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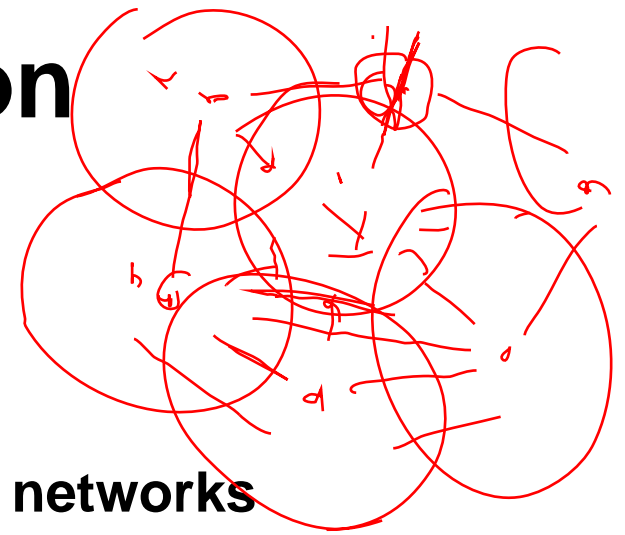
# Algorithms for Radio Networks

## Localization

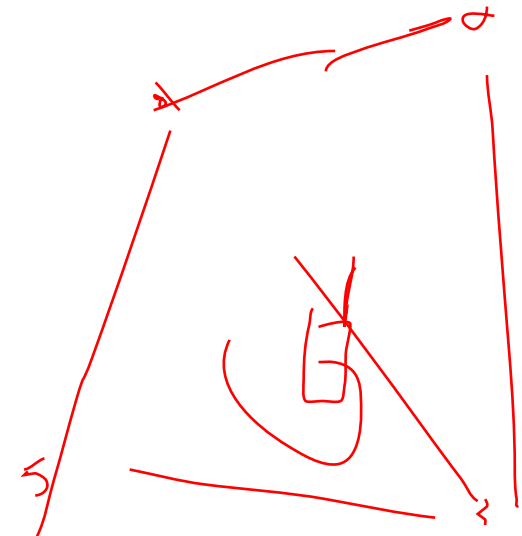
University of Freiburg Technical Faculty  
Computer Networks and Telematics  
Prof. Christian Schindelhauer



# Coarse Localization Techniques



- Hop-distance
  - in dense ad hoc networks or wireless sensor networks
  - approximate position by the number of hops to anchor points
- Overlapping connections
  - position at the intersection of the received transmission circuits
- Localization point in the triangle
  - determination of triangles of anchor points
    - in which the node lies
  - overlap provides approximate position
- “Fingerprinting” of signal strength measures



# Localization methods

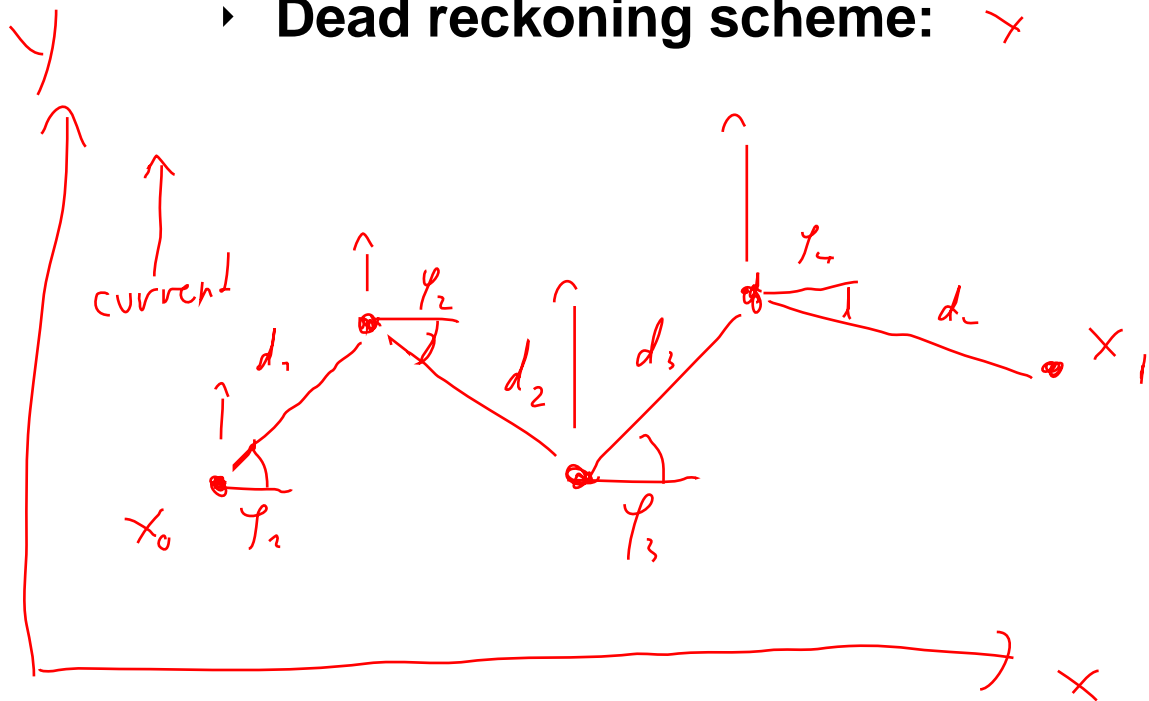
- Dead Reckoning: Relative localization depending on course and traveled distance
- Triangulation: Calculate the intersection of angular bearings
- Trilateration: Calculate the intersection of three range measurements (circles)
- Multilateration with *absolute* ranges: Calculate the intersection of *at least four* range measurements
  - In the plane: circles, in space: spheres
  - May be over-determined equation system
- Multilateration with *relative* ranges: Hyperbolic multilateration
  - Multilateration with unknown send time
  - Calculate intersection of hyperbolas / hyperboloids

