Wireless Sensor Networks
21st Lecture
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Options for topology control

- Control node activity – deliberately turn on/off nodes
- Control link activity – deliberately use/not use certain links

Flat network – all nodes have essentially same role
Power control

Hierarchical network – assign different roles to nodes; exploit that to control node/link activity
Backbones
Clustering
Geometric Spanner Graphs

- A Graph \( G = (V, E) \) with \( V \subseteq \mathbb{R} \) where for all \( u, v \in V \) there exists a path \( P = (u = u_1, u_2, \ldots, u_\ell = v) \) with

\[
\begin{align*}
\text{limited length:} & \\
||P|| := & \sum_{i=2}^{\ell} |u_i - u_{i-1}| \\
\leq & c \cdot |u - v|
\end{align*}
\]

- in a limited radius:

\[
\begin{align*}
\max_{i=1,\ldots,\ell} |u - u_i| & \leq c \cdot |u - v|
\end{align*}
\]

\[
\begin{align*}
\text{limited energy costs:} & \\
||P||^\delta := & \sum_{i=2}^{\ell} |u_i - u_{i-1}|^\delta \\
\leq & c \cdot |u - v|^\delta
\end{align*}
\]

(c,\( \delta \))-Power-Spanner Graph
Proximity Graphs

- **Relative Neighborhood Graph (RNG):**
  - There is an edge between $u$ and $v$ only if there is no vertex $w$ such that $d(u,w)$ and $d(v,w)$ are both less than $d(u,v)$
  - No spanners
  - Small degree

- **Gabriel Graph (GG):**
  - There is an edge between $u$ and $v$ if there is no vertex $w$ in the circle with diameter chord $(u,v)$
  - Perfect $(1,2)$-power spanners
  - No strong spanners
  - Possibly high degree
Useful Structures for Multi-hop Networks

- **Global structures:**
  - Minimum spanning trees & minimum broadcast trees

- **Local structures:**
  - Dominating sets: distributed algorithms and tradeoffs

- **Hierarchical structures:**
  - Sparse neighborhood covers
Applications of Spanning Trees

- Forms a backbone for routing
- Forms the basis for certain network partitioning techniques
- Subtrees of a spanning tree may be useful during the construction of local structures
- Provides a communication framework for global computation and broadcasts
Arbitrary Spanning Trees

➢ Trivial algorithm:
  – A designated node starts the “flooding” process
  – When a node receives a message, it forwards it to its neighbors the first time
  – Maintain sequence numbers to differentiate between different computations

➢ Properties
  – Nodes can operate asynchronously
  – Number of messages is $O(m)$; worst-case time,
  – for synchronous control, is $O(Diam(G))$
Minimum Spanning Trees

- **Centralized algorithms**
  - Prim’s algorithm [57]
    - Increase the connected component by adding the lightest adjacent edge not in the component
  - Kruskal’s algorithm [56]
    - Add small weight edges (leaving out edges within a connected component)

- **The basic algorithm** [Gallagher-Humblet-Spira 83]
  - $O(m + n \log n)$ messages and $O(n \log n)$ time

- **Improved time and/or message complexity** [Chin-Ting 85, Gafni 86, Awerbuch 87]

- **First sub-linear time algorithm** [Garay-Kutten-Peleg 93]:
  - Improved to $O(D + n^{0.61} \log^* n)$

- **Taxonomy and experimental analysis**
  - [Faloutsos-Molle 96] $O(D + \sqrt{n \log^* n})$
  - lower bound [Rabinovich-Peleg 00] $\Omega(D + \sqrt{n} / \log n)$
The Basic Algorithm

- Distributed implementation of Boruvka’s algorithm [Boruvka 26]
- Each node is initially a fragment
- Fragment $F_1$ repeatedly finds a min-weight edge leaving it and attempts to merge with the neighboring fragment, say
  - If fragment $F_2$ also chooses the same edge, then merge $F_2$
  - Otherwise, we have a sequence of fragments, which together form a fragment
Subtleties in the Basic Algorithm

- All nodes operate asynchronously.
- When two fragments are merged, we should “relabel” the smaller fragment.
- Maintain a level for each fragment and ensure that fragment with smaller level is relabeled:
  - When fragments of same level merge, level increases; otherwise, level equals larger of the two levels.
- Inefficiency: A large fragment of small level may merge with many small fragments of larger levels.
Asymptotic Improvements to the Basic Algorithm

- The fragment level is set to log of the fragment size [Chin-Ting 85, Gafni 85]
  - Reduces running time to $O(n \log^* n)$
- Improved by ensuring that computation in level fragment is blocked for time $O(2^\ell)$
  - Reduces running time to $O(n)$
A Sublinear Time Distributed Algorithm

- All previous algorithms perform computation over fragments of MST, which may have diameter $\Omega(n)$
- Two phase approach [GKP 93, KP 98]
  - Controlled execution of the basic algorithm, stopping when fragment diameter reaches a certain size
  - Execute an edge elimination process that requires processing at the central node of a BFS tree
- Running time is $O(\text{Diam}(G) + \sqrt{n} \log^* n)$
- Requires a fair amount of synchronization
Minimum Energy Broadcast Routing

- Given a set of nodes in the plane, need to broadcast from a source to other nodes
- In a single step, a node may broadcast within a range by appropriately adjusting transmit power
- Energy consumed by a broadcast over range $r$ is proportional to
- Problem: Compute the sequence of broadcast steps that consume minimum total energy $r^\alpha$
  - Optimum structure is a directed tree rooted at the source
Energy-Efficient Broadcast Trees

- NP-hard for general graphs, complexity for the plane still open
- Greedy heuristics proposed [Wieselthier et al 00]
  - MST: Minimum spanning tree with edge weights equal to energy required to transmit over the edge
  - SPT: Shortest path tree with same weights
  - BIP: Bounded Incremental Power: Add next node into broadcast tree, that requires minimum extra power
- MST and BIP have constant-factor approximation ratios
- SPT (Shortest-Path Tree) has ratio $\Omega(n)$ [Wan et al 01]
  - If weights are square of Euclidean distances, then MST for any point set in unit disk is at most 12
Thank you

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