
  - $|\delta S| \ge \beta\, |S|$

- Since for $d \in \omega(1)$ a random connected d-regular graph is a $\theta(d)$ expander asymptotically almost surely (a.a.s: in the limit with probability 1), we have
- Theorem:
  - For $d > 2$ consider any d-regular connected Graph G0. Then in the limit the 1-Flipper operation establishes an expander graph after a sufficiently large number of applications a.a.s.

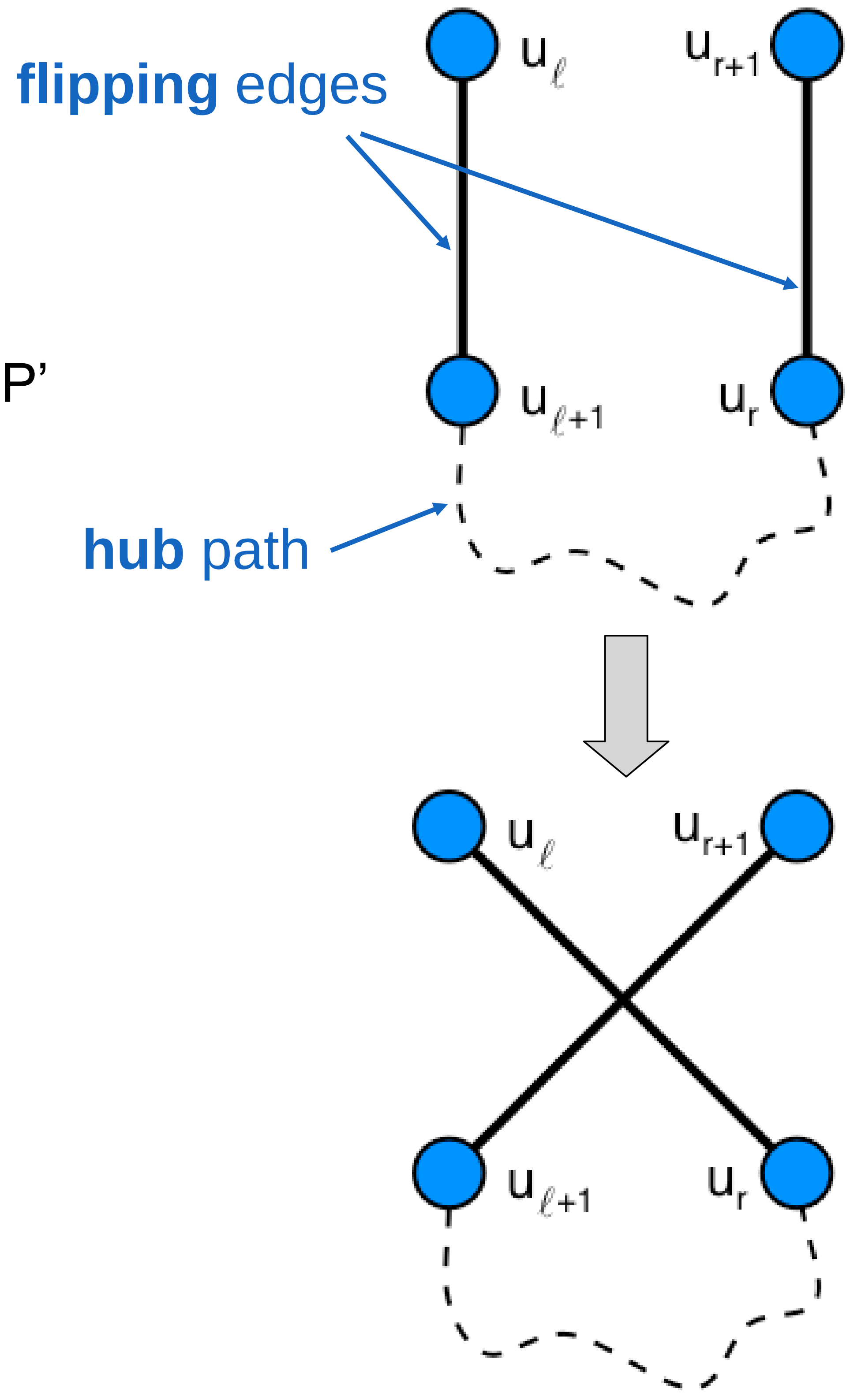
Flipper

	Flipper
Graphs	Undirected Graphs
Soundness	✓
Generality	✓
Feasibility	✓
Convergence	?

- **Flipper involves 4 nodes**
- **Generates truly random graphs**
- **Open Problems:**
  - convergence rate is polynomial
  - conjecture: $O(dn \log n)$

The k-Flipper (Fk)

- The operation
  - choose random node
  - random walk P‘ in G
  - choose hub path with nodes
    - $\{u_l, u_r\}$, $\{u_{l+1}, u_{r+1}\}$ occur only once in P’
  - if $\{u_l, u_r\}, \{u_{l+1}, u_{r+1}\} \notin E$
    - add edges $\{u_l, u_r\}$, $\{u_{l+1}, u_{r+1}\}$ to E
    - remove $\{u_l, u_{l+1}\}$ and $\{u_r, u_{r+1}\}$ from E