# Dead Reckoning

- Relative vector navigation, vectors of orientation  $\phi_i$  and distance  $d_i$
- Animals: “path integration” by special regions in hippocampus of desert ants (Wehner, 2003)
- Dead reckoning scheme: ✕

Relative angle

$$\phi_i = \phi_{i-1} + \alpha_i$$



Recursive

$$x_i = x_{i-1} + d_i \cos \phi_i$$

$$y_i = y_{i-1} + d_i \sin \phi_i$$

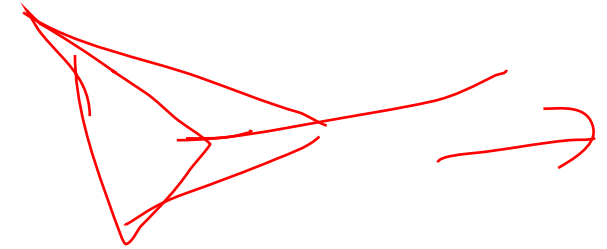
Direct

$$x_i = x_0 + \sum_{j=1}^n d_j \cos \phi_j$$

$$y_i = y_0 + \sum_{j=1}^n d_j \sin \phi_j$$

# Dead Reckoning

- **Example: Navigation of ships / airplanes**
  - if course is known (compass)
  - if traveled distance is known (ship log, pitot tube)
- **Prone to drift (water current, wind, wheel slip)**
- **Errors add up over time**

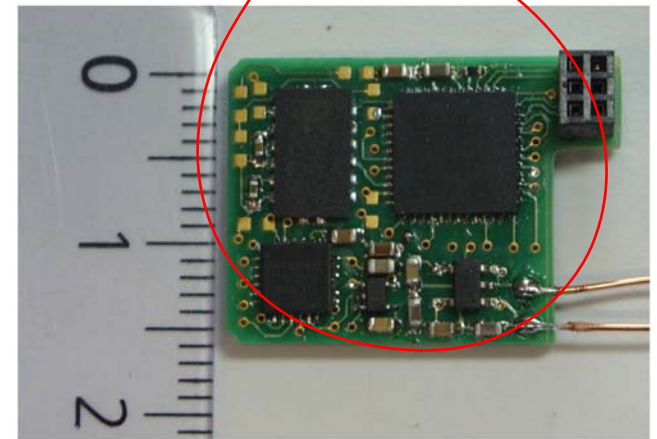


# Inertial Navigation

- Consider orientation and traveled distance as direction vector  $s_t$  at time  $t$ .
- What if only acceleration  $a_t$  is measured?
  - *Inertial navigation*, double integration

$$\underline{\vec{s}(t)} = \int \int \underline{\vec{a}(t)} dt^2 + \underline{\vec{s}_0} + \underline{\vec{v}_0} \cdot t$$

