ALBERT-LUDWIGS-
UNIVERSITÄT FREIBURG

Network Protocol Design and Evaluation

Exercise 8

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Task 1

Task 1 *Empirical distributions*

You observe the remote access to your server for 30 min and count the requests for each minute. The result is the following:

3 3 2 3 1 2 7 0 2 0 1 6 3 1 3 3 4 5 4 7 2 6 4 2 2 3 3 5 4 1

1. Calculate mean, median and quartiles for the number of requests per minute. Calculate the empirical distribution function.
2. Does this data fit to a distribution you know? Show the goodness of fit graphically.
3. Perform a χ^2 test of the sample data and a Poisson distribution.

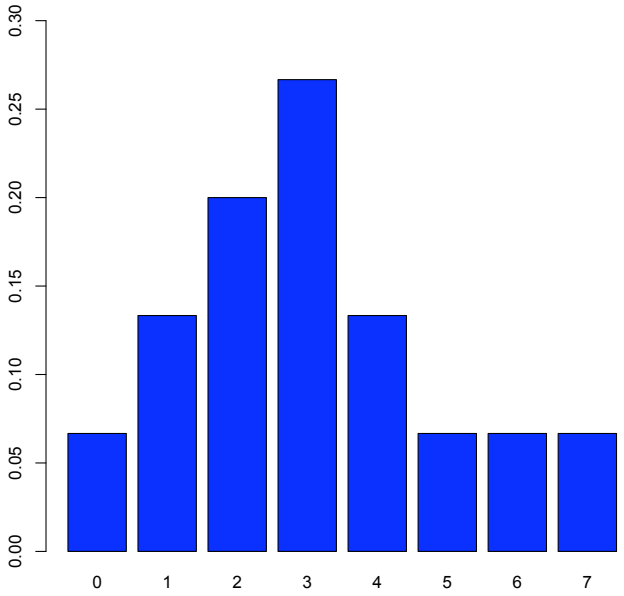
Task 1

- **Data:**

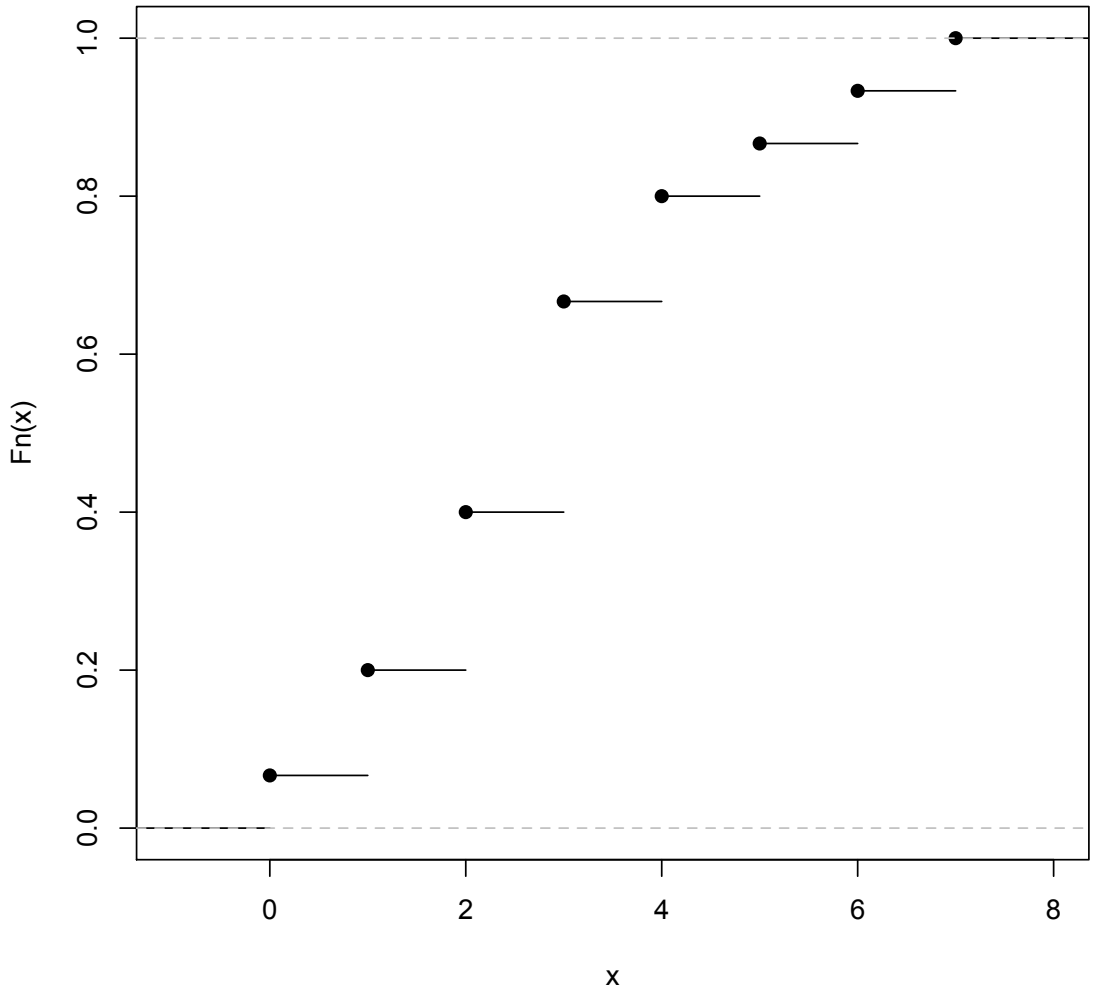
  3 3 2 3 1 2 7 0 2 0 1 6 3 1 3 3 4 5 4 7 2 6 4 2 2 3 3 5 4 1

