ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG

Algorithms for Radio Networks

Orthogonal Frequency Division Multiplexing

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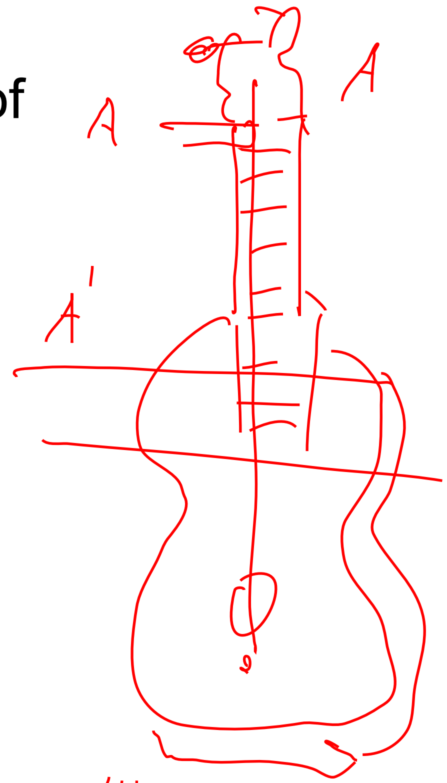
Repetition

- **Multiplexed**
  - Spatial Multiplexing
  - Frequency division multiplexing
  - Time division multiplexing
  - Code division multiplexing
  - Multiple-input multiple-output (next ~~lecture~~ Year)
- **Modulation**
  - Amplitude modulation
  - Phase modulation
  - Frequency modulation

Principle of OFDM

- **OFDM (Orthogonal Frequency Division Multiplex)**
  - Signals are divided into parallel signal streams
  - Parallel signals are modulated on carrier waves of different frequencies, phase / amplitude
  - e.g. 16-QAM
  - The carrier signals are combined and transmitted simultaneously
- **Special form of frequency-division multiplexing**
- **The carrier waves using orthogonal frequency:**
  - frequencies f, 2f, 3f, 4f, 5f, ...



A A' E'' A'' #C E'''

440Hz 880Hz 3.440Hz

2f 3f 4f 5f 6f 7f

Repitition: Complex Numbers

$z \cdot z^* = |z|$

- **i: imaginary number with**
  - $i^2 = -1$
- **A complex number is a linear combination of a real part a and imaginary b**
  - $z = a + bi$
- **Calculation rules:**
  - $(a+bi)+(c+di) = (a+c) + (b+d)\, i$
  - $(a+bi)\,(c+di) = (ac - bd) + (ad + bc)\, i$
  - $1/\,(a+b\,i) = (a-bi)/(a^2+b^2)$
- **Complex conjugate**
  - $(a+bi)^* = (a - bi)$

$bd\,i^2 = -bd$

$$\frac{1}{a+bi} = \frac{a-bi}{\underbrace{(a+bi)(a-bi)}_{\underbrace{a^2-b^2i^2}_{a^2+b^2}}}$$

Exponentiation of Complex Numbers

- **Important equation**
  - $e^{i\pi} = -1$
  - $e^{i\varphi} = \cos \varphi + i \sin \varphi$
- **Exponentiation of a complex number**
  - $e^{a+bi} = e^a e^{bi} = e^a (\cos b + i \sin b)$
- **Therfore**
  - real part $e^{i\varphi}$: $\mathrm{Re}(e^{i\varphi}) = \cos \varphi$
  - imaginary of $e^{i\varphi}$: $\mathrm{Im}(e^{i\varphi}) = \sin \varphi$

a^b

$a + bi$

dist. → $r \cdot e^{i\varphi}$ ← angle

← polar coordinates

$$(r \cdot e^{ia})(r \cdot e^{ia})^* = r \cdot e^{ia} \cdot r \cdot e^{-ia}$$

$$= r^2 e^{\underbrace{ia + (-ia)}_{0}} = r^2$$

Equivalent Representations of the FFT

$z_k = a_k - b_k i$

$Re\left[e^{i2\pi kt/T}\right] = \cos i2\pi kt/T$

- **Real number representation**
  - Sine and cosine functions of different frequencies

$$g(x) = \sum_{k=0}^{N-1} a_k cos \frac{2\pi kt}{T} + b_k sin \frac{2\pi kt}{T} \iff$$

- **Computation of the inverse by cosine/sine integral product**

$$\boxed{\begin{aligned} a_k &= \frac{2}{T} \int_0^T g(t) \cos(2\pi n f t) dt \\ b_k &= \frac{2}{T} \int_0^T g(t) \sin(2\pi n f t) dt \end{aligned}}$$