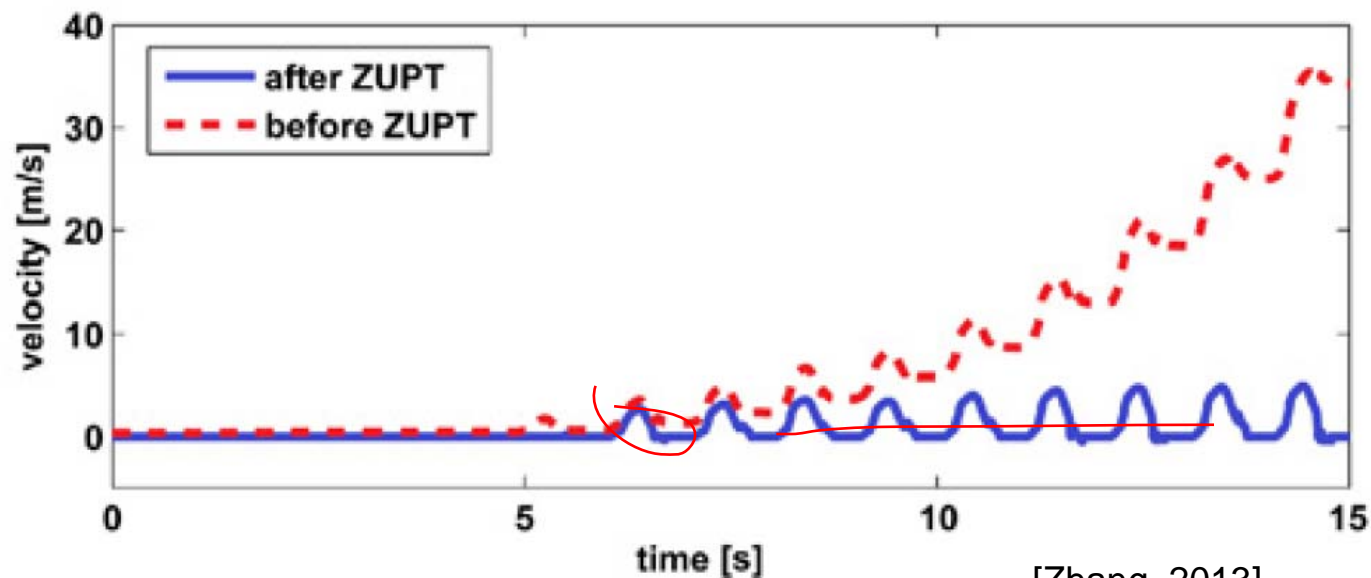
- Often also rotation is measured (angular velocity)
- Combine accelerometer, gyroscope, and compass:
  - Inertial Measurement Unit (IMU)



[F. Höflinger, 2013]

# Inertial Navigation

- **Foot-mounted MEMS-IMU**
  - Errors add up over time
- **Compensation: Zero velocity update**
  - Detect footstep
  - Translation velocity is zero at this moment!

