



ALBERT-LUDWIGS-
UNIVERSITÄT FREIBURG

Algorithms for Radio Networks

**Wireless Sensor Networks: Sensor Coverage &
Lifetime**

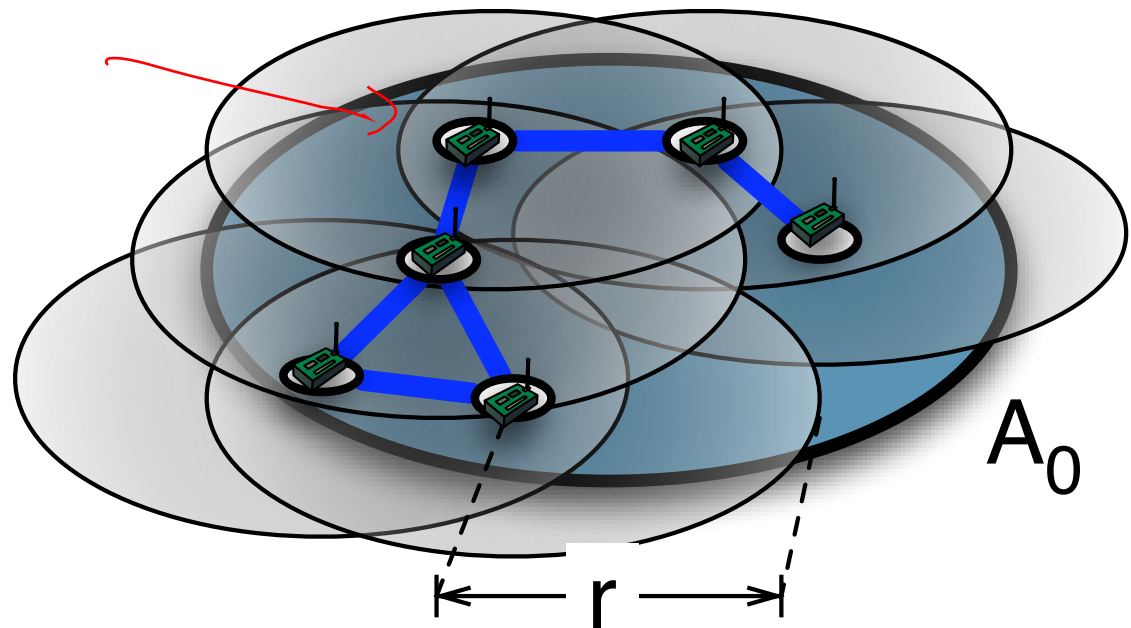
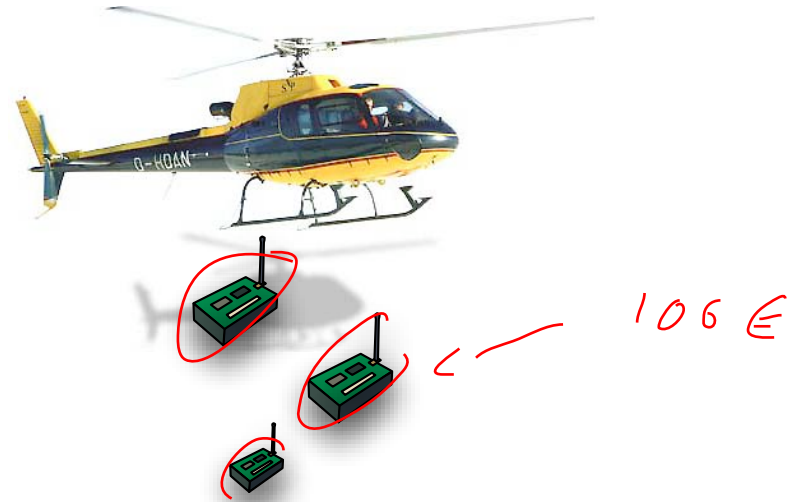
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Computer Networks and Telematics
Christian Schindelhauer



