ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG

Algorithm Theory

2 Divide and Conquer: Line Intersection

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Algorithms Theory

Chapter 2

Divide and Conquer: Line Intersection

The Principle of Divide and Conquer

- **Remember Quicksort?**
- **Formulation and analysis of the principle**
- **Geometric Divide-and-Conquer**
  - Closest-Pair
  - Line segment intersection
  - Voronoi diagram
- **Fast Fourier Transformation**

Divide and Conquer Paradigm

Divide-and-conquer method for solving a problem of size n

1. Divide:

n > c: Divide the problem into k sub-problems of sizes $n_1,\ldots,n_k$ ($k \geq 2$)

n ≤ c: Use direct solution

2. Conquer:

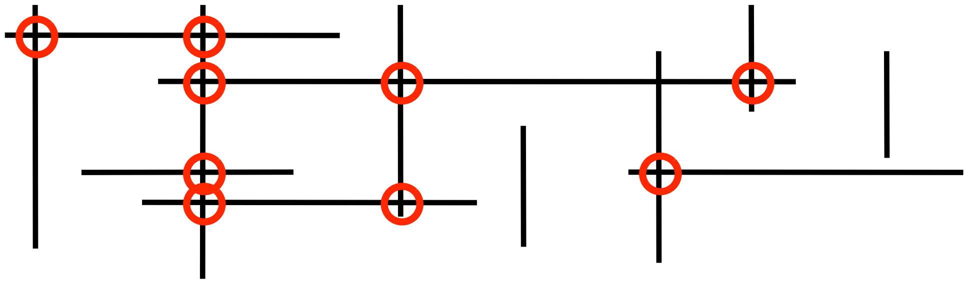
Recursively solve the k sub-problems (using Divide and Conquer)