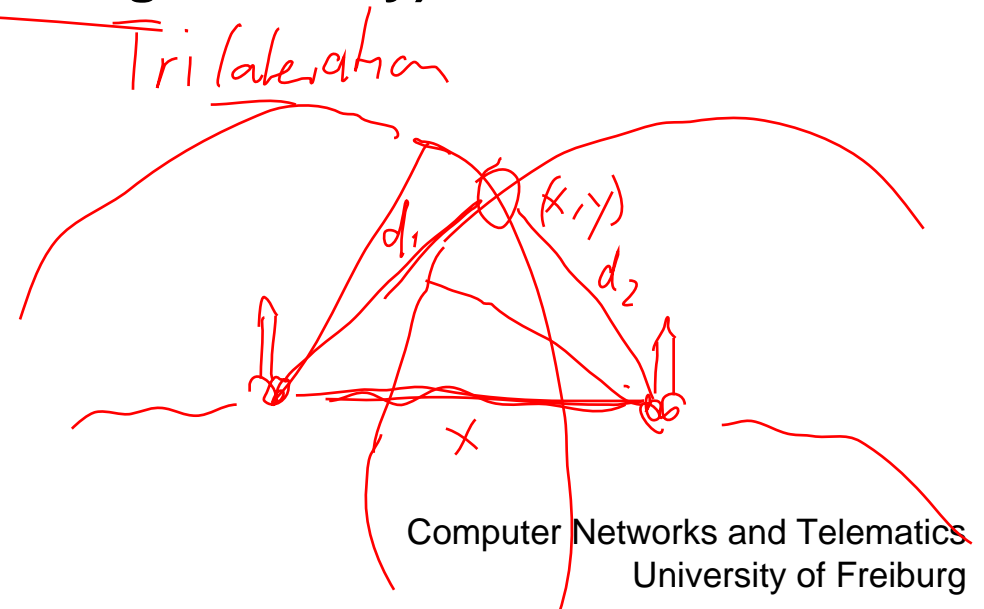
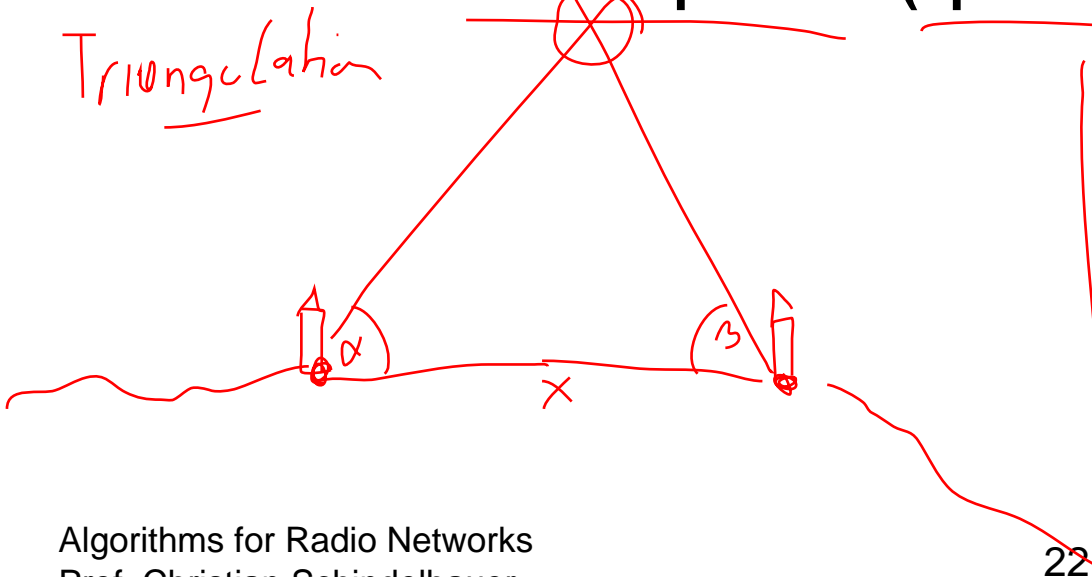
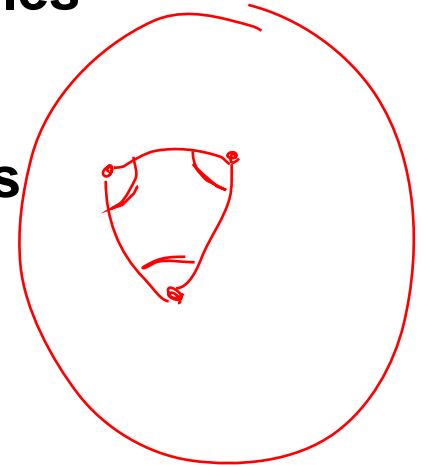


[Zhang, 2013]

# Triangulation

- Given a side of known length and two adjacent angles
- In the plane:
  - Calculate the intersection point of the other sides
  - Duality with trilateration: Triangle congruency (angle-side-angle)  $\leftrightarrow$  (side-side-side)
- On earth surface:
  - More complicated (spherical trigonometry)

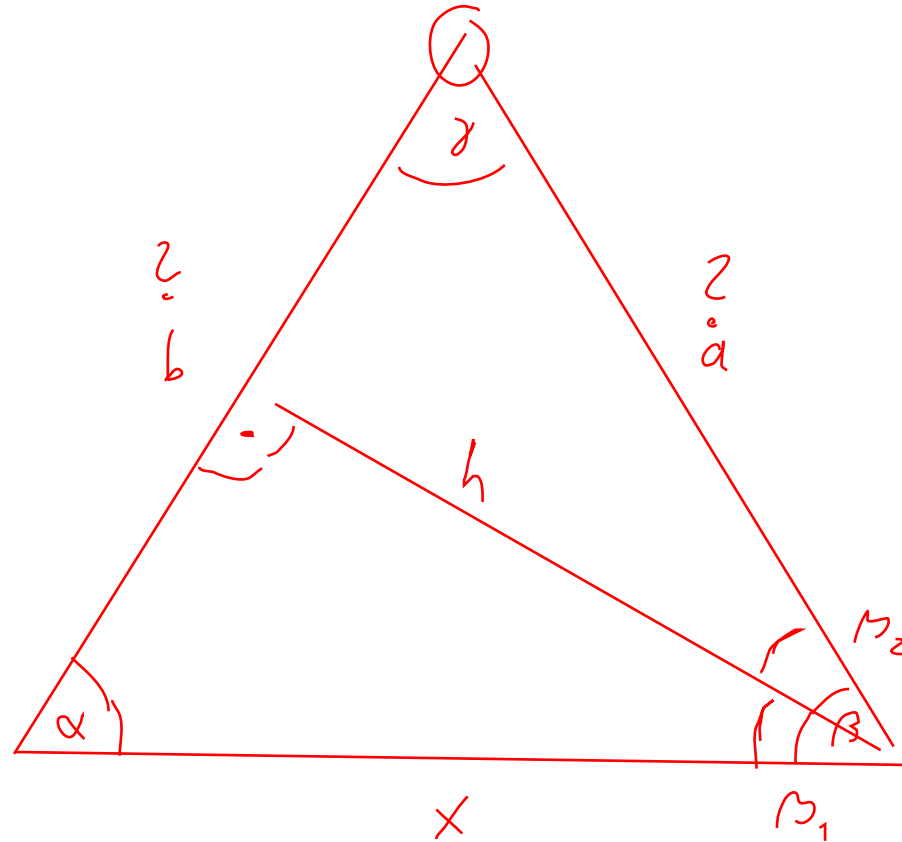
Earth.