Literature

- **Handbook on Theoretical and Algorithmic Aspects of Sensor, Ad Hoc Wireless and Peer-to-Peer-Networks (Editor: Jie Wu)**
 - Chapter 27: Models and Algorithms for Coverage Problems in Wireless Sensor Networks

Sensor Coverage



Sensor Coverage



► Problem

- Given an area
- Cover the area with the smallest possible number of sensor nodes

► Variants

- Circle Covering
 - 2-dimensional surface, sensor coverage is given by circles
- Art Gallery Problem
 - Angled rooms: Sensor coverage and line of sight angle
 - * e.g. camera surveillance
- Arbitrarily complex variants conceivable

Random Circle Covering

► Naive approach

- Given a square of area A
- How many randomly positioned the sensors with unit disk cover the the square?

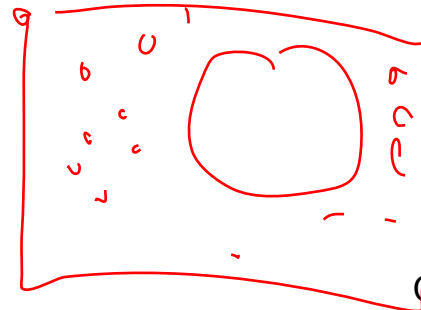
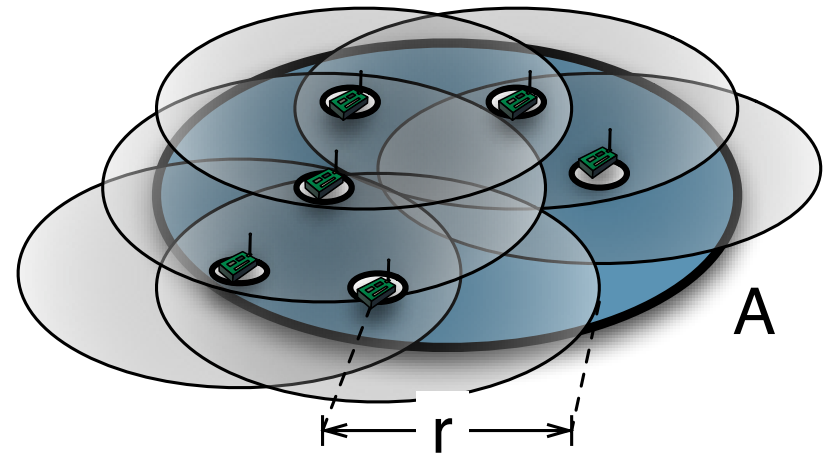
► Naive calculation

- Area of the unit circle: $r^2\pi$
- Number of sensors required: $n = A / (r^2\pi)$

► Intuition

- $O(A/r^2)$ should be sufficient

► But: intuition is wrong!



Random Circle Covering

► Naive approach

- Given a square of area A
- How many randomly positioned sensors with unit disk cover the square?

► Theorem

- Let $n = A / (r^2\pi)$
 - where A denotes the area of the square
 - and r denotes the sensor radius
- To cover such a square of $\Theta(\underline{n \log n})$ randomly placed sensors are necessary and sufficient

Random Circle Covering

► Theorem

- Let $n = A / (r^2\pi)$
 - where A denotes the area of the square
 - and r denotes the sensor radius
- To cover such a square of $\Theta(n \log n)$ randomly placed sensors are necessary and sufficient

► Proof sketch (lower bound):