3. Merge:

Combine the k partial solutions to get the overall solution

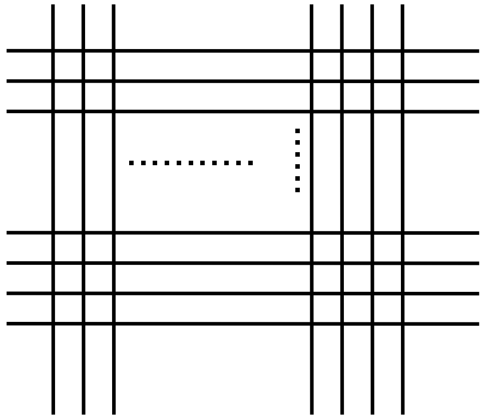
Line Segment Intersection

Find all pairs of intersection line segments



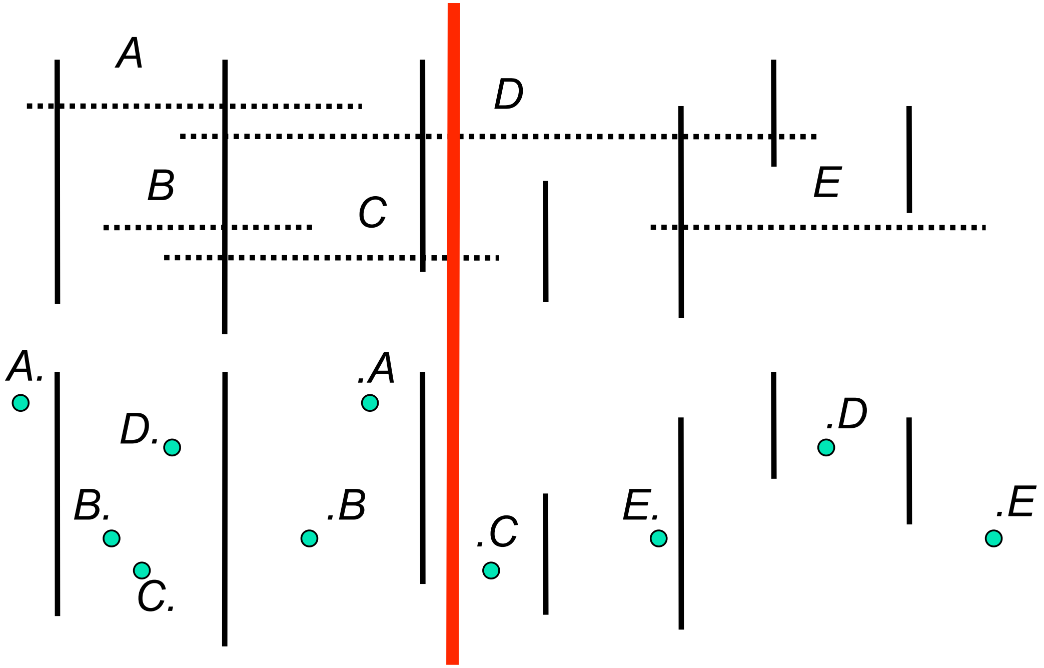
Naive approach:
$O(n^2)$

Output sensitive:
$O(n \log n + \#\text{intersections})$



Line Segment Intersection

Find all pairs of intersection line segments



We represent horizontal line segments by their endpoints. Then we introduce a vertical partitioning of all objects.

ReportCuts

Input: Set S of vertical line segments and endpoints of horizontal line segments

Output: All intersections of vertical line segments with horizontal line segments for which at least one endpoint is in S.

1. Divide

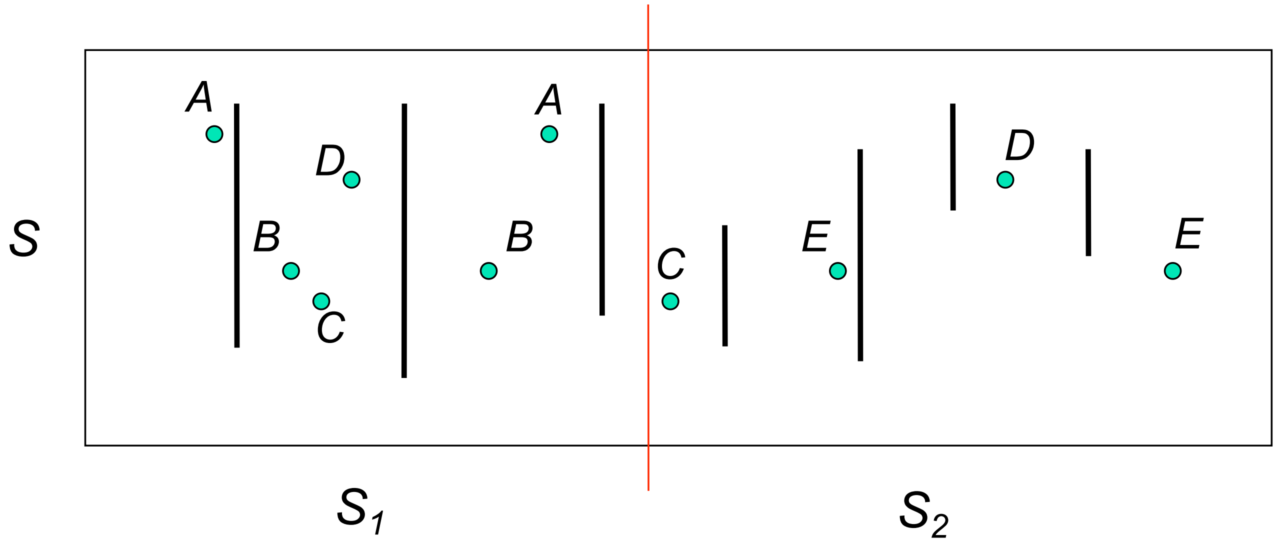
if $|S| > 1$

then use a vertical line L to divide S into equally sized sets S_1 (left of L) and S_2 (right of L)

