Wireless Sensor Networks
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Amplitude Representation

- Amplitude representation of a sinus curve
  - \[ s(t) = A \sin(2\pi ft + \varphi) \]
  - \( A \): amplitude
  - \( \varphi \): phase shift
  - \( f \): frequency = \( 1/T \)
  - \( T \): period

![Amplitude representation of a sinus curve](image)
Fourier Transformation

- Fourier transformation of a periodic function:
  - Decomposition into sinus curves

- Dirichlet’s conditions for a periodic function:
  - \( f(x) = f(x+2\pi) \)
  - \( f(x) \) is continuous and monotone in finitely many intervals of \((−\pi, \pi)\)
  - If is non-continuous in \( x_0 \), then \( f(x_0) = (f(x_0^-)+f(x_0^+))/2 \)

- Theorem of Dirichlet:
  - \( f(x) \) satisfies Dirichlet’s conditions. Then the Fourier coefficients \( a_0, a_1, a_2, \ldots, b_1, b_2, \ldots \) exist such that:

\[
\lim_{n \to \infty} \frac{a_0}{2} + \sum_{k=1}^{n} a_k \cos kx + b_k \sin kx = f(x) .
\]
Computation of Fourier coefficients

➢ Fourier coefficients \( a_i, b_i \) can be computed as follows

- For \( k = 0, 1, 2, \ldots \)
  \[
  a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos kx \, dx
  \]

- For \( k = 1, 2, 3, \ldots \)
  \[
  b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin kx \, dx
  \]

➢ Example: saw tooth curve

\[
f(x) = x, \text{ für } 0 < x < 2\pi
\]

\[
f(x) = \pi - 2 \left( \frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \ldots \right)
\]
Fourier-Analysis

• Theorem of Fourier for period $T=1/f$:
  - The coefficients $c, a_n, b_n$ can be computed as follows

$$g(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(2\pi k ft) + b_k \sin(2\pi k ft)$$

$$a_k = \frac{2}{T} \int_{0}^{T} g(t) \cos(2\pi n ft) dt$$

$$b_k = \frac{2}{T} \int_{0}^{T} g(t) \sin(2\pi n ft) dt$$

• The square of the sum of the $k$-th terms is proportional to the energy in this frequency

$$(a_k)^2 + (b_k)^2$$
Frequency Bands

10^0  10^2  10^4  10^6  10^8  10^10  10^12  10^14  10^16  10^18  10^20  10^22  10^24

Radio  Microwave  Infrared  UV  X-ray  Gamma Ray

f (Hz) 10^1  10^5  10^6  10^7  10^8  10^9  10^10  10^11  10^12  10^13  10^14  10^15  10^16

Twisted pair  Coax  Satellite  Terrestrial microwave  Fiber optics

Band  LF  MF  HF  VHF  UHF  SHF  EHF  THF

LF  Low Frequency  MF  Medium Freq.  HF  High Freq.
VHF  Very High Freq.  UHF  Ultra High F.  SHF  Super High Fr.
EHF  Extra High Frequency  UV  Ultra Violet

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Radio Propagation

- Propagation on straight line
- Signal strength is proportional to $1/d^2$ in free space
  - In practice can be modeled by $1/d^c$, for $c$ up to 4 or 5
- Energy consumption
  - for transmitting a radio signal over distance $d$ in empty space is $d^2$
- Basic properties
  - Reflection
  - Refraction (between media with slower speed of propagation)
  - Interference
  - Diffraction
  - Attenuation in air (especially HV, VHF)
Radio Propagation

- **VLF, LF, MF**
  - follow the curvature of the globe (up zu 1000 kms in VLF)
  - pass through buildings
- **HF, VHF**
  - absorbed by earth
  - reflected by ionosphere in a height of 100-500 km
- **>100 MHz**
  - No passing through walls
  - Good focus
- **> 8 GHz absorption by rain**
Radio Propagation

- **Multiple Path Fading**
  - Because of reflection, diffraction and diffusion the signal arrives on multiple paths
  - Phase shifts because of different path length causes interferences

- **Problems with mobile nodes**
  - Fast Fading
    - Different transmission paths
    - Different phase shifts
  - Slow Fading
    - Increasing or decreasing the distance between sender and receiver
Signal Interference Noise Ratio

- Receiving-power = Transmission-power \cdot \text{path-loss}
  - path loss \sim 1/r^\beta
  - \beta \in [2,5]