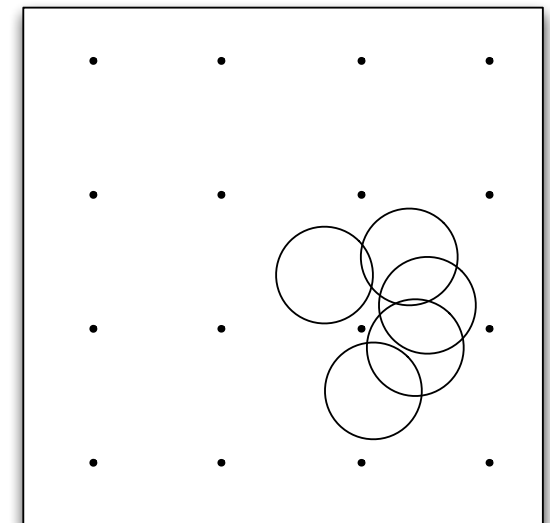
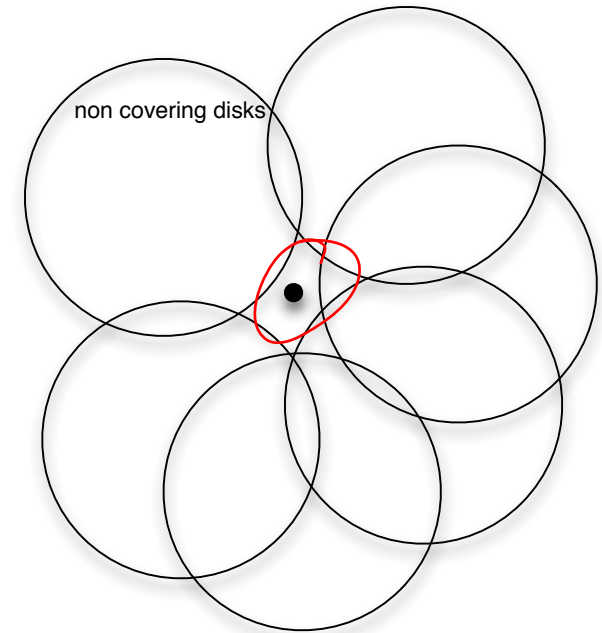
- The probability that a given point is not covered by a sensor is at least

$$1 - r^2\pi/A = 1 - 1/n$$

- Consider n such points with distance r
- The probability that at least $1/n \log n$ sensors do **not** cover one of these point is therefore

$$\left(1 - \frac{1}{n}\right)^{\frac{1}{2}n \log n} \geq \left(\frac{1}{4}\right)^{\frac{1}{2} \log n} = \frac{1}{n}$$

- Hence, the expected number of uncovered points is 1



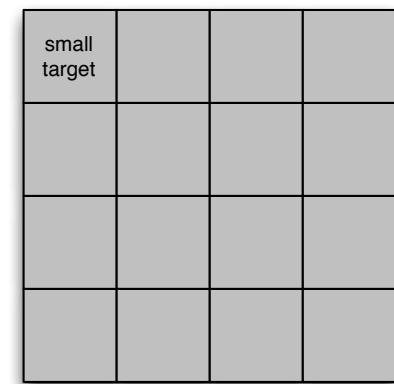
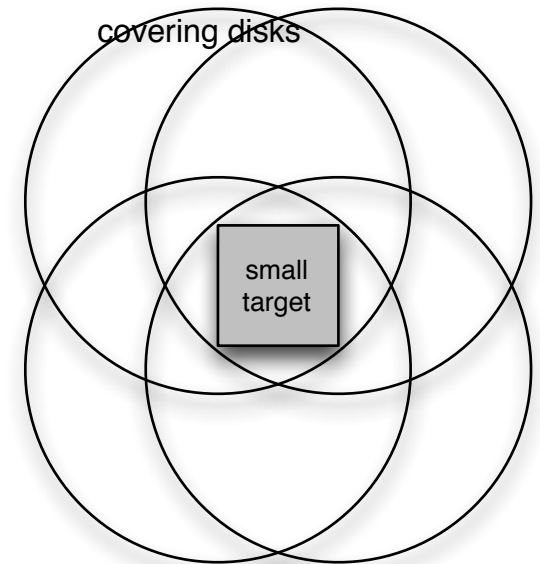
Random Circle Covering