else S contains no intersections

ReportCuts

1. Divide-Step



2. Conquer

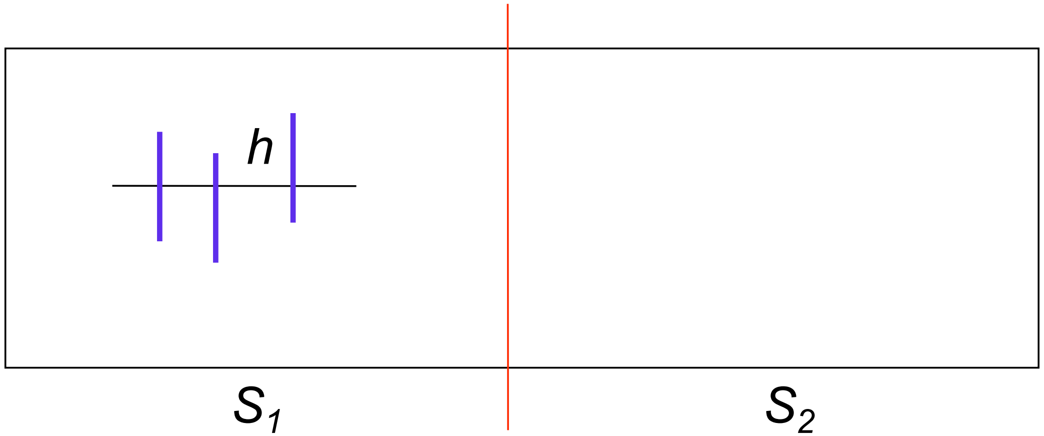
ReportCuts(S_1); ReportCuts(S_2)