- **Signal to Interference + Noise Ratio = SINR**
  - S = receiving power from desired sender
  - I = receiving power from interfering senders
  - N = other interfering signals (e.g. noise)

- **Necessary for recognizing the signal:**

\[
\text{SINR} = \frac{S}{I+N} \geq \text{Threshold}
\]
Frequency allocation

➢ Some frequencies are allocated to specific uses
   – Cellular phones, analog television/radio broadcasting, DVB-T, radar, emergency services, radio astronomy, …

➢ Particularly interesting: ISM bands (“Industrial, scientific, medicine”) – license-free operation

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.553-13.567 MHz</td>
<td></td>
</tr>
<tr>
<td>26.957 – 27.283 MHz</td>
<td></td>
</tr>
<tr>
<td>40.66 – 40.70 MHz</td>
<td></td>
</tr>
<tr>
<td>433 – 464 MHz</td>
<td>Europe</td>
</tr>
<tr>
<td>900 – 928 MHz</td>
<td>Americas</td>
</tr>
<tr>
<td>2.4 – 2.5 GHz</td>
<td>WLAN/WPAN</td>
</tr>
<tr>
<td>5.725 – 5.875 GHz</td>
<td>WLAN</td>
</tr>
<tr>
<td>24 – 24.25 GHz</td>
<td></td>
</tr>
</tbody>
</table>
Example: US frequency allocation

Transceivers and the Physical Layer

- Frequency bands
- *Modulation*
- Signal distortion – wireless channels
- From waves to bits
- Channel models
- Transceiver design
Modulation and keying

- How to manipulate a given signal parameter?
  - Set the parameter to an arbitrary value: **analog modulation**
  - Choose parameter values from a finite set of legal values: **digital keying**
  - Simplification: When the context is clear, modulation is used in either case

- Modulation?
  - Data to be transmitted is used to select transmission parameters as a function of time
  - These parameters modify a basic sine wave, which serves as a starting point for **modulating** the signal onto it
  - This basic sine wave has a **center frequency** \( f_c \)
  - The resulting **signal** requires a certain **bandwidth** to be transmitted (centered around center frequency)
Modulation (keying!) examples

- Use data to modify the amplitude of a carrier frequency → Amplitude Shift Keying

- Use data to modify the frequency of a carrier frequency → Frequency Shift Keying

- Use data to modify the phase of a carrier frequency → Phase Shift Keying
Amplitude Shift Keying (ASK)

- Let $E_i(t)$ be the symbol energy at time $t$

$$s_i(t) = \sqrt{\frac{2E_i(t)}{T}} \cdot \sin(\omega_0 t + \phi)$$

- The first term is a convention such that $E_i$ denotes the energy
- Example: $E_0(t) = 1$, $E_1(t)=2$ for all $t$
Phase Shift Keying (PSK)

- For phase signals $\phi_i(t)$

$$s_i(t) = \sqrt{\frac{2E}{T}} \cdot \sin(\omega_0 t + \phi_i(t))$$

- Example:

![Graph showing phase shift keying (PSK) example](image.png)
Frequency Shift Keying (FSK)

- For frequency signals $\omega_i(t)$

$$ s_i(t) = \sqrt{\frac{2E}{T}} \cdot \sin(\omega_i(t) \cdot t + \phi) $$

- Example:

![Graph of Frequency Shift Keying (FSK) example](image)
Receiver: Demodulation

The receiver looks at the received wave form and matches it with the data bit that caused the transmitter to generate this wave form
  – Necessary: one-to-one mapping between data and wave form
  – Because of channel imperfections, this is at best possible for digital signals, but not for analog signals

Problems caused by
  – Carrier synchronization: frequency can vary between sender and receiver (drift, temperature changes, aging, …)
  – Bit synchronization (actually: symbol synchronization): When does symbol representing a certain bit start/end?
  – Frame synchronization: When does a packet start/end?
  – Biggest problem: Received signal is not the transmitted signal!
Overview

- Frequency bands
- Modulation
- Signal distortion – wireless channels
- From waves to bits
- Channel models
- Transceiver design
Transmitted signal ≠ received signal!