► Theorem

- Let $n = A / (r^2\pi)$
 - where A denotes the area of the square
 - and r denotes the sensor radius
- To cover such a square of $\Theta(n \log n)$ randomly placed sensors are necessary and sufficient

► Proof sketch (upper bound):

- By $c n \log n$ random sensors every square of size $r/3 \times r/3$ is covered with probability $1-n^{-k}$
 - where k grows linear with c
- Then the whole square is covered with probability $1-n^{-k-1}$



Optimal Deterministic Bound

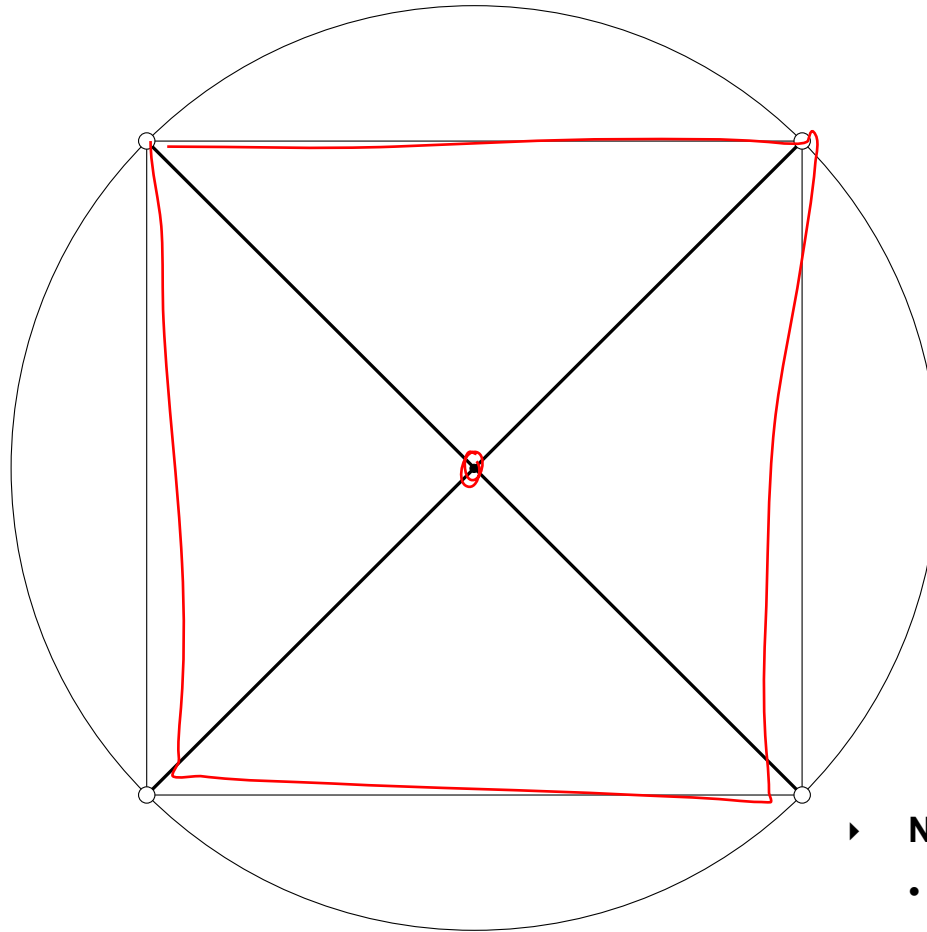
- ▶ **Nurmela, Östergard**

- Covering a square with up to 30 equal circles
(Teknillisen korkeakoulun tietojenkäsittelyteorian laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

- ▶ **How many circles can cover a square?**

- A closed form solution is unknown
- However for a small number of circles the problem can be solved by exhaustive search