- **Complex representation**
  - real part of the exponential function of differnent frequencies

$$Re\left[f(x)\right] = \boxed{\sum_{k=0}^{N-1} z_k \, e^{i2\pi kt/T}}$$

- **Computation of the inverse by the integral over the product with the complex conjugated carrier wave**

$$z_k = \frac{1}{T} \int_0^T \left(e^{i2\pi kt/T}\right)^* f(x) dt$$

$(e^{ia})^* = (\cos a + i \sin a)^*$

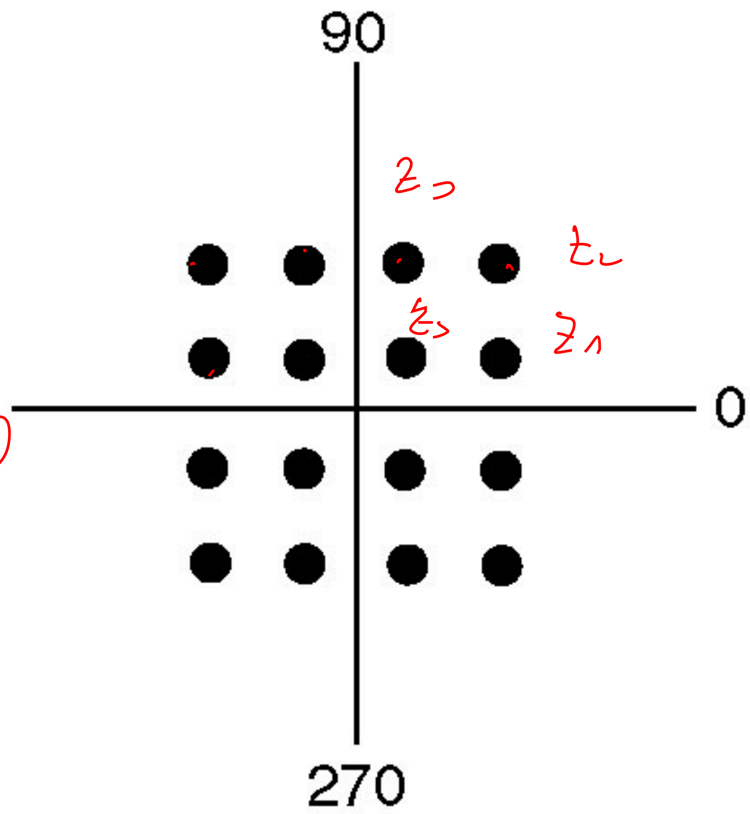
$= \cos a - i \sin a$

$= \cos(-a) + i(\sin(-a)) = e^{-ia}$

Advantage of the Complex Representation

- **Each of the QAM symbols can be represented directly as a complex number**

$$f(x) = \sum_{k=0}^{N-1} z_k \; e^{i2\pi kt/T}$$



Application OFDM

- **Wired**
  - Broadband Internet (ADSL, VDSL)
  - Powerline communications networks (power line communication)
- **Wireless**
  - WLAN: 802.11 a,g,n
  - Terrestrial digital television DVB-T
  - Mobile communication
    - 802.16 WiMAX (Worldwide Interoperability for Microwave Access)
  - WPAN 802.15.3a

Pros and Cons

- **Pro**
  - High bandwidth at low SINR
  - Simple and efficient method
  - proven technology
  - Robust to Multiple Path Fading
  - Efficient use of frequency bands

- **Contra**
  - Susceptible to Doppler effect
  - High power consumption
  - Synchronization reduces efficiency

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$b_1\ b_2$

0 1 1 1 0 1 0 1 1 0 1 1 1 0 1 1 0 . . .

string

$b_1\ b_3$

2

b_2 :

3

b_3

4

b_4

$$\boxed{e^{i\pi} + 1 = 0}$$

$x^2 = 4 \rightarrow x = 2$

$x = -2$

$x^2 = 2 \rightarrow x = \underline{\sqrt{2}} = \underline{1.41\ldots}$

$x^2 = \underline{\underline{-1}}$

$42 \cdot 42 = + \ldots$

$-42 \cdot (-42) = + \ldots$

$x = i$

$$\boxed{i^2 = -1}$$

$\cancel{i < 2}$

$\cancel{2i < 3i}$

2

$1+i$

$a+$

$\sqrt{-1} = i \qquad x^2 = i \qquad (-i)^2 = -1$

$\sqrt{i}\ 2 \qquad x = \frac{1+i}{\sqrt{2}}$

$$\frac{(1+i)^2}{(\sqrt{2})^2} = \frac{1 + 2i + \overbrace{i^2}^{-1}}{2} = \frac{2i}{2} = \underline{i}$$

$x^2 + a + + b = 0 \Rightarrow$ always 2 solut

$a + b i$

Re Im

$\textcircled{7}$: $x^7 + 7x^6 + 3x^5 + 17i = 0$

$\rightarrow$ 7 solutions

$$\boxed{e^{i \cdot x} = \cos x + i \sin x}$$

$$e^{i\pi} = \underbrace{\cos \pi}_{-1} + i \underbrace{\sin \pi}_{0}$$