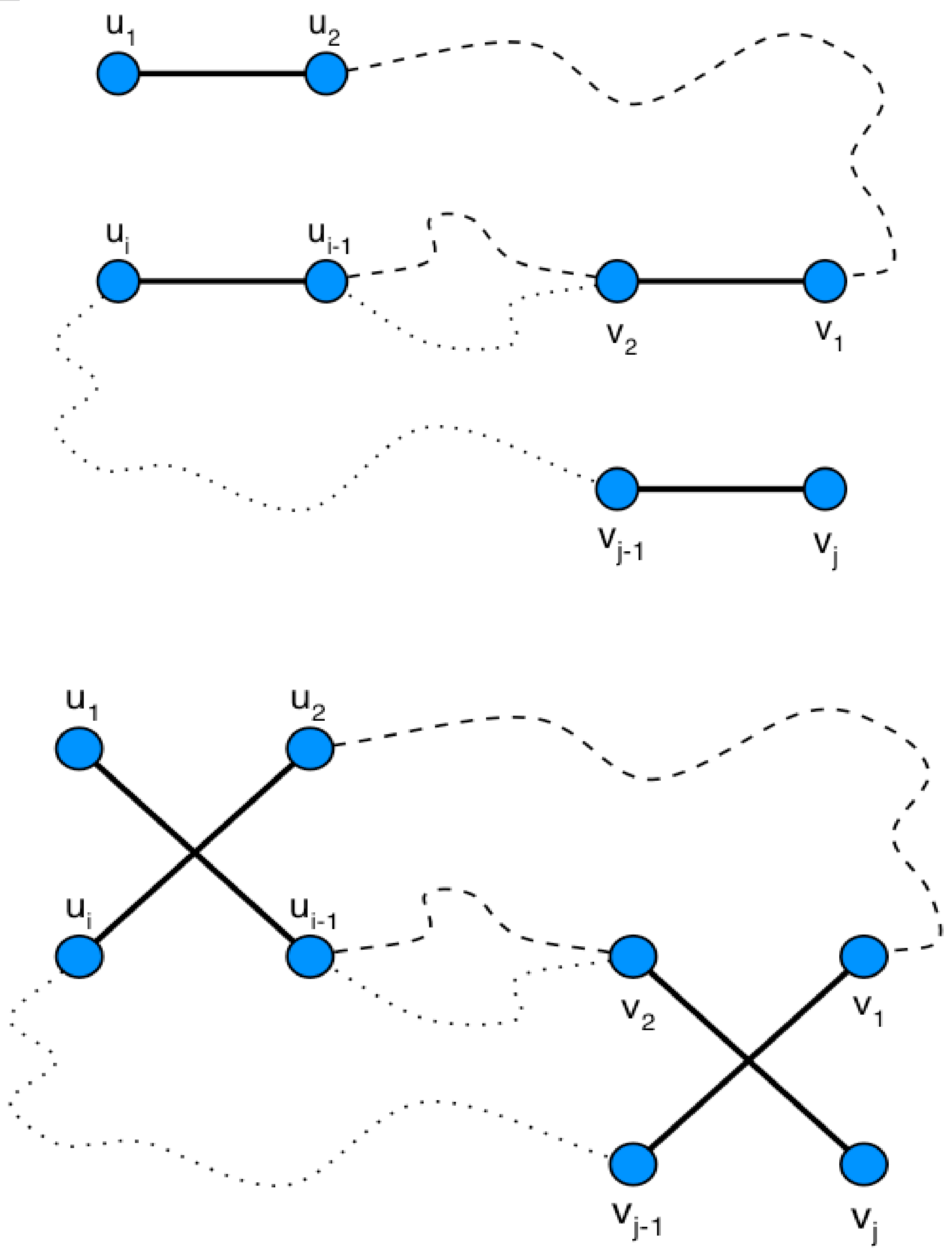


k-Flipper: Properties ...

- k-Flipper preserves connectivity and d-regularity
  - proof analogously to the 1-Flipper
- k-Flipper provides reachable,
  - since the 1-Flipper provides reachability
  - k-Flipper can emulate 1-Flipper
- But: k-Flipper is not symmetric:
  - a new proof for expansion property is needed

Concurrency ...

- In a P2P-network there are concurrent Flipper operations
  - No central coordination
  - Concurrent Flipper operations can speed up the convergence process
  - However concurrent Flipper operations can disconnect the network



k-Flipper

	k-Flipper large k	k-Flipper small k
Graphs	Undirected Graphs	Undirected Graphs
Soundness	✓	✓
Generality	✓	✓
Feasibility	↯	✓
Convergence	✓	?

- Convergence only proven for too long paths
  - Operation is not feasible then.
  - Does k-Flipper quickly converge for small k?
- Open problem:
  - Which k is optimal?

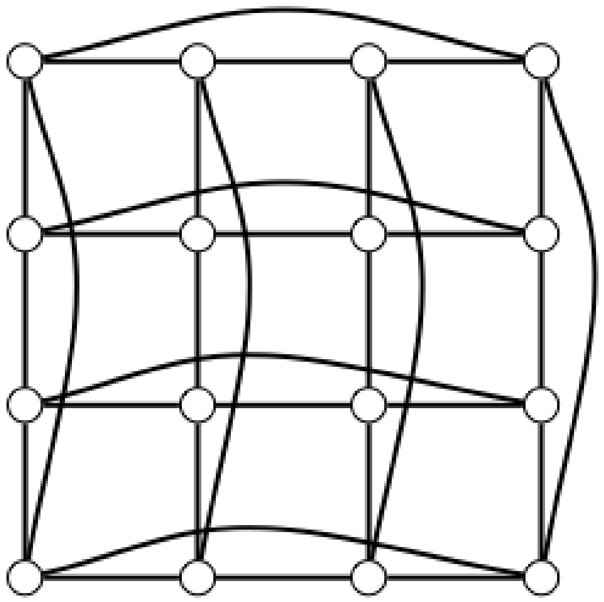
All Graph Transformation

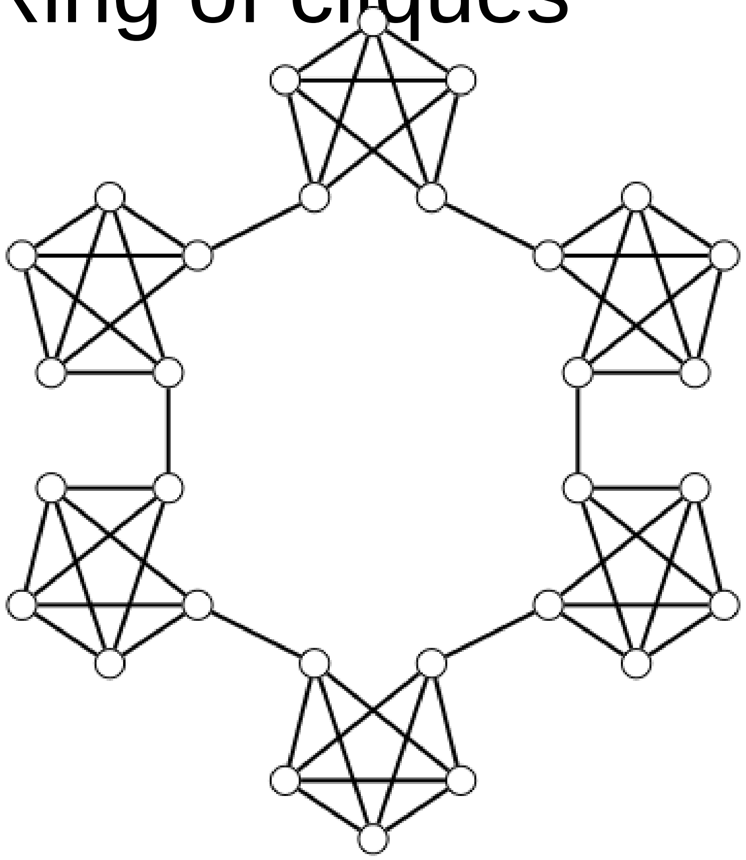
	Simple-Switching	Flipper	Pointer-Push&Pull	k-Flipper small k	k-Flipper large k
Graphs	Undirected Graphs	Undirected Graphs	Directed Multigraphs	Undirected Graphs	Undirected Graphs
Soundness	?	✓	✓	✓	✓
Generality	↘	✓	✓	✓	✓
Feasibility	✓	✓	✓	✓	↘
Conver-gence	✓	?	?	?	✓