- **Wireless transmission** distorts any transmitted signal
  - Received <> transmitted signal; results in *uncertainty at receiver* about which bit sequence originally caused the transmitted signal
  - Abstraction: *Wireless channel* describes these distortion effects

- **Sources of distortion**
  - Attenuation – energy is distributed to larger areas with increasing distance
  - Reflection/refraction – bounce of a surface; enter material
  - Diffraction – start “new wave” from a sharp edge
  - Scattering – multiple reflections at rough surfaces
  - Doppler fading – shift in frequencies (loss of center)
Attenuation results in path loss

- Effect of attenuation: received signal strength is a function of the distance \( d \) between sender and transmitter

- Captured by \textit{Friis free-space equation}
  - Distance: \( R \)
  - Wavelength: \( \lambda \)
  - \( P_r \): power at receive antenna
  - \( P_t \): power at transmit antenna
  - \( G_t \): transmit antenna gain
  - \( G_r \): receive antenna gain

\[
\frac{P_r}{P_t} = G_t G_r \left( \frac{\lambda}{4\pi R} \right)^2
\]

\[
P_r(d) = P_r(d_0) \cdot \left( \frac{d_0}{d} \right)^2
\]
Suitability of different frequencies – Attenuation

- Attenuation depends on the used frequency
- Can result in a frequency-selective channel
  - If bandwidth spans frequency ranges with different attenuation properties

http://www.geographie.uni-muenchen.de/igtf/Multimedia/Klimatologie/physik_arbeit.htm
Distortion effects: Non-line-of-sight paths

- Because of reflection, scattering, ..., radio communication is not limited to direct line of sight communication
  - Effects depend strongly on frequency, thus different behavior at higher frequencies

- Different paths have different lengths = propagation time
  - Results in *delay spread* of the wireless channel
  - Closely related to frequency-selective fading properties of the channel
  - With movement: *fast fading*
Wireless signal strength in a multi-path environment

- Brighter color = stronger signal
- Obviously, simple (quadratic) free space attenuation formula is not sufficient to capture these effects

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To take into account stronger attenuation than only caused by distance (e.g., walls, ...), use a larger exponent $\gamma > 2$

- $\gamma$ is the **path-loss exponent**

\[ P_{\text{recv}}(d) = P_{\text{recv}}(d_0) \cdot \left(\frac{d_0}{d}\right)^\gamma \]

- Rewrite in logarithmic form (in dB):

\[ PL(d)[\text{dB}] = PL(d_0)[\text{dB}] + 10\gamma \log_{10} \left(\frac{d}{d_0}\right) \]

Take obstacles into account by a random variation

- Add a Gaussian random variable $X_\sigma$ with 0 mean, variance $\sigma^2$ to dB representation

- Equivalent to multiplying with a lognormal distributed r.v. in metric units → **lognormal fading**

\[ PL(d)[\text{dB}] = PL(d_0)[\text{dB}] + 10\gamma \log_{10} \left(\frac{d}{d_0}\right) + X_\sigma[\text{dB}] \]
Transceivers and the Physical Layer

- Frequency bands
- Modulation
- Signal distortion – wireless channels
- *From waves to bits*
- Channel models
- Transceiver design
Noise and interference

➢ So far: only a single transmitter assumed
  - Only disturbance: self-interference of a signal with multi-path “copies” of itself

➢ In reality, two further disturbances
  - **Noise** – due to effects in receiver electronics, depends on temperature
    • Typical model: an additive Gaussian variable, mean 0, no correlation in time
  - **Interference** from third parties
    • Co-channel interference: another sender uses the same spectrum
    • Adjacent-channel interference: another sender uses some other part of the radio spectrum, but receiver filters are not good enough to fully suppress it

➢ Effect: Received signal is distorted by channel, corrupted by noise and interference
  - What is the result on the received bits?
Symbols and bit errors

- Extracting symbols out of a distorted/corrupted wave form is fraught with errors
  - Depends essentially on strength of the received signal compared to the corruption
  - Captured by \textit{signal to noise and interference ratio (SINR) given in decibel}:
    \[
    \text{SINR} = 10 \log_{10} \left( \frac{P_{\text{recv}}}{N_0 + \sum_{i=1}^{k} I_i} \right)
    \]

- SINR allows to compute \textit{bit error rate (BER) for a given modulatio}:
  - Also depends on data rate (# bits/symbol) of modulation

![Graph showing BER vs. SNR]

Figure 4.7 Bit error rate for coherently detected binary PSK and FSK
Overview

- Frequency bands
- Modulation
- Signal distortion – wireless channels
- From waves to bits
- Channel models
- Transceiver design
Channel models – analog signal