Disk Coverage of a Square

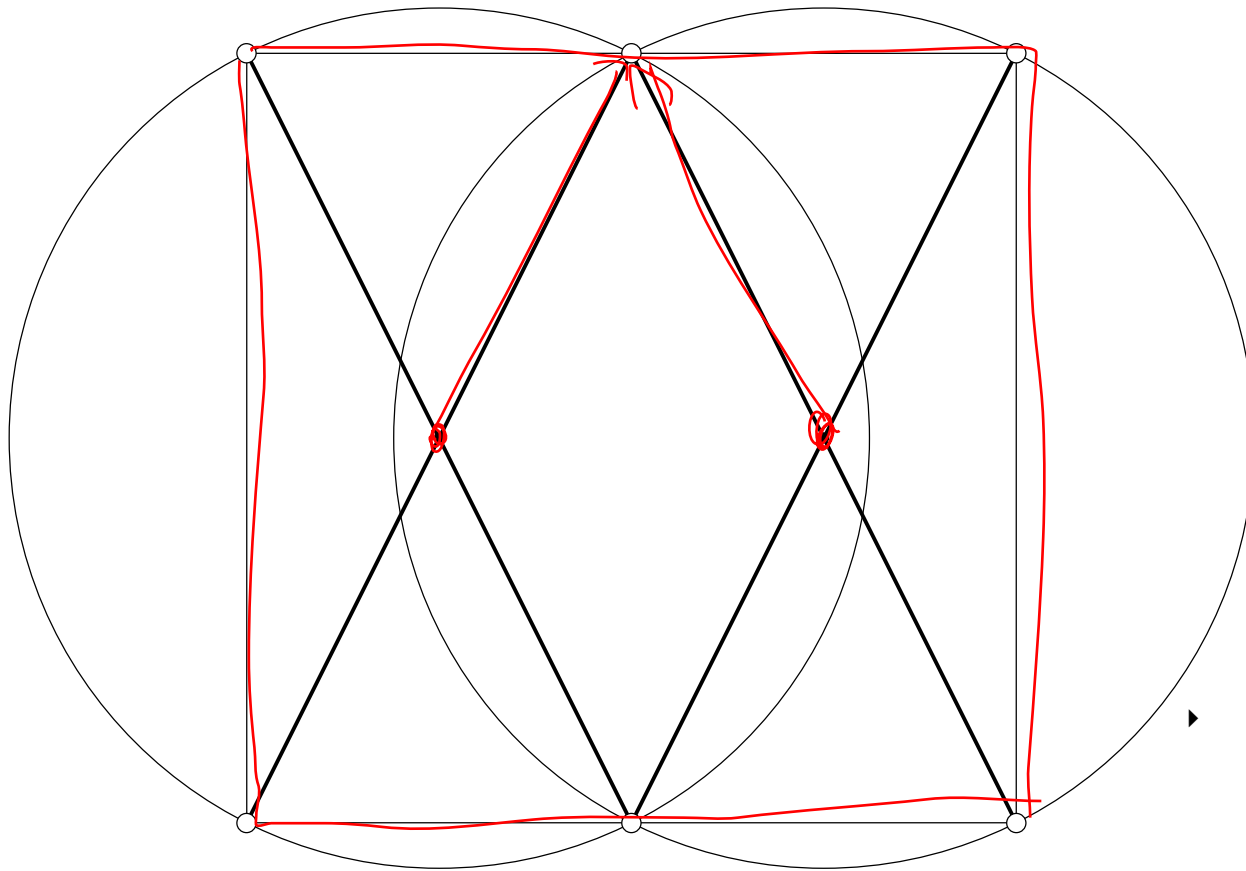


$$n = 1$$

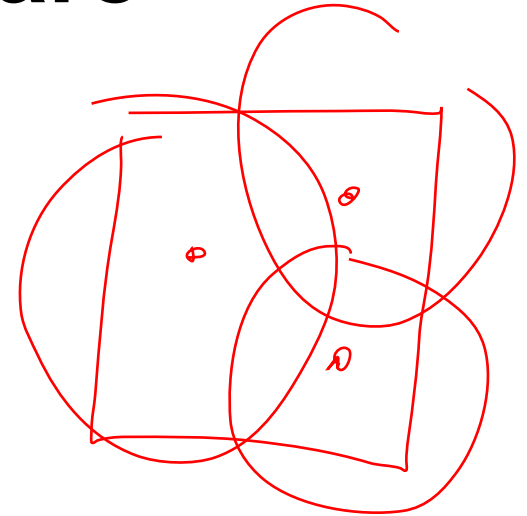
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Disk Coverage of a Square



