Goals of this chapter

- Means for a node to determine its physical position (with respect to some coordinate system) or symbolic location
- Using the help of
  - Anchor nodes that know their position
  - Directly adjacent
  - Over multiple hops
- Using different means to determine distances/angles locally
Overview

- Basic approaches
- Trilateration
- Multihop schemes
Localization & positioning

- **Determine** physical position or logical location
  - Coordinate system or symbolic reference
  - Absolute or relative coordinates

- **Options**
  - Centralized or distributed computation
  - Scale (indoors, outdoors, global, ...)
  - Sources of information

- **Metrics**
  - Accuracy (how close is an estimated position to the real position?)
  - Precision (for repeated position determinations, how often is a given accuracy achieved?)
  - Costs, energy consumption, ...
Main approaches (information sources)

- **Proximity**
  - Exploit finite range of wireless communication
  - E.g.: easy to determine location in a room with infrared room number announcements

- **(Tri-/Multi-)lateration and angulation**
  - Use distance or angle estimates, simple geometry to compute position estimates

- **Scene analysis**
  - Radio environment has characteristic "signatures"
  - Can be measured beforehand, stored, compared with current situation
Estimating distances – RSSI

- Received Signal Strength Indicator
  - Send out signal of known strength, use received signal strength and path loss coefficient to estimate distance

\[ P_{\text{recv}} = c \frac{P_{\text{tx}}}{d^\alpha} \Leftrightarrow d = \sqrt[\alpha]{\frac{cP_{\text{tx}}}{P_{\text{recv}}}} \]

- Problem: Highly error-prone process – Shown: PDF for a fixed RSSI
Estimating distances – other means

➢ **Time of arrival (ToA)**
  - Use time of transmission, propagation speed, time of arrival to compute distance
  - Problem: Exact time synchronization

➢ **Time Difference of Arrival (TDoA)**
  - Use two different signals with different propagation speeds
  - Example: ultrasound and radio signal
    - Propagation time of radio negligible compared to ultrasound
  - Compute difference between arrival times to compute distance
  - Problem: Calibration, expensive/energy-intensive hardware
Determining angles

- Directional antennas
  - On the node
  - Mechanically rotating or electrically “steerable”
  - On several access points
    - Rotating at different offsets
    - Time between beacons allows to compute angles
Some range-free, single-hop localization techniques

- **Overlapping connectivity**: Position is estimated in the center of area where circles from which signal is heard/not heard overlap.

- **Approximate point in triangle**
  - Determine triangles of anchor nodes where node is inside, overlap them.
  - Check whether inside a given triangle – move node or simulate movement by asking neighbors.
  - Only approximately correct.
Overview

- Basic approaches
- Trilateration
- Multihop schemes
Trilateration

- Assuming distances to three points with known location are exactly given
- Solve system of equations (Pythagoras!)
  - \((x_i, y_i)\) : coordinates of anchor point \(i\), \(r_i\) distance to anchor \(i\)
  - \((x_u, y_u)\) : unknown coordinates of node
  \[
  (x_i - x_u)^2 + (y_i - y_u)^2 = r_i^2 \quad \text{for } i = 1, \ldots, 3
  \]
  - Subtracting eq. 3 from 1 & 2:
    \[
    (x_1 - x_u)^2 - (x_3 - x_u)^2 + (y_1 - y_u)^2 - (y_3 - y_u)^2 = r_1^2 - r_3^2
    \]
    \[
    (x_2 - x_u)^2 - (x_2 - x_u)^2 + (y_2 - y_u)^2 - (y_2 - y_u)^2 = r_2^2 - r_3^2.
    \]
  - Rearranging terms gives a linear equation in \((x_u, y_u)\):
    \[
    2(x_3 - x_1)x_u + 2(y_3 - y_1)y_u = (r_1^2 - r_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2)
    \]
    \[
    2(x_3 - x_2)x_u + 2(y_3 - y_2)y_u = (r_2^2 - r_3^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2)
    \]
Trilateration as matrix equation

- Rewriting as a matrix equation:

\[
2 \begin{bmatrix}
x_3 - x_1 & y_3 - y_1 \\
x_3 - x_2 & y_3 - y_2
\end{bmatrix}
\begin{bmatrix}
x_u \\
y_u
\end{bmatrix}
= \begin{bmatrix}
(r_1^2 - r_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2) \\
(r_2^2 - r_3^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2)
\end{bmatrix}
\]

- Example: \((x_1, y_1) = (2,1), (x_2, y_2) = (5,4), (x_3, y_3) = (8,2), r_1 = 10^{0.5}, r_2 = 2, r_3 = 3\)

\[
2 \begin{bmatrix}
6 & 1 \\
3 & -2
\end{bmatrix}
\begin{bmatrix}
x_u \\
y_u
\end{bmatrix}
= \begin{bmatrix}
64 \\
22
\end{bmatrix}
\]

\rightarrow (x_u, y_u) = (5,2)
Thank you

(and thanks go also to Holger Karl for providing slides)