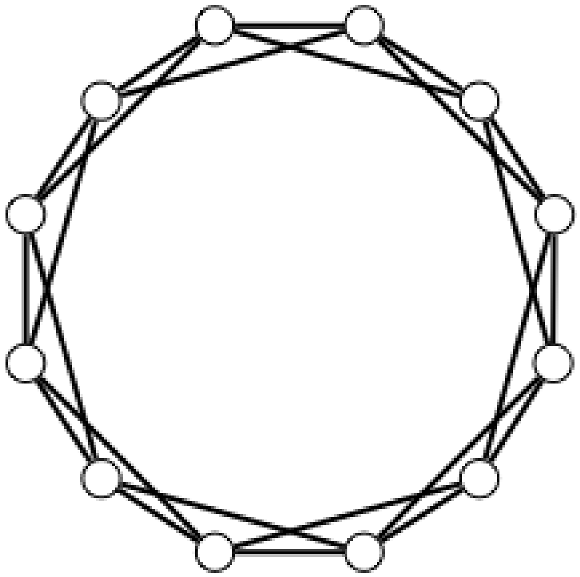
- **Open Problems**
  - Conjecture: Flipper converges in after O(dn log n) operations to a truly random graph
  - Conjecture: k-Flipper converges faster, but involves more nodes and flags
  - Conjecture: k-Flipper does not pay out
- **Empirical Simulations**
  - Estimate expansion by eingenvalue gap
  - Estimate eigenvalue gap by iterated multiplication of a start vector

Start Graphs

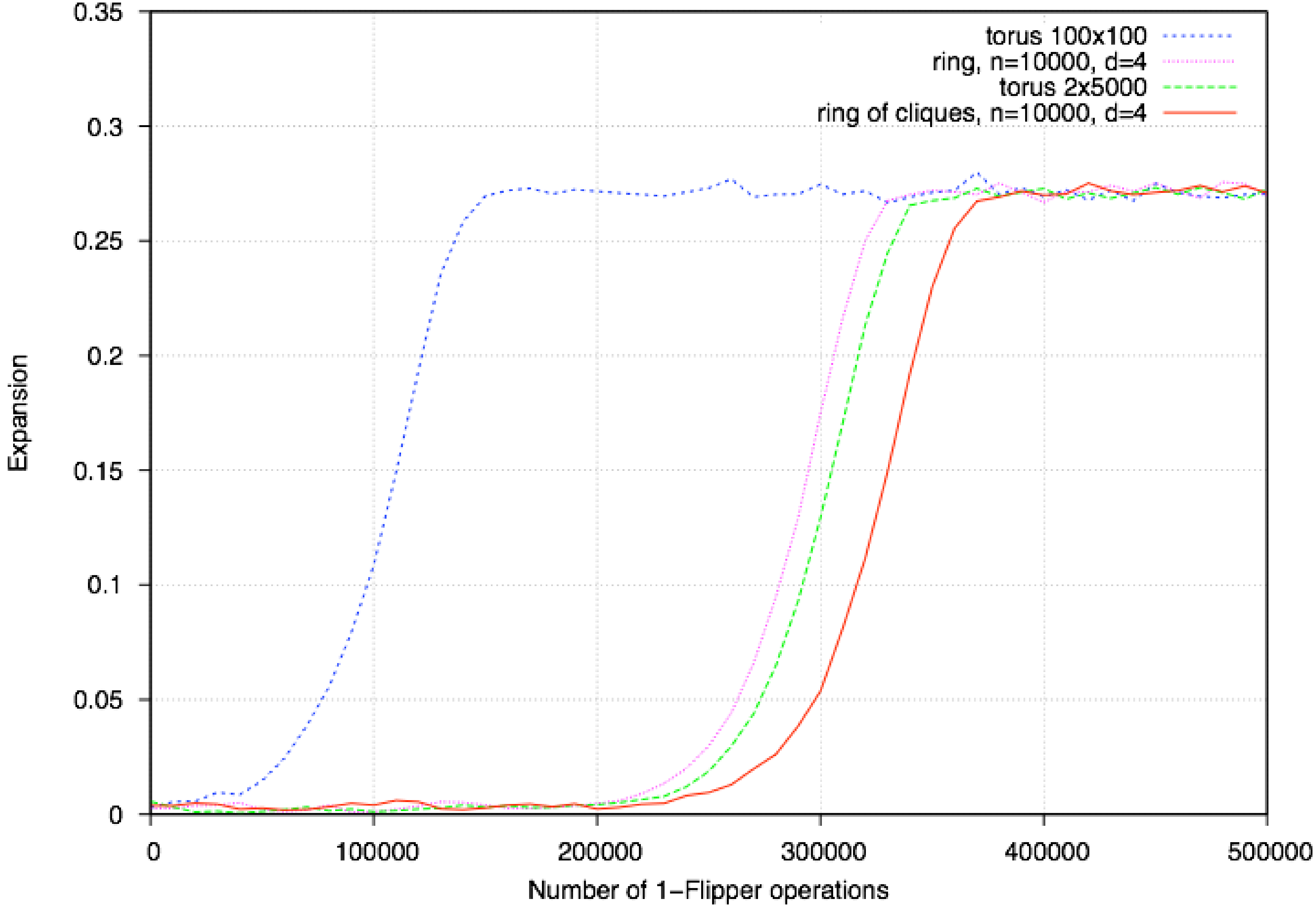
- Ring with neighbor edges
- Torus
- Ring of cliques