Triangulation

$\Leftrightarrow$

Trilateration

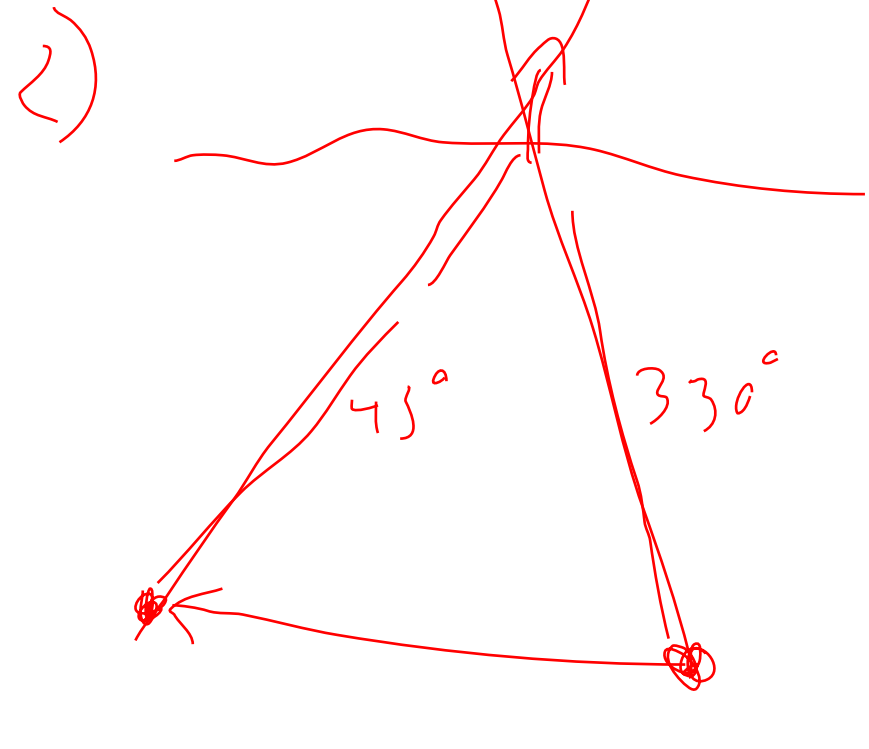
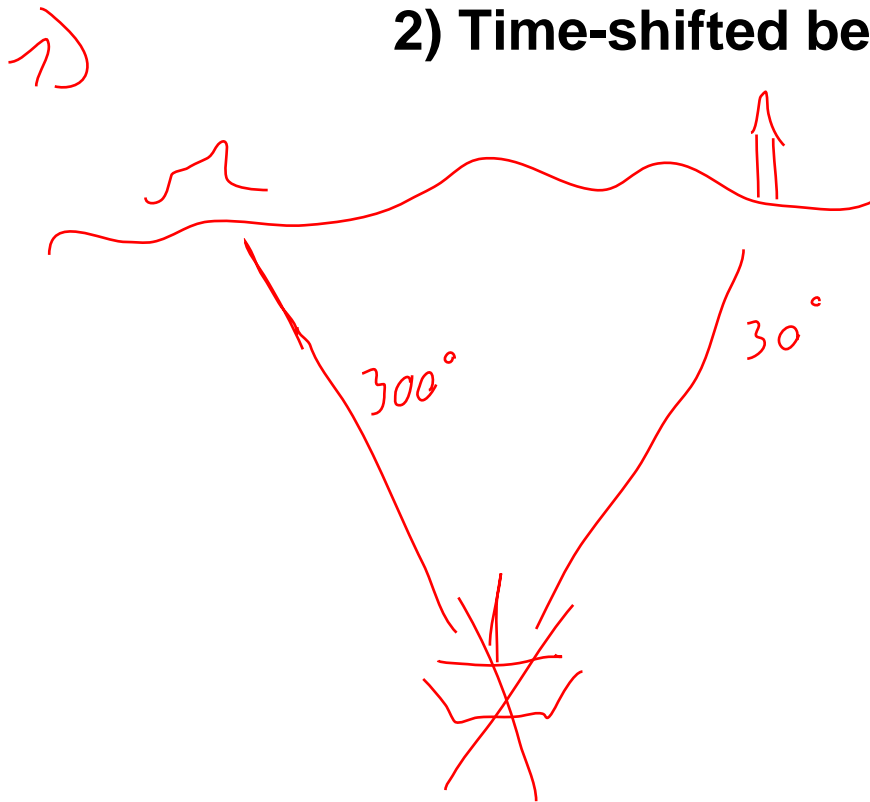