ReportCuts

3. Merge: ???

Possible intersection of a horizontal line segment *h* in S_1

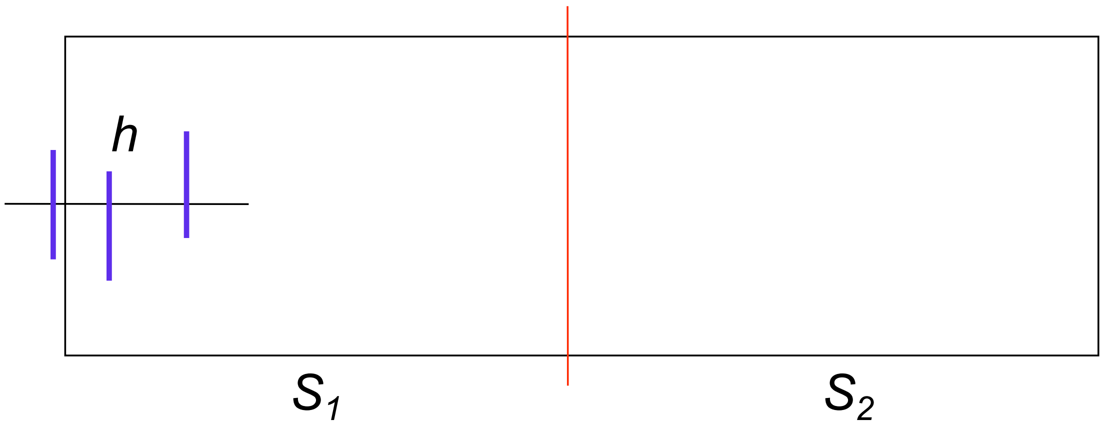
Case 1: both endpoints in S_1



ReportCuts

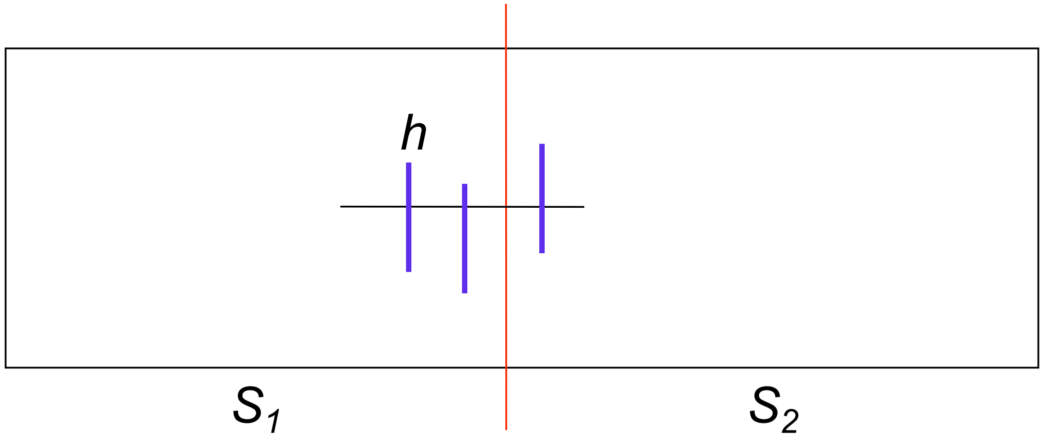
Case 2: only one endpoint of h in S_1

2 a) right endpoint in S_1

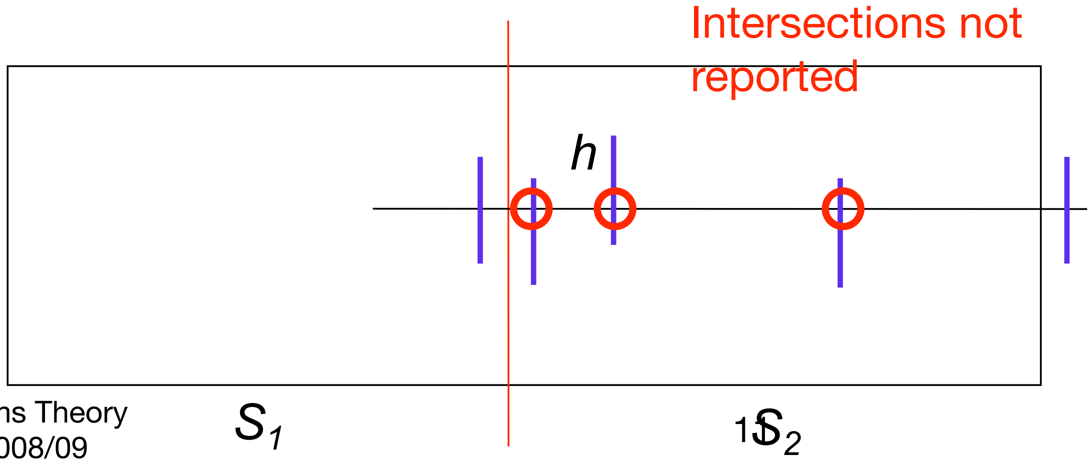


ReportCuts

2 b) left endpoint of h in S_1



right endpoint in S_2



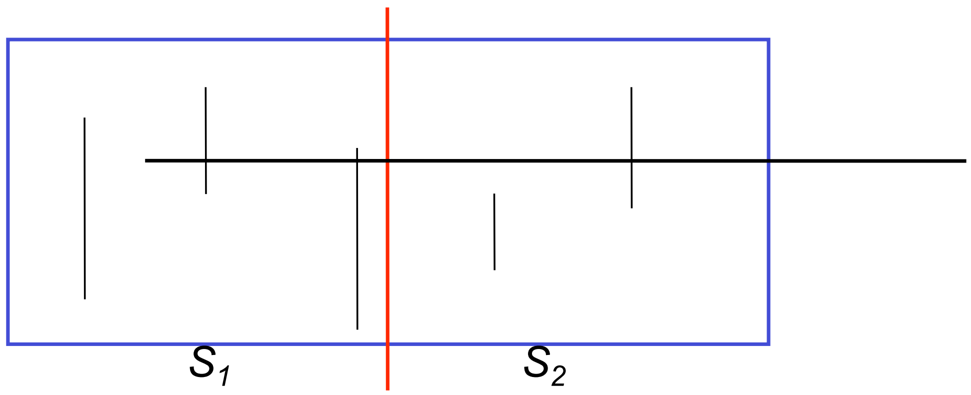
right endpoint not in S_2

Procedure: ReportCuts(S)

3. Merge:

Return the intersection of vertical line segments in S_2 with horizontal line segments in S_1, for which the left endpoint is in S_1 and the right endpoint is neither in S_1 nor S_2

Proceed analogously for S_1



Implementation

Set S

L(S): y-coordinates of all left endpoints in *S* for which the corresponding right endpoints is not in *S*

R(S): y-coordinates of all right endpoints in *S* for which the corresponding left end point is not in *S*