- How to stochastically capture the behavior of a wireless channel
  - Main options: model the SNR or directly the bit errors

- Signal models
  - Simplest model: assume transmission power and attenuation are constant, noise an uncorrelated Gaussian variable
    - Additive White Gaussian Noise model, results in constant SNR
    - For expectation $\mu$ and standard deviation $\sigma$ the density function is defined as:
      \[
      f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2 \right)
      \]
  - Situation with no line-of-sight path, but many indirect paths: Amplitude of resulting signal has a Rayleigh distribution (Rayleigh fading)
    - $\Omega = E(R^2)$. Then the density function is
      \[
      p_R(r) = \frac{r}{\Omega} e^{-r^2/\Omega}
      \]
  - One dominant line-of-sight plus many indirect paths: Signal has a Rice distribution (Rice fading)
Channel models – digital

➢ Directly model the resulting bit error behavior
  - Each bit is erroneous with constant probability, independent of the other bits → binary symmetric channel (BSC)
  - Capture fading models’ property that channel be in different states → Markov models – states with different BERs
    • Example: Gilbert-Elliot model with “bad” and “good” channel states and high/low bit error rates

Fractal channel models describe number of (in-)correct bits in a row by a heavy-tailed distribution
WSN-specific channel models

- Typical WSN properties
  - Small transmission range
  - Implies small delay spread (nanoseconds, compared to micro/milliseconds for symbol duration)
  - Frequency-non-selective fading, low to negligible inter-symbol interference
    - Coherence bandwidth often > 50 MHz

- Some example measurements
  - \( \gamma \) path loss exponent
  - Shadowing variance \( \sigma^2 \)
  - Reference path loss at 1 m

<table>
<thead>
<tr>
<th>Location</th>
<th>Average of ( \gamma )</th>
<th>Average of ( \sigma^2 )[dB]</th>
<th>Range of PL(1m)[dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engineering Building</td>
<td>1.9</td>
<td>5.7</td>
<td>[−50.5, −39.0]</td>
</tr>
<tr>
<td>Apartment Hallway</td>
<td>2.0</td>
<td>8.0</td>
<td>[−38.2, −35.0]</td>
</tr>
<tr>
<td>Parking Structure</td>
<td>3.0</td>
<td>7.9</td>
<td>[−36.0, −32.7]</td>
</tr>
<tr>
<td>One-sided Corridor</td>
<td>1.9</td>
<td>8.0</td>
<td>[−44.2, −33.5]</td>
</tr>
<tr>
<td>One-sided patio</td>
<td>3.2</td>
<td>3.7</td>
<td>[−39.0, −34.2]</td>
</tr>
<tr>
<td>Concrete canyon</td>
<td>2.7</td>
<td>10.2</td>
<td>[−48.7, −44.0]</td>
</tr>
<tr>
<td>Plant fence</td>
<td>4.9</td>
<td>9.4</td>
<td>[−38.2, −34.5]</td>
</tr>
<tr>
<td>Small boulders</td>
<td>3.5</td>
<td>12.8</td>
<td>[−41.5, −37.2]</td>
</tr>
<tr>
<td>Sandy flat beach</td>
<td>4.2</td>
<td>4.0</td>
<td>[−40.8, −37.5]</td>
</tr>
<tr>
<td>Dense bamboo</td>
<td>5.0</td>
<td>11.6</td>
<td>[−38.2, −35.2]</td>
</tr>
<tr>
<td>Dry tall underbrush</td>
<td>3.6</td>
<td>8.4</td>
<td>[−36.4, −33.2]</td>
</tr>
</tbody>
</table>
Sharing the Medium

- **Space-Multiplexing**
  - Spatial distance
  - Directed antennae

- **Frequency-Multiplexing**
  - Assign different frequencies to the senders

- **Time-Multiplexing**
  - Use time slots for each sender

- **Spread-spectrum communication**
  - Direct Sequence Spread Spectrum (DSSS)
  - Frequency Hopping Spread Spectrum (FHSS)

- **Code Division Multiplex**
Frequency Hopping
Spread Spectrum

- Change the frequency while transferring the signal
  - Invented by Hedy Lamarr, George Antheil

- Slow hopping
  - Change the frequency slower than the signals come

- Fast hopping
  - Change the frequency faster
Thank you
(and thanks go also to Holger Karl for providing some slides)