$$\frac{h}{a} = \cos \beta_2$$

$$\frac{h}{x} = \cos \beta_1$$

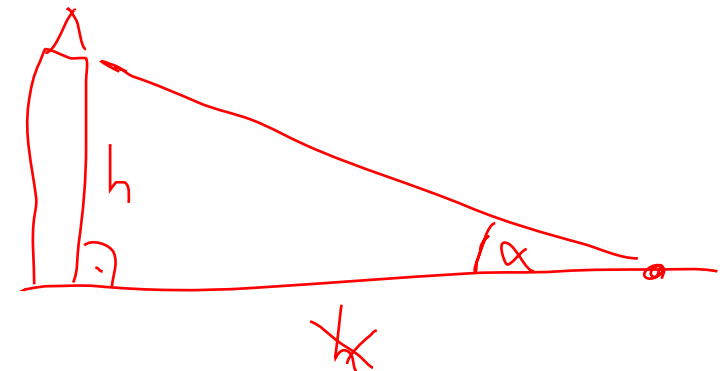
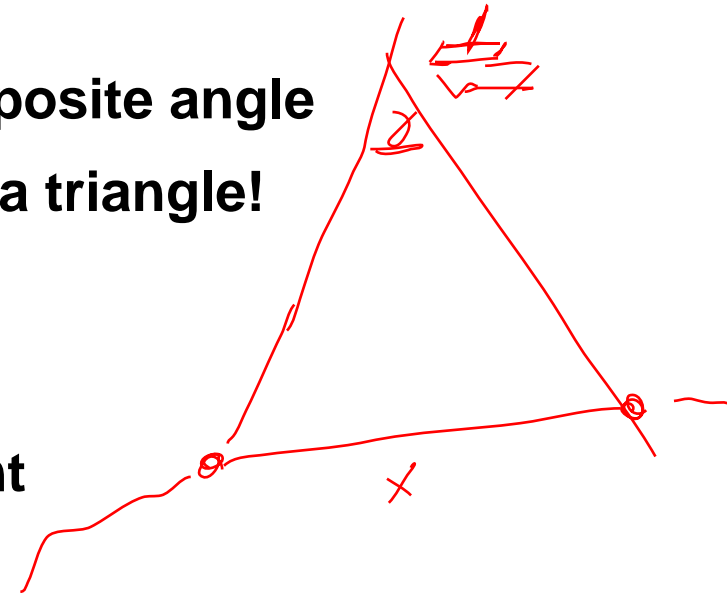
# Triangulation

- **Example: Navigation of ships / airplanes (cross bearing triangulation)**
  - **1) Bearings of two objects on a map**
  - 2) Time-shifted bearings of the same object**



# Triangulation

- Given a side of known length and the opposite angle
  - Triangle congruency: Does not define a triangle!
  - What else is possible?
- Given a lighthouse of known height  $h$ 
  - Measurement of angle  $\phi$ , use a sextant
  - Calculation of distance  $d = h / \tan(\phi)$
  - Measurement of lighthouse bearing  
     → position in polar coordinates
- Height of lighthouse not known
  - Sail towards lighthouse



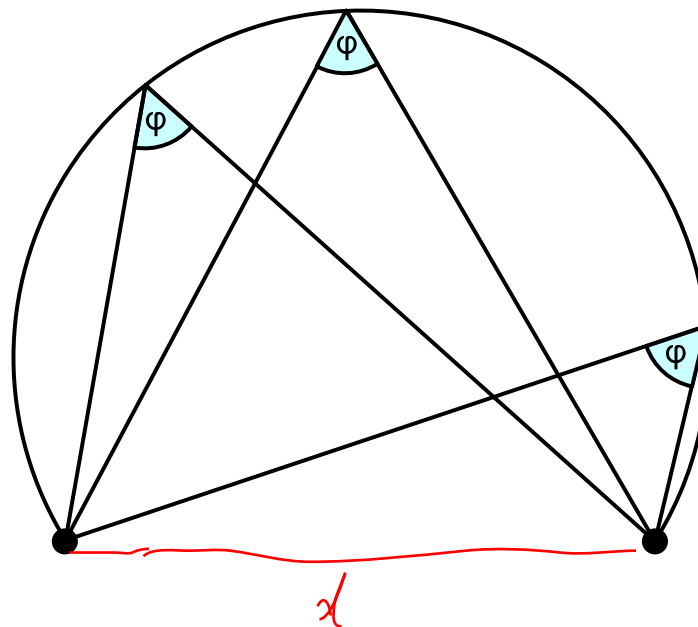
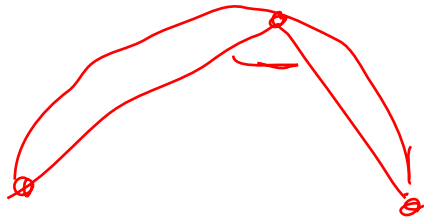
$$d = \frac{h}{\tan \alpha_1}$$