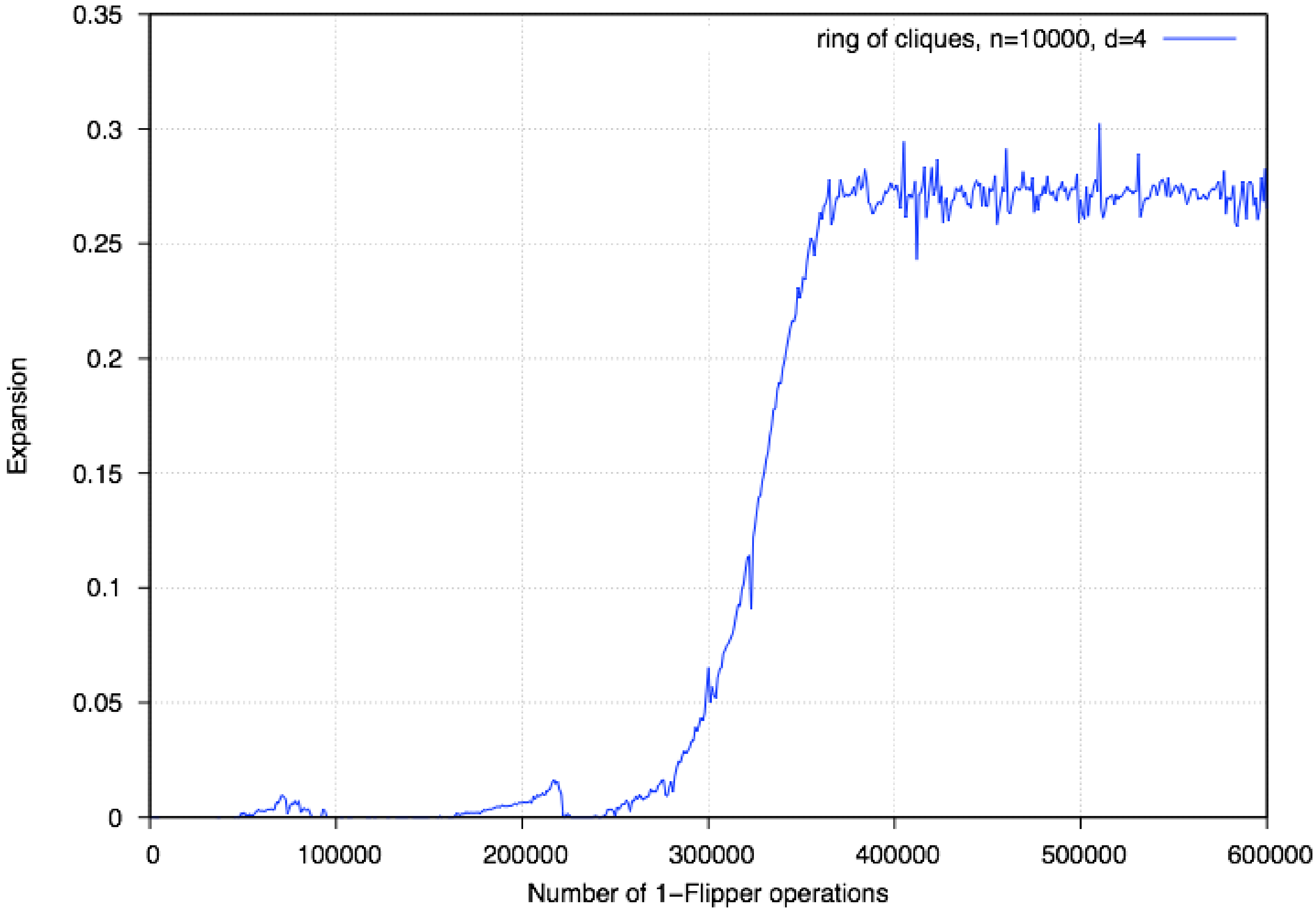




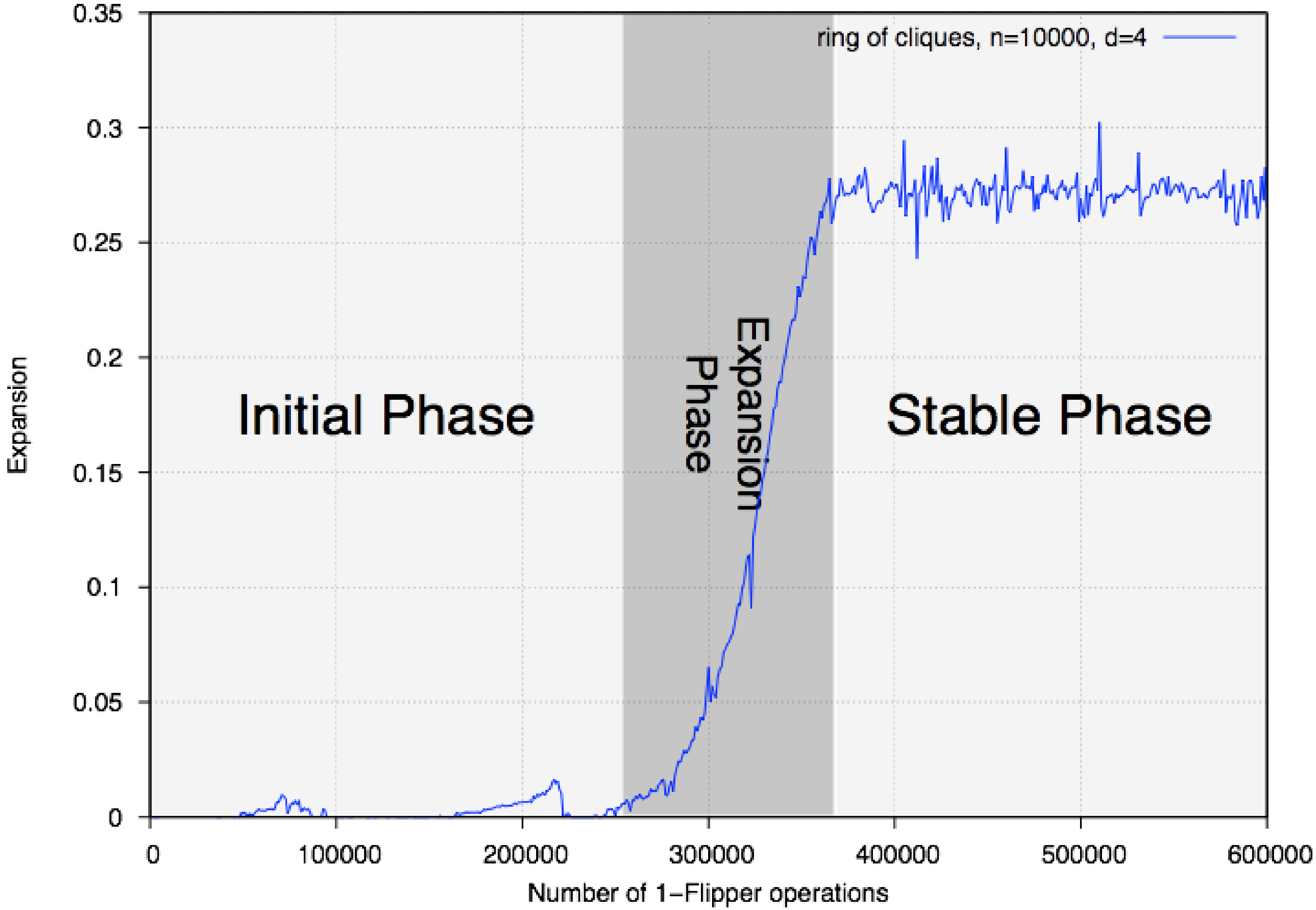
Flipper
Influence of the Start Graph



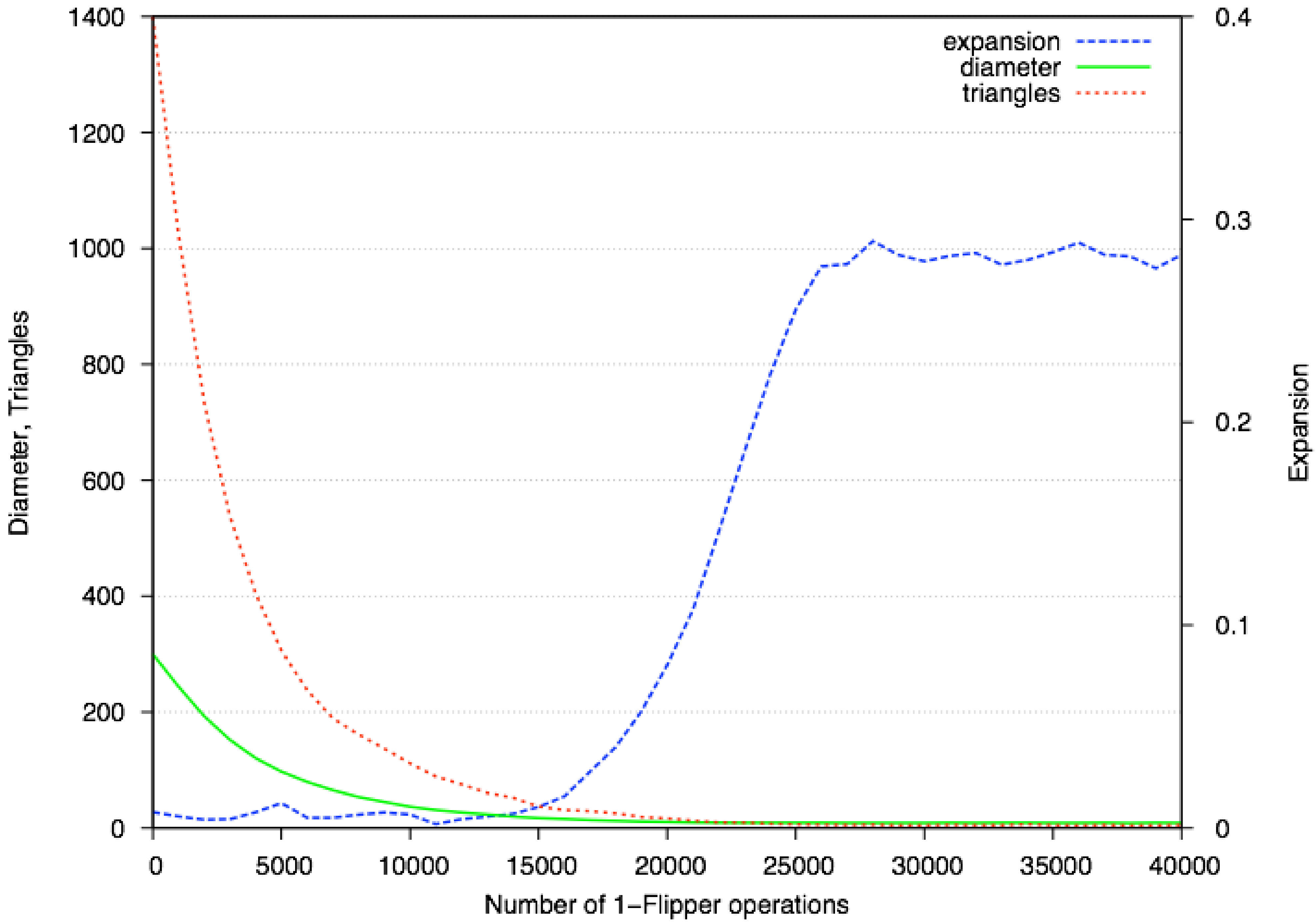
Development of Expansion



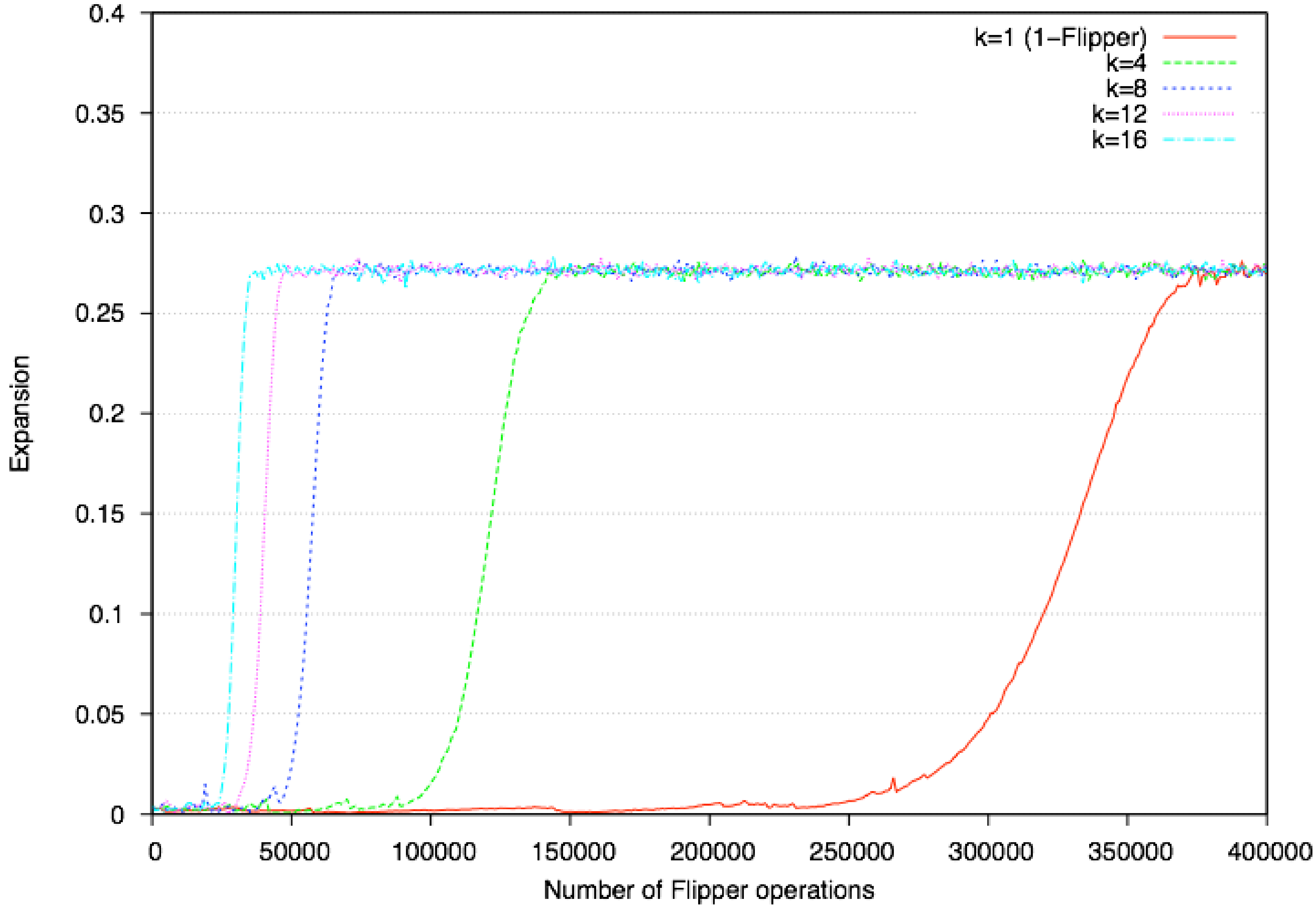