V(S): y-intervals of all vertical line-segments in *S*

Base cases

S **contains only one elements** s

Case 1: $s = (x,y)$ is a left endpoint

$$L(S) = \{y\} \qquad R(S) = \emptyset \qquad V(S) = \emptyset$$

Case 2: $s = (x,y)$ is a right endpoint

$$L(S) = \emptyset \qquad R(S) = \{y\} \qquad V(S) = \emptyset$$

Case 3: $s = (x,y_1,y_2)$ is a vertical line-segment

$$L(S) = \emptyset \qquad R(S) = \emptyset \qquad V(S) = \{[y_1,y_2]\}$$

Merge-Step

Assume that $L(S_i), R(S_i), V(S_i)$ **are known** $i=1,2$

$S = S_1 \cup S_2$

$L(S) = (L(S_1) \setminus R(S_2)) \cup L(S_2)$

$R(S) = (R(S_2) \setminus L(S_1)) \cup R(S_1)$

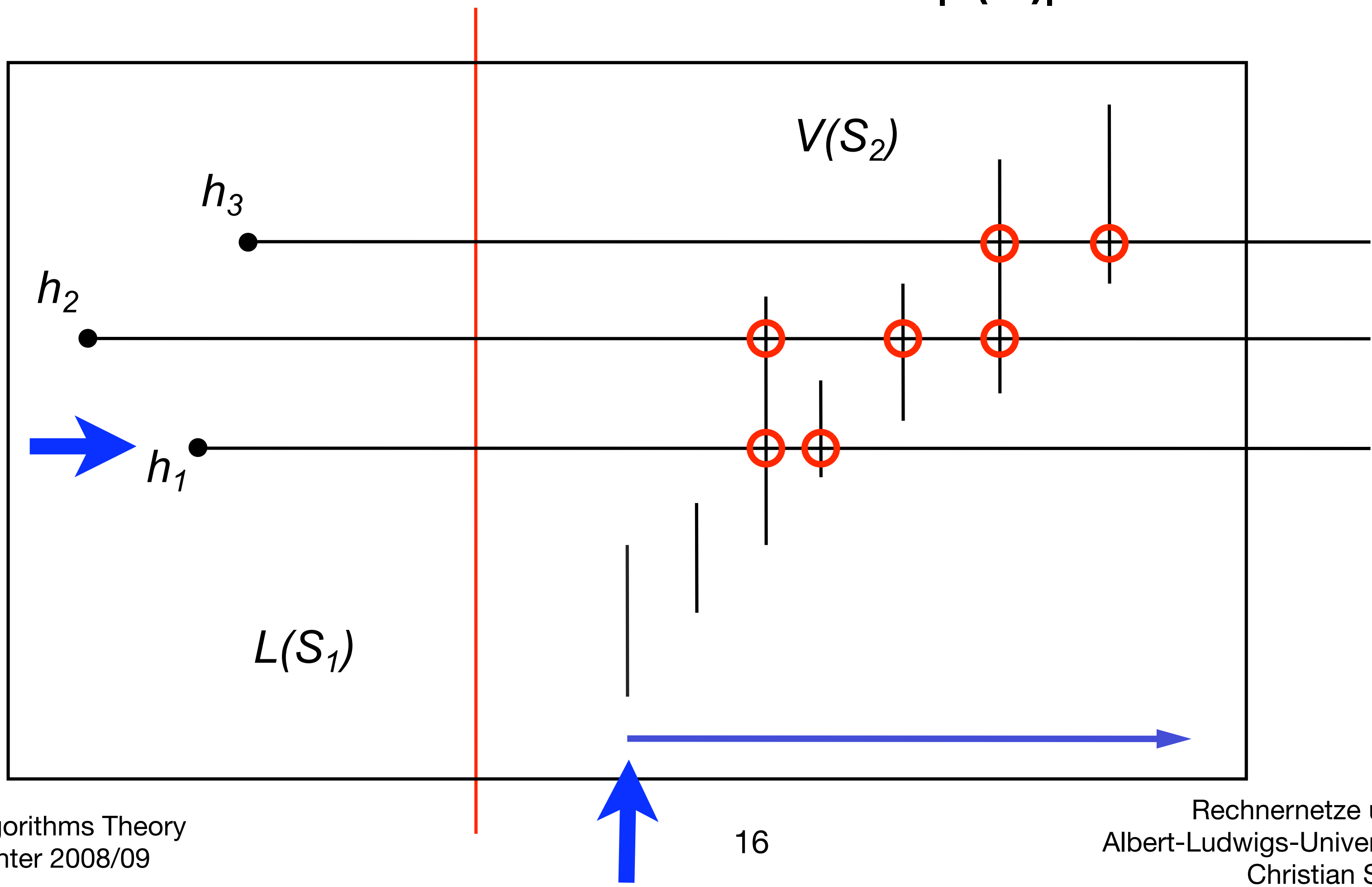