$$d = \frac{h}{\tan \alpha_2}$$



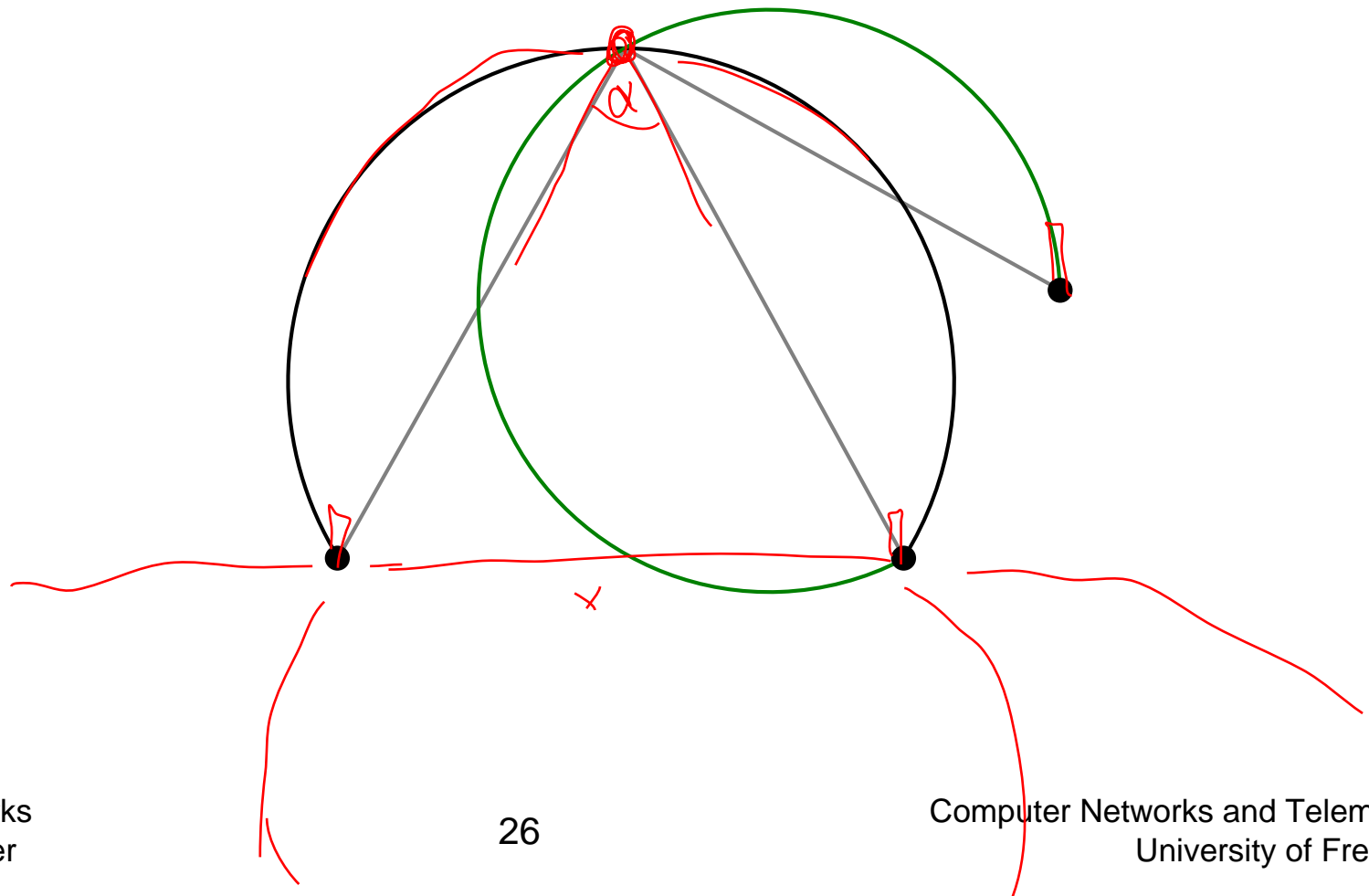
# Triangulation

- Given a side of known length and the opposite angle
  - Measure angle  $\phi$  of two landmarks (by theodolite or by laser scanner)
  - If  $\phi = 90^\circ$  : Ship's position resides on Thales' circle
  - Other angles: generalization of Thales' circle
  - Circle of equal angles  
("Fasskreisbogen")