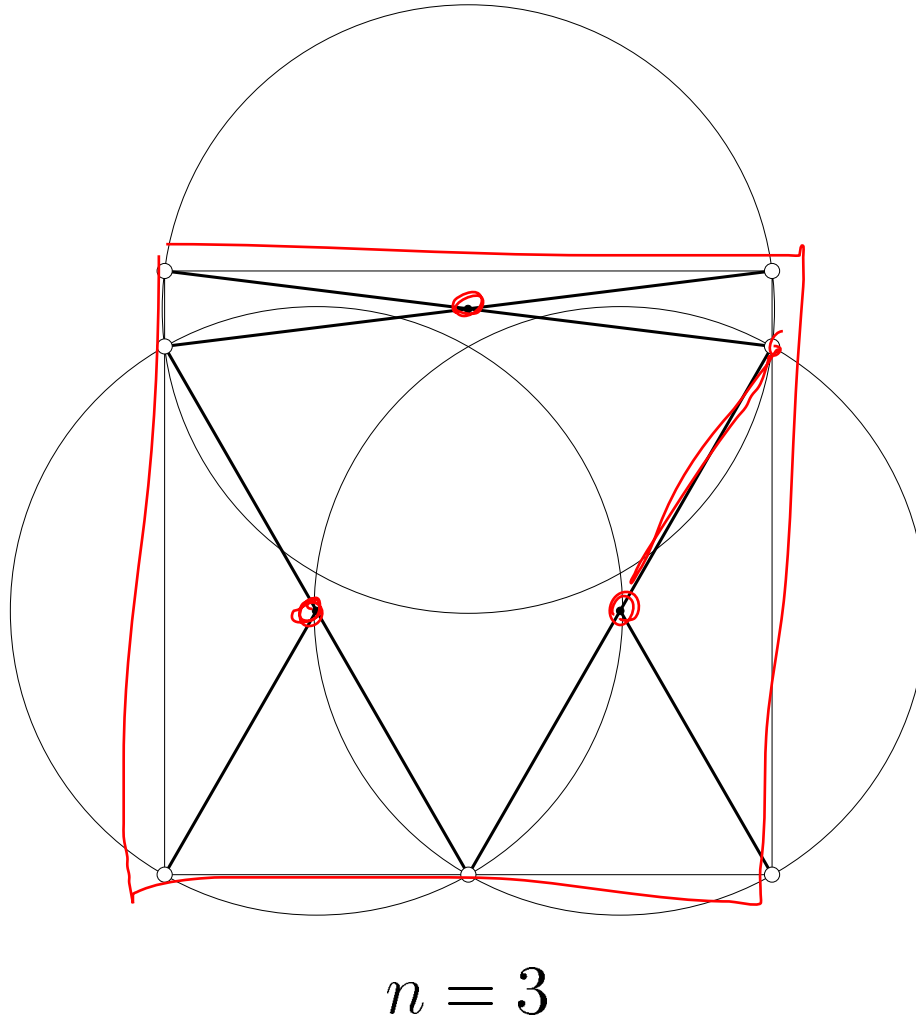
$$n = 2$$



► **Nurmela, Östergård**

- Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteoria laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