$V(S) = V(S_1) \cup V(S_2)$

L,R: ordered by increasing y-coordinates
linked lists

V: ordered by increasing lower endpoints
linked lists

Output of the intersections

Runtime: |V(S₂)| + #intersections



Running Time

Initally, the input (vertical line segments, left/right eindpoints of horizontal line segments) has to be sorted and stored in an array.

Divide-and-Conquer:

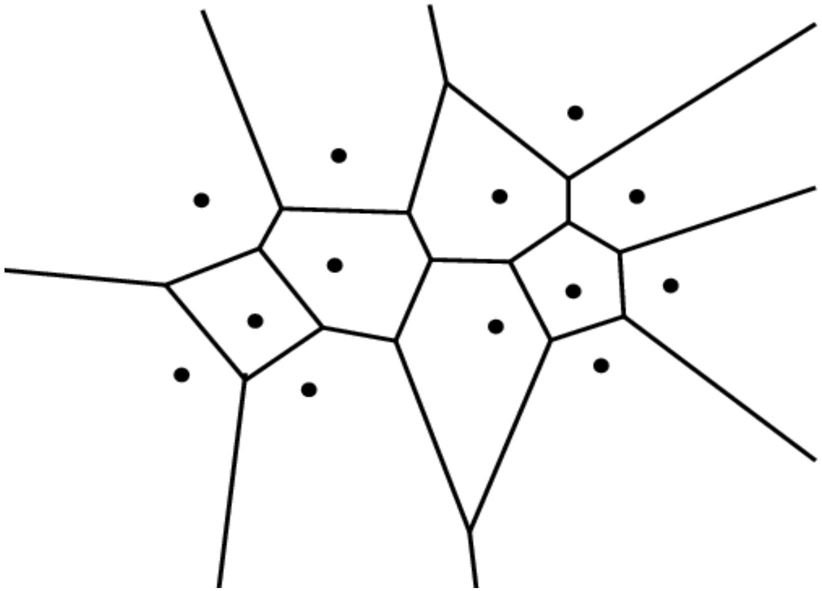
$T(n) = 2T(n/2) + an$ + size of output
$T(1) = O(1)$

$O(n \log n + k)$ k = #intersections

Computation of a Voronoi Diagram

Input: A set of sets

Output: Partition of the plane into regions, each consisting of the points closer to a site than any other site



