Deterministic Leader Election in Anonymous Radio Networks

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Seminar Algorithms for Computer Networks
WS 2020
Based on Paper by A. Miller, A. Pelc, R. N. Yadav
Problem: Leader Election

Model:
- undirected graph

Goal:
- Exactly one node decides leader

Complexity:
- Time
- Message
Model Extension

Synchronous rounds

\[ \sigma = \text{largest\_wake-up\_tag} - \text{smallest\_wake-up\_tag} \]

Augmente Graph $\rightarrow$ Graph $G$ including the wake-up tags
Distributed Radio Interaction Protocol (DRIP)

Transmit message M

Stay silent & receive nothing

Stay silent & receive message M

Stay silent & receive noise

Next action for v: based on history[v]

Leader Election Algorithm $\leftrightarrow$ DRIP D and decision function $f$

patient DRIP: no messages send in global rounds 0 ... $\sigma$
Classifier Algorithm

- Input: configuration $G_{aug}$
- Goal: is leader election problem feasible for given configuration

- Centralized algorithm
- Phase-based protocol

1. Initialize
2. Execute 1 phase
   1. Compute label for each node
3. Refine the equivalence classes
4. Check for termination
Classifier (Init)

Phase 0

<table>
<thead>
<tr>
<th>Number of equivalence Classes</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>$= 5 - 0 = 5$</td>
</tr>
<tr>
<td>Classlabel[1]</td>
<td>null</td>
</tr>
</tbody>
</table>
Before Phase j:

<table>
<thead>
<tr>
<th>Number of equivalence Classes</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>5</td>
</tr>
<tr>
<td>Tuple computation</td>
<td>(class_id, local_round_v)</td>
</tr>
</tbody>
</table>

Computed tuples:
- $(1, \emptyset)$
- $(2, \emptyset)$
- $(1, \emptyset)$
- $(1, 2)$
- $(1, 2)$

Node Label:
- $(1, 2, 1)(1, 5, \ast)(2, 7, 1)$
**Classifier (Phase j)**

**Before Phase j:**

<table>
<thead>
<tr>
<th>Number of equivalence Classes</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma )</td>
<td>5</td>
</tr>
<tr>
<td>Tuple computation</td>
<td>(class_id, local_round_v)</td>
</tr>
<tr>
<td>Computed tuples:</td>
<td></td>
</tr>
<tr>
<td>( (1, 3) )</td>
<td></td>
</tr>
<tr>
<td>( (2, 8) )</td>
<td></td>
</tr>
<tr>
<td>( (2, 8) )</td>
<td></td>
</tr>
<tr>
<td>Node Label:</td>
<td></td>
</tr>
<tr>
<td>( (1, 3, 1)(2, 8, *) )</td>
<td></td>
</tr>
</tbody>
</table>

**After Phase j:**

<table>
<thead>
<tr>
<th>Number of equivalence Classes</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Termination after this round</td>
<td></td>
</tr>
</tbody>
</table>
 Classifier Algorithm

- Input: configuration $G_{\text{aug}}$
- Goal: is leader election problem feasible for given configuration

- Centralized algorithm
- Phase-based protocol
- Terminates when
  - At least one class only contains exactly one node
  - No changes in two consecutive rounds
- Time Complexity:
  - Time Complexity of a Phase: $O(n^2 \Delta)$
  - Number of Phases: at most $n/2$ iterations
  - $\rightarrow$ overall $O(n^3 \Delta)$ time, worst case $O(n^4)$

- can be used to solve Leader Election Problem
canonical DRIP

- Create lists of tuples according to execution of Classifier
  - $L_j[k] = \text{(class id k previous phase, label current phase)}$
  - give to nodes all lists $L_j$

- Decision function $f$
  - Defined according to Classifier

- Node $v$ executes:
  1. Check equivalence class using $v_{\text{LBL}}$ and $L_j$ of current round $j$
  2. Wait for Transmission Block of $v_{\text{CLASS}}$
  3. Transmit in local round $\sigma + 1$ to all neighbors
  4. Compute new label $v_{\text{LBL}}$
canonical DRIP (Time Complexity)

✧ Phase j:
  ◦ #Transmission Blocks equal to the number of equivalence classes
  ◦ Each transmission block has $2\sigma + 1$ rounds
  ◦ And additional $\sigma$ rounds, where each node is quiet

✧ Same number of phases as Classifier

✧ Time complexity: $O(n^2 \sigma)$
Conclusion & Outlook

- Centralized decision algorithm in $O(n^2\Delta)$
- Distributed dedicated leader election algorithm in $O(n^2\sigma)$
  - Only for feasible configurations previously shown by Classifier

- Negative results:
  - Lower bound: $\Omega(n)$ even if $\sigma = 1$
  - Lower bound: $\Omega(\sigma)$ even bounded $n$

- Proved:
  - Nonexistence of distributed algorithm deciding whether a configuration is feasible
  - Nonexistence of universal distributed leader election working for all feasible configurations
  - feasibility of leader election in anonymous radio networks is solved

- Open Problems:
  - Can complexity of Classifier be improved?
  - What is optimal complexity for centralized algorithm?
  - Exists a $O(n + \sigma)$ dedicated leader election algorithm for all feasible configurations?