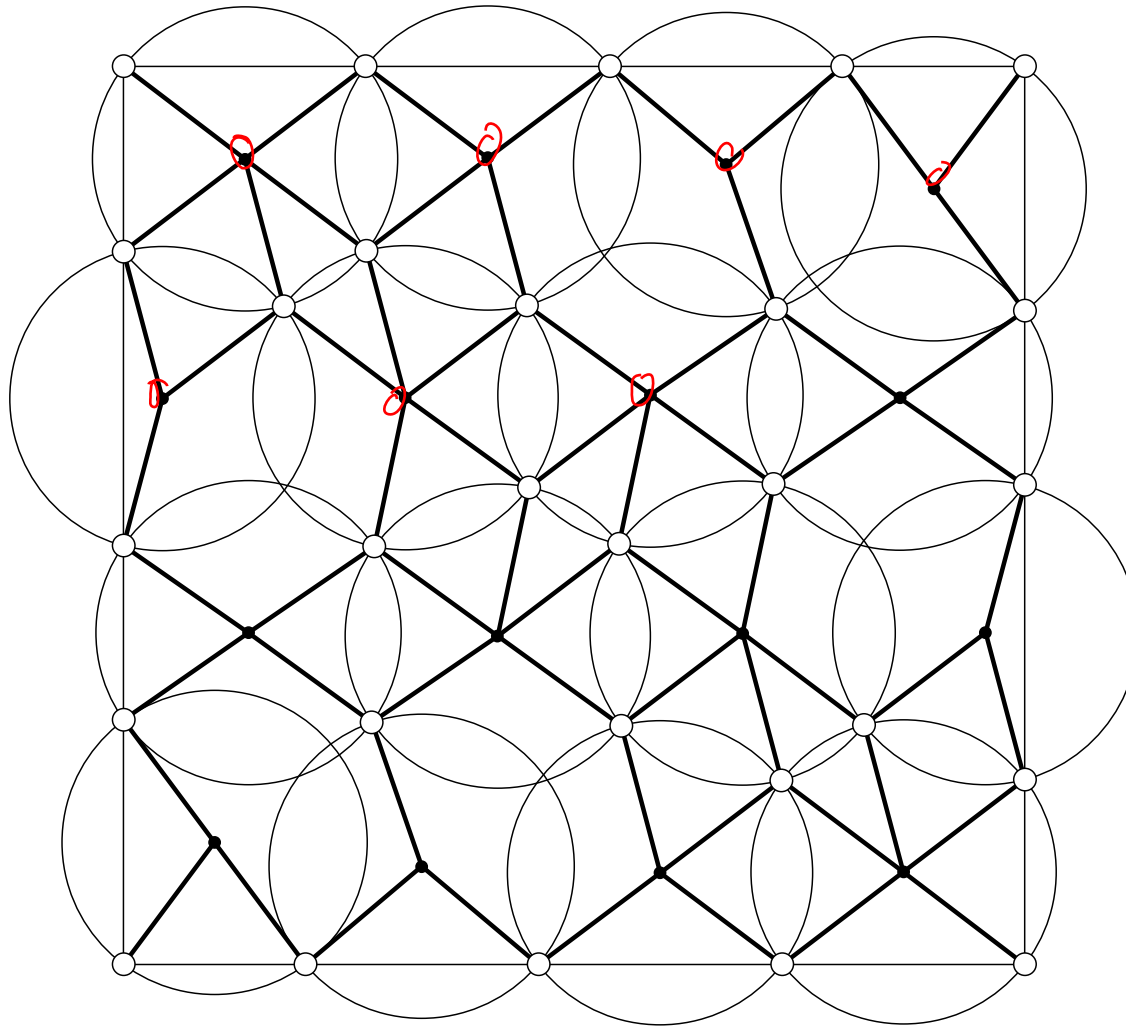
Disk Coverage of a Square



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- Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteoria laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

Disk Coverage of a Square