Development of Expansion



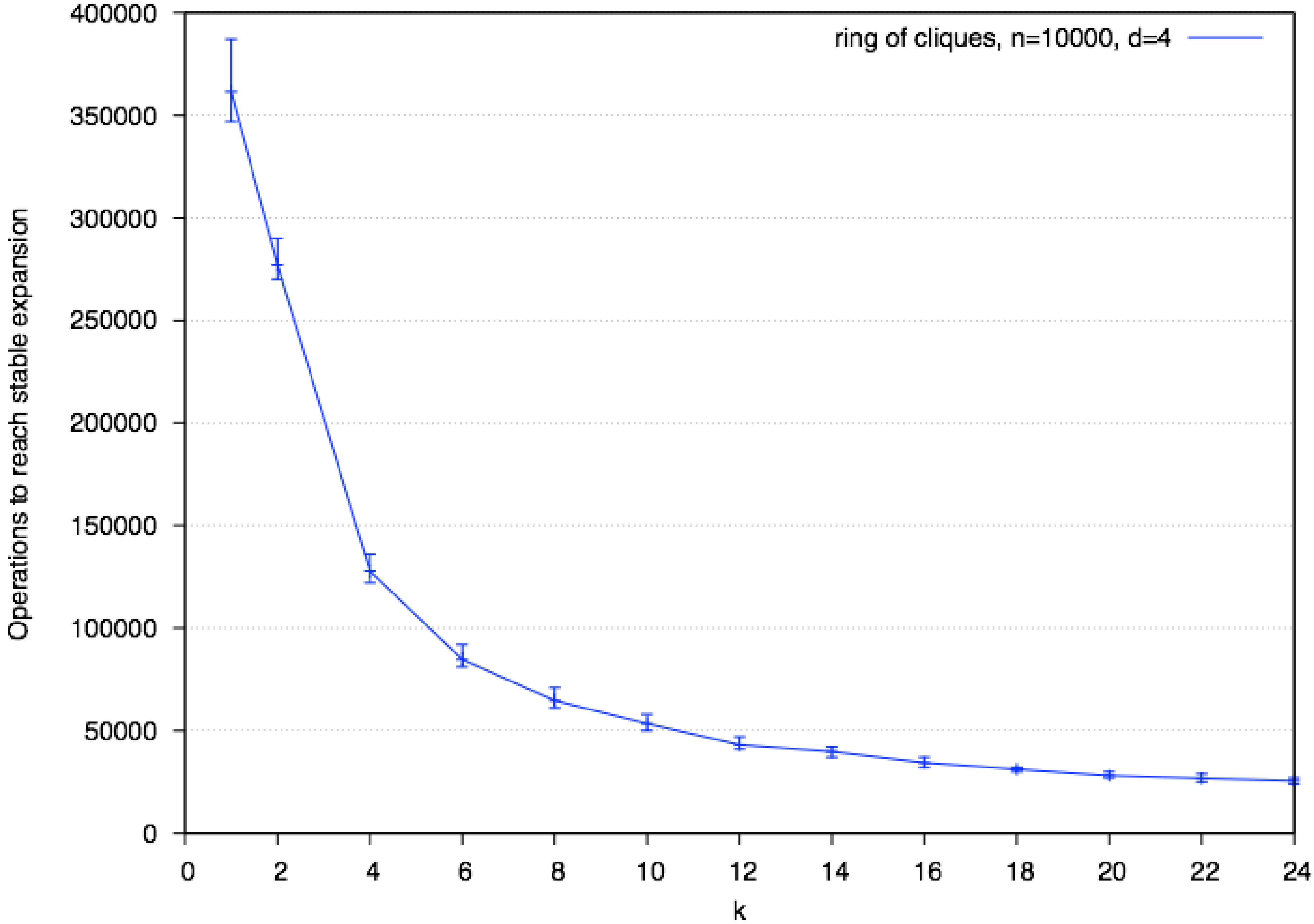
Expansion, Diameter & Triangles



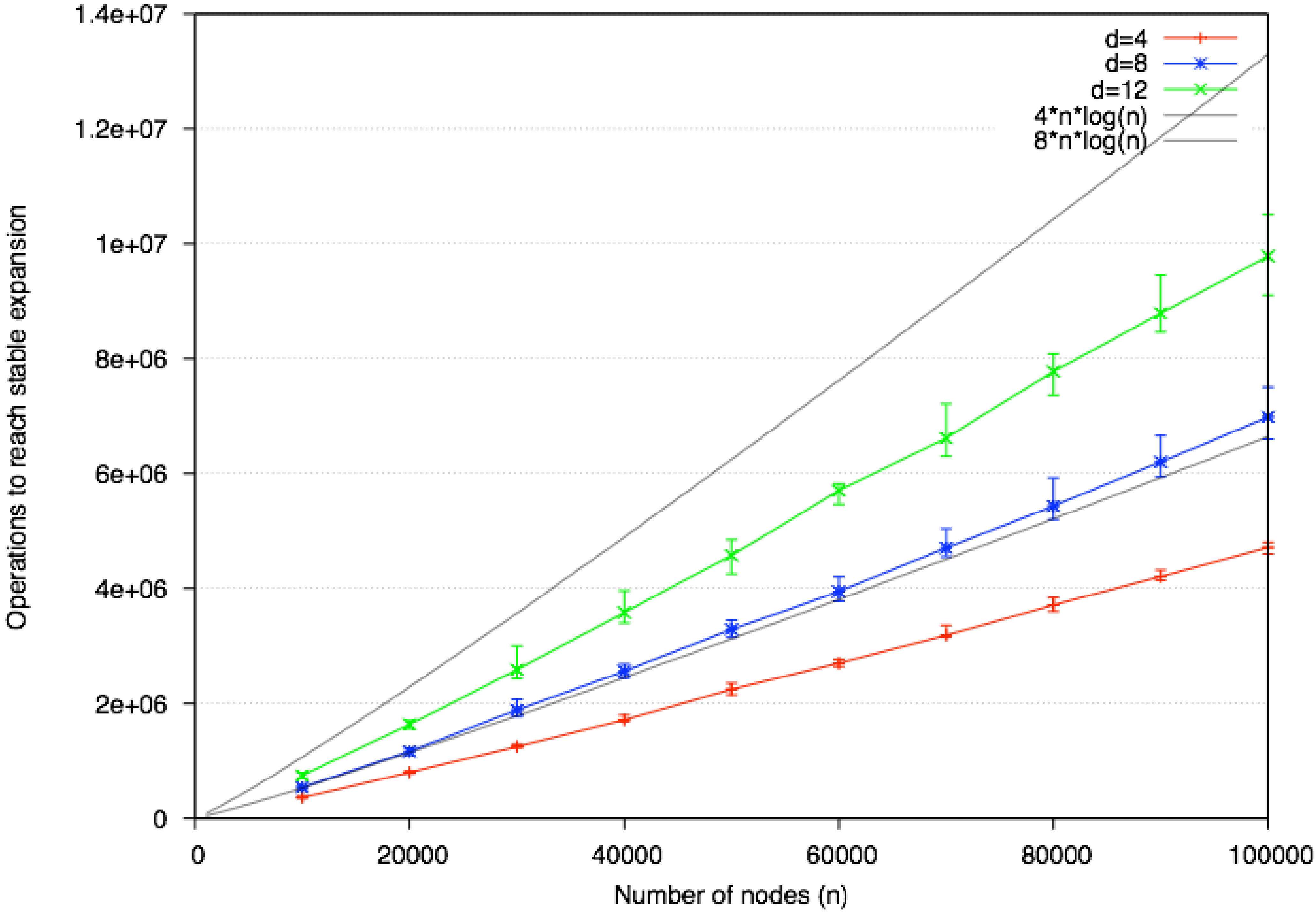
k-Flipper
Start Graph: Ring of Cliques



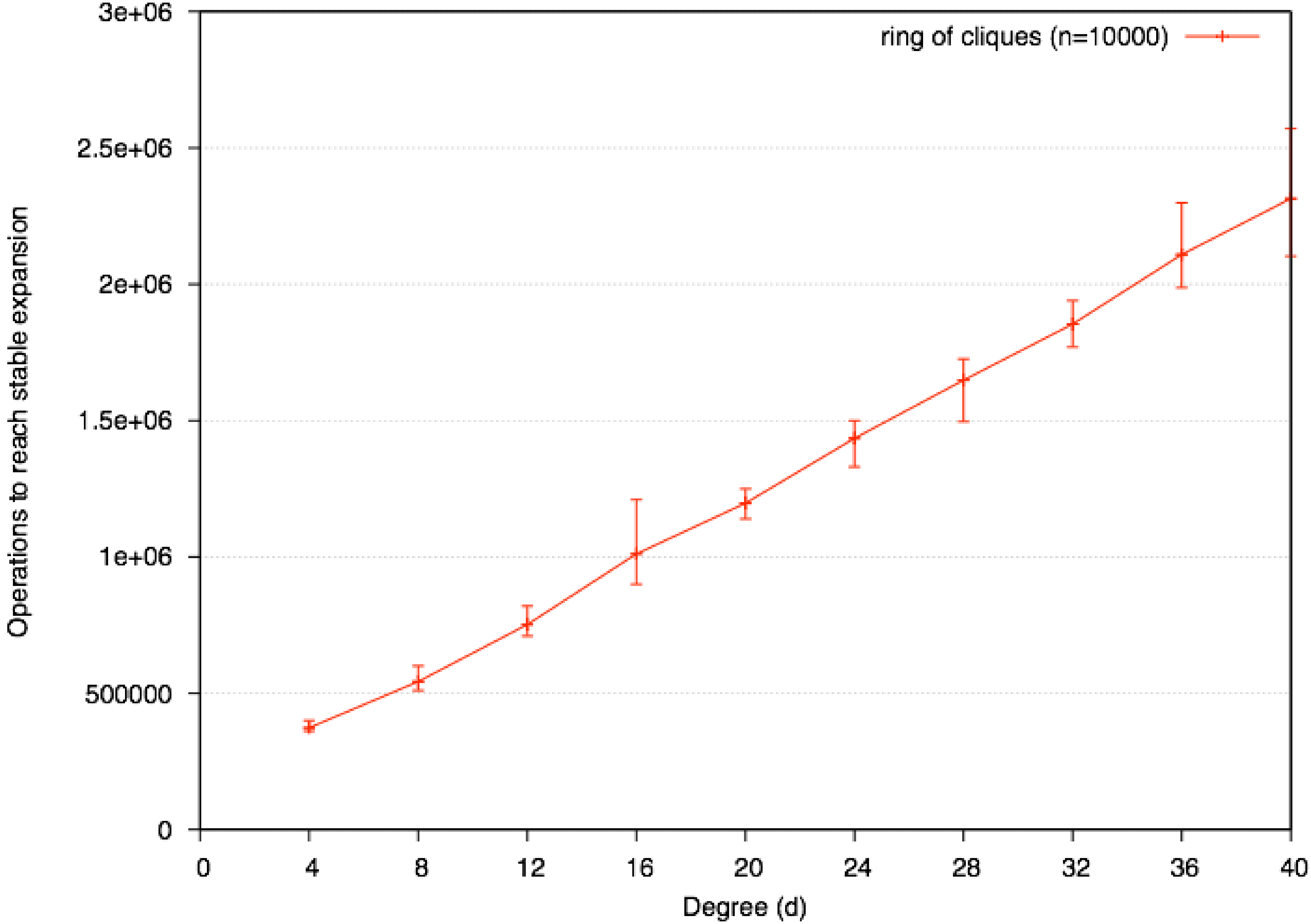