- **Summary:**

```
Min.     0.000
1st Qu.  1.000
Median   3.000
Mean     3.067
3rd Qu.  4.000
Max.     7.000
```

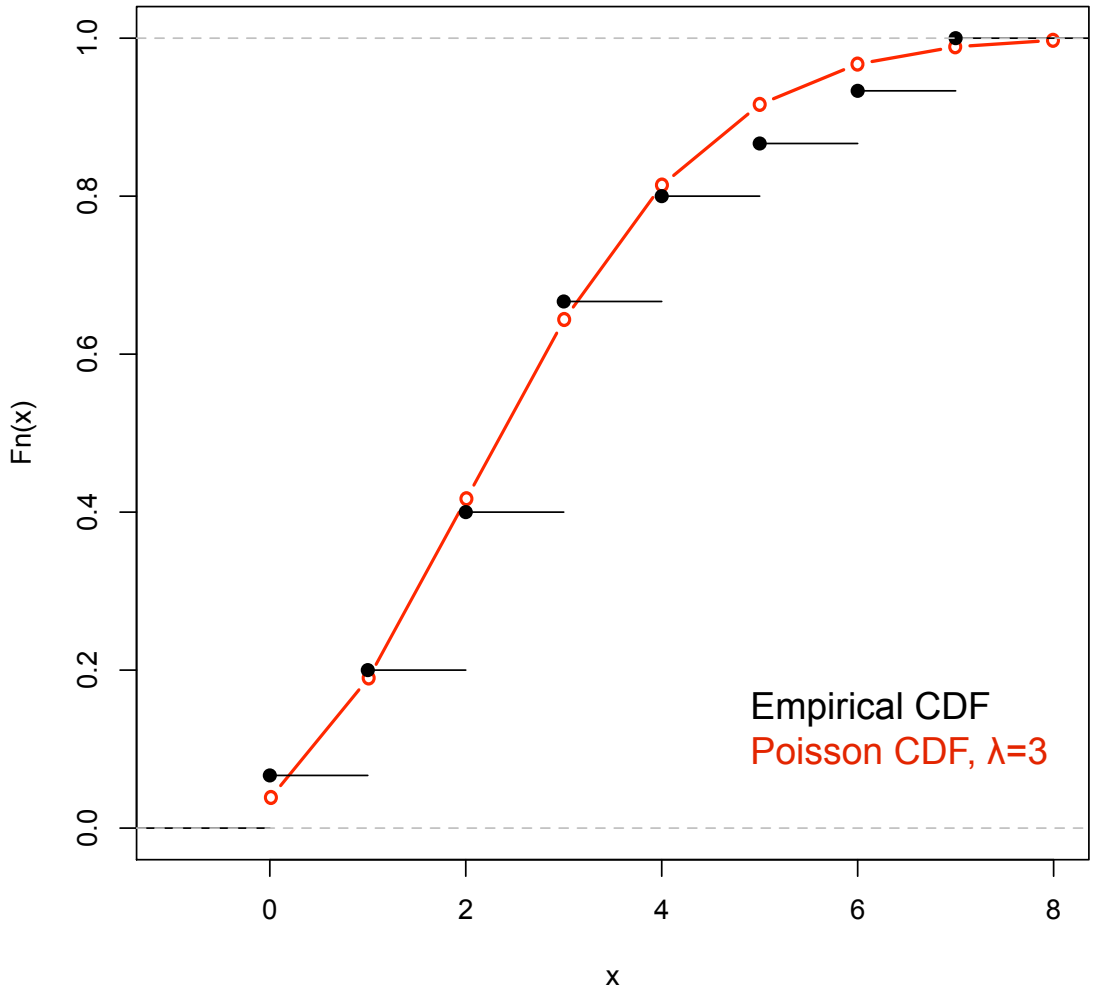


Empirical Distribution Function

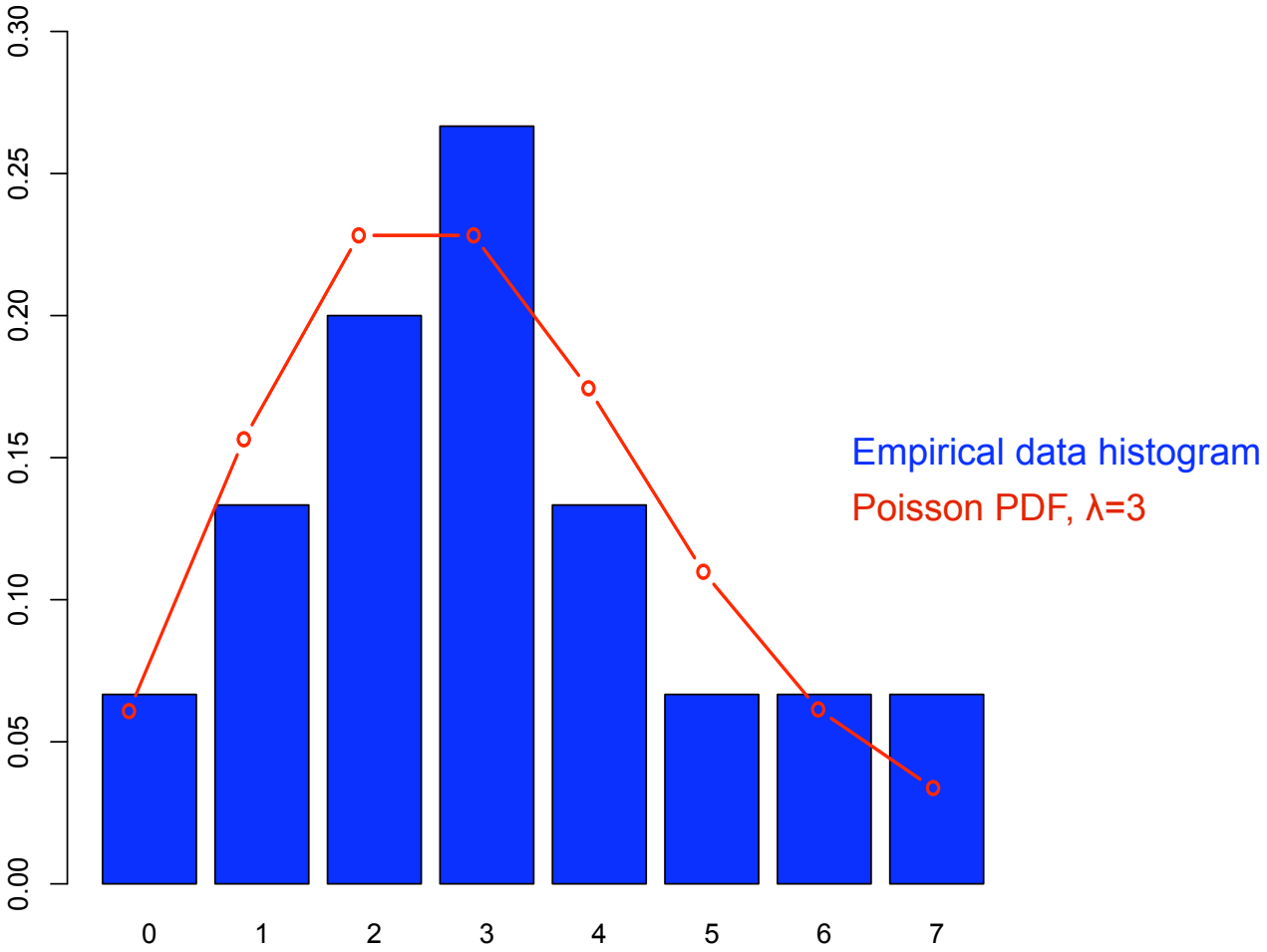


Plotted with R using "plot(ecdf(d))", where d is the data vector

Empirical and Poisson CDF



Density Functions



Task 1

- **Chi square test:**
  - Under a significance level of 0,05 reject the hypothesis, if the result $\chi^2 > 14.07$
  - $\chi^2 = 2.2476$, df = 7, p-value = 0.9449
  - The result means: the test does not indicate a bad fit

Statistics with R

(see www.r-project.org)

```
> d=c(3,3,2,3,1,2,7,0,2,0,1,6,3,1,3,3,4,5,4,7,2,6,4,2,2,3,3,5,4,1)
> table(d)
d
0 1 2 3 4 5 6 7
2 4 6 8 4 2 2 2