# Triangulation

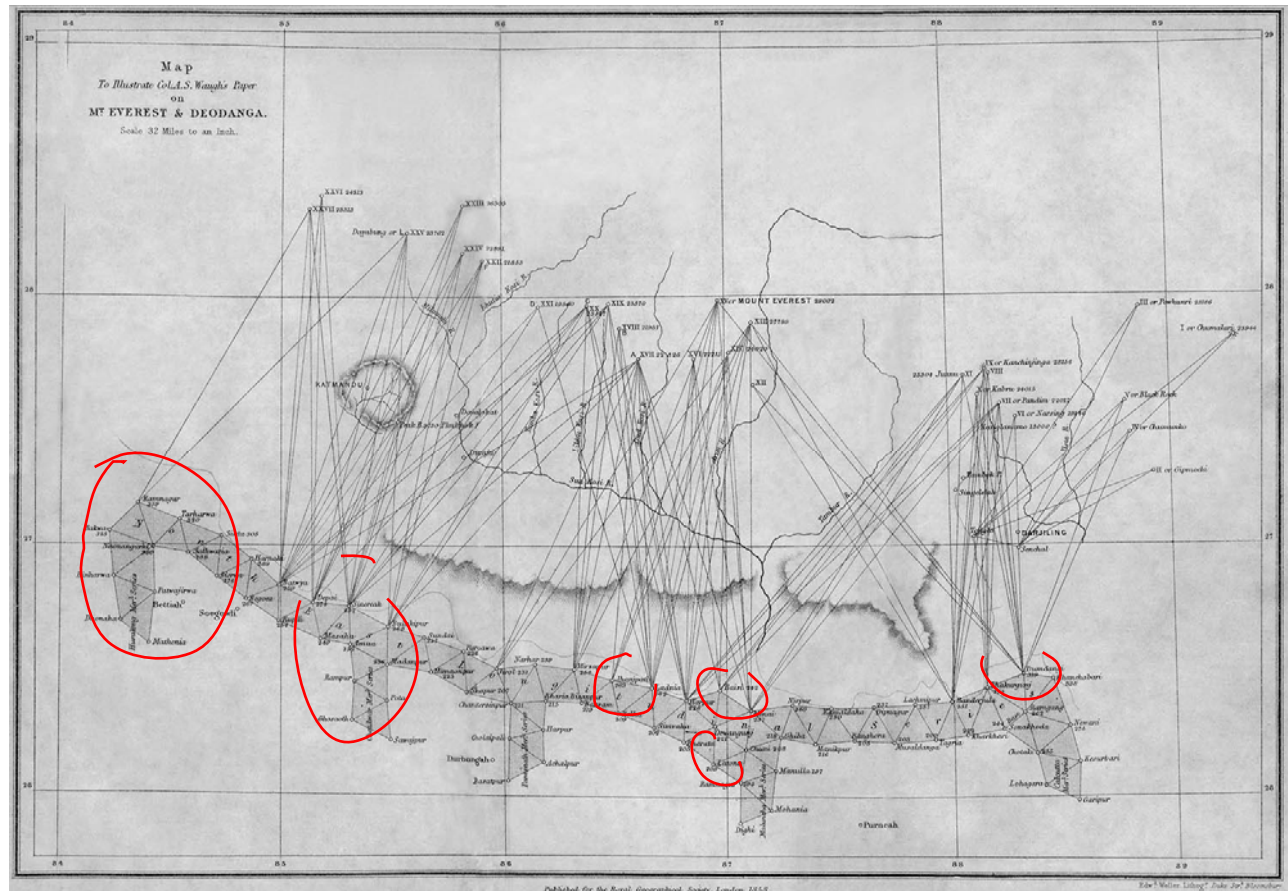
- **Given a side of known length and the opposite angle**
  - **Calculate position by a third landmark**



# Triangulation

## ▸ Height of Mt. Everest

- 8,840 m above NN (Sickdhar, 1856)
- 8,848 m (Survey of India, 1955)
- 8,850 m (GPS, 1999)
- 8,849 m (Radar reflectors, 2004)
- ...



[A. Waugh, Mt. Everest & Deodanga, 1862.]



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## Localization

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