k-Flipper
Start Graph: Ring of Cliques



Convergence of Flipper



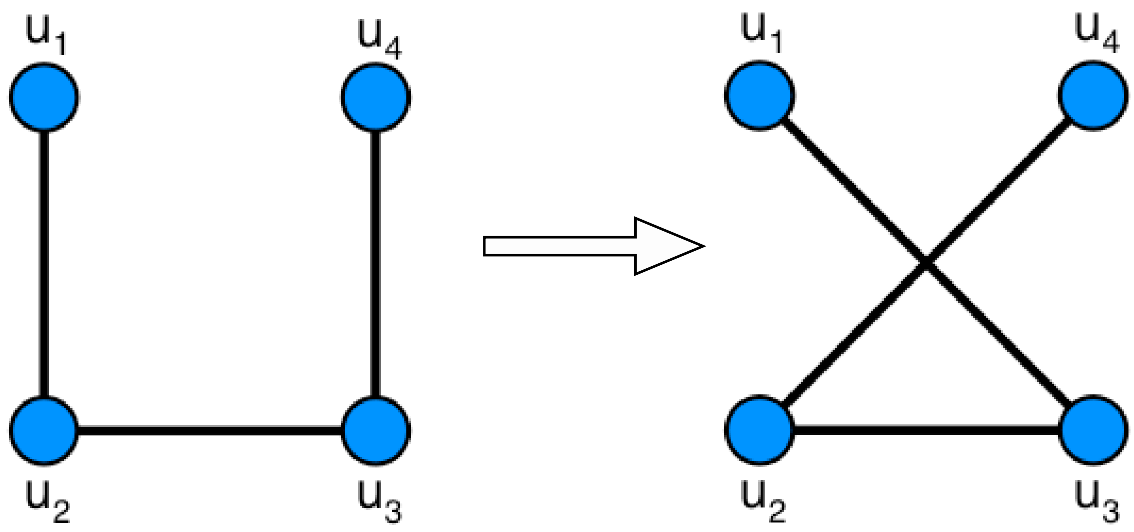
Convergence of Flipper
Varying Degree

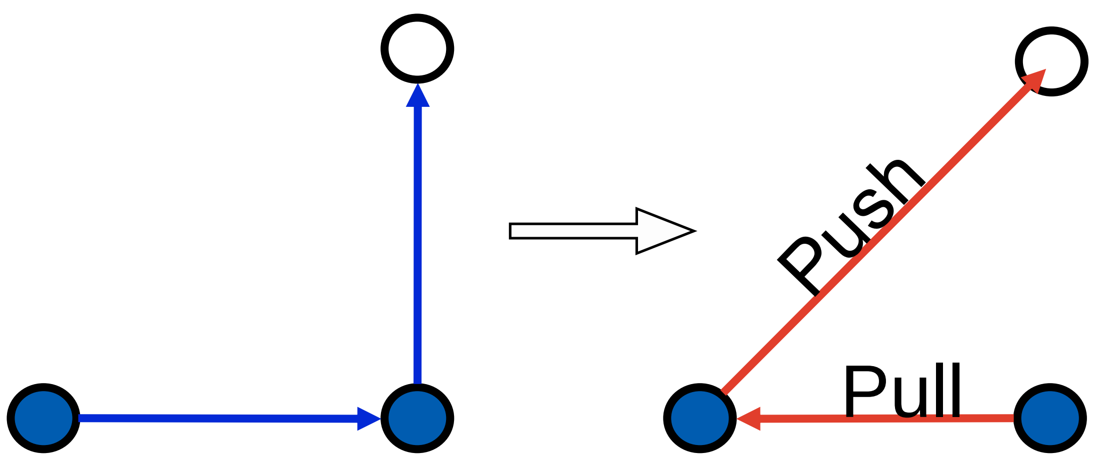


All Graph Transformation

	Simple-Switching	Flipper	Pointer-Push&Pull	k-Flipper small k	k-Flipper large k
Graphs	Undirected Graphs	Undirected Graphs	Directed Multigraphs	Undirected Graphs	Undirected Graphs
Soundness	?	✓	✓	✓	✓
Generality	↯	✓	✓	✓	✓
Feasibility	✓	✓	✓	✓	↯
Convergence	✓	✓	?	✓	✓

Good Peer-to-Peer-Operations





Peer-to-Peer Networks

09 Random Graphs for Peer-to-Peer-Networks

Christian Ortolf
Technical Faculty
Computer-Networks and Telematics
University of Freiburg