> tabulate(d)
[1] 4 6 8 4 2 2 2
> f=c(2,tabulate(d))
> f
[1] 2 4 6 8 4 2 2 2
> p=f/sum(f)
> p
[1] 0.06666667 0.13333333 0.20000000 0.26666667 0.13333333 0.06666667
[7] 0.06666667 0.06666667
> x=c(0,1,2,3,4,5,6)
> q = c(dpois(x, 3),1-sum(dpois(x, 3)))
> q
[1] 0.04978707 0.14936121 0.22404181 0.22404181 0.16803136 0.10081881
[7] 0.05040941 0.03350854
> chisq.test(f,p=q)
        Chi-squared test for given probabilities
data:  f
X-squared = 2.2476, df = 7, p-value = 0.9449
>
```

Usually data is read from a file, here we set the data vector directly

← tabulate omits the count for x=0, we add 2 manually

← frequencies (histogram data)

← normalize the data

← prob for x=0..6 and for x > 6 from the Poisson distribution w. λ=3

← how good does the sample data match the probability vector?

Task 2

Task 2 *Arrival Processes*

Assume that the arrival of students in the cafeteria follows a Poisson process and that 3 students arrive on average per minute.

1. You decide to eat there if it is likely (let's say with probability of more than 0.75) that the number of students arriving per minute is less than 5. Will you go there?
2. What is the probability that more than 10 students arrive?
3. Plot the probability mass function and the cumulative distribution function.

Task 2

- Poisson process with λ=3 arrivals per unit time (t=1)

$$\Pr_\lambda[X = k] = \frac{\lambda^k}{k!} e^{-\lambda}$$

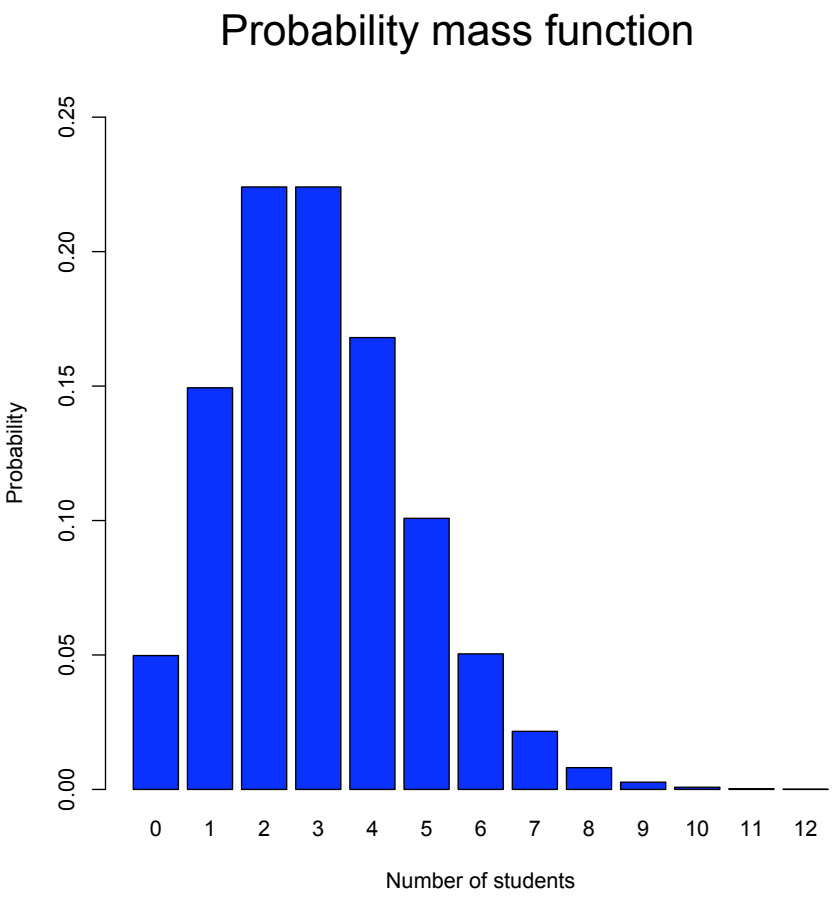
- Probability that less than 5 students arrive:

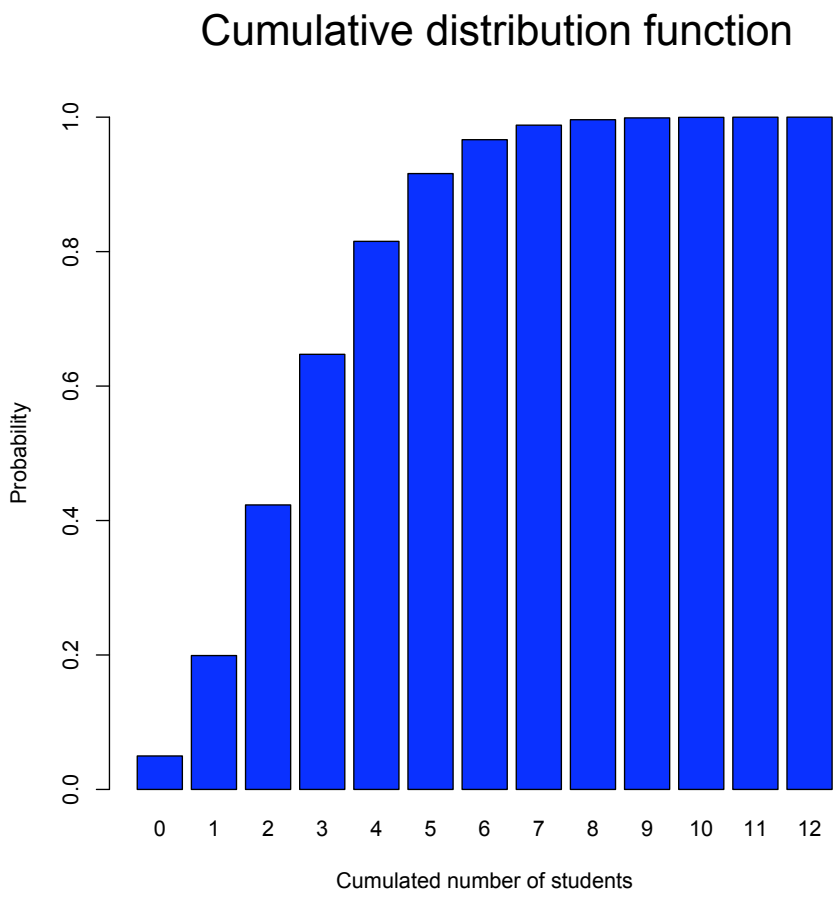
$$\Pr[X \leq 4] = \sum_{k=0}^{4} \Pr_\lambda[X = k] \approx 0.815$$

- Probability that more than 10 students arrive:

$$\Pr[X > 10] = 1 - \Pr[X \leq 10] \approx 0.00029$$

Task 2


Probability mass function
Probability
0.00
0.05
0.10
0.15
0.20
0.25
0 1 2 3 4 5 6 7 8 9 10 11 12
Number of students


Cumulative distribution function
Probability
0.0
0.2
0.4
0.6
0.8
1.0
0 1 2 3 4 5 6 7 8 9 10 11 12
Cumulated number of students