Definition of a Voronoi Diagram

P **: Set of sites**

$H(p \mid p') = \{x \mid x$ **is closer to** p **than to** $p'\}$

Voronoi region of p

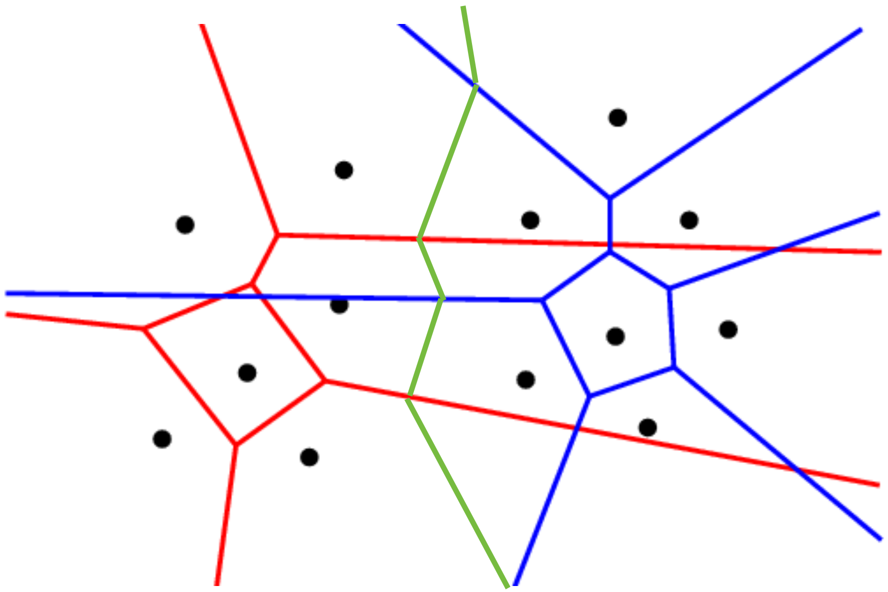
$$VR(p) = \bigcap_{p' \in P \setminus \{p\}} H(p \mid p')$$

Computation of a Voronoi Diagram

Divide : Partition the set of sites into equally sized sets

Conquer: Recursive computation of the two smaller Voronoi diagrams

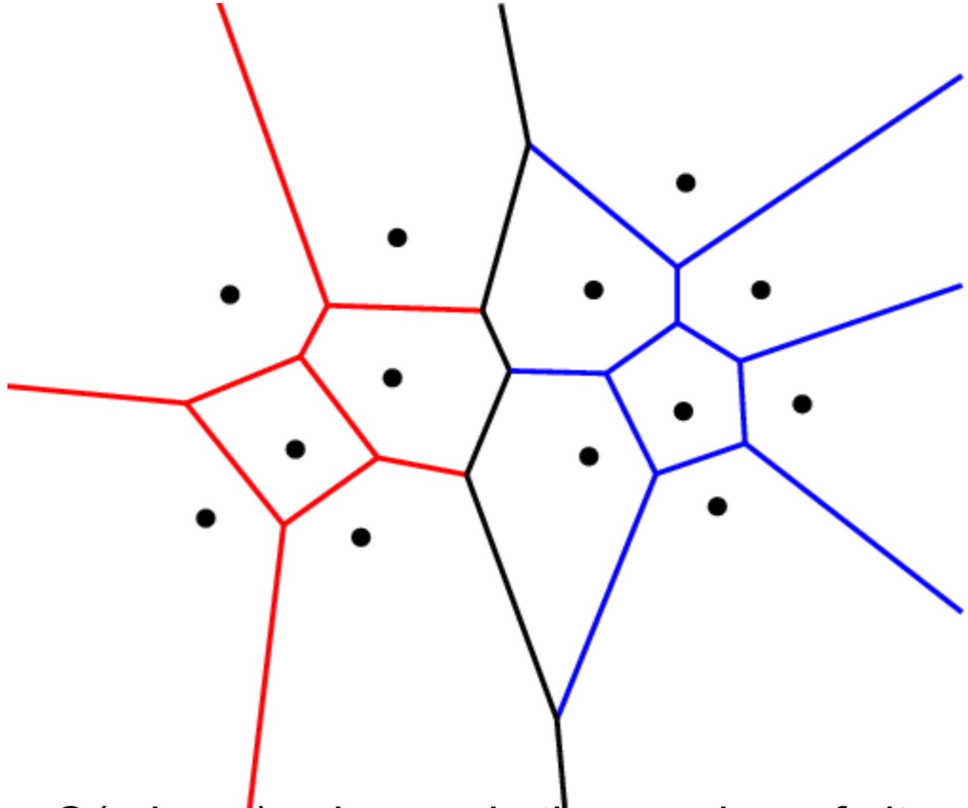
Stopping conditions: The Voronoi diagram of a single is the whole plane



Merge: Connect the dagrams by adding new edges

Computation of the Voronoi Diagram

Output: **The complete Voronoi diagram**



Running time: *O(n log n)*, where n is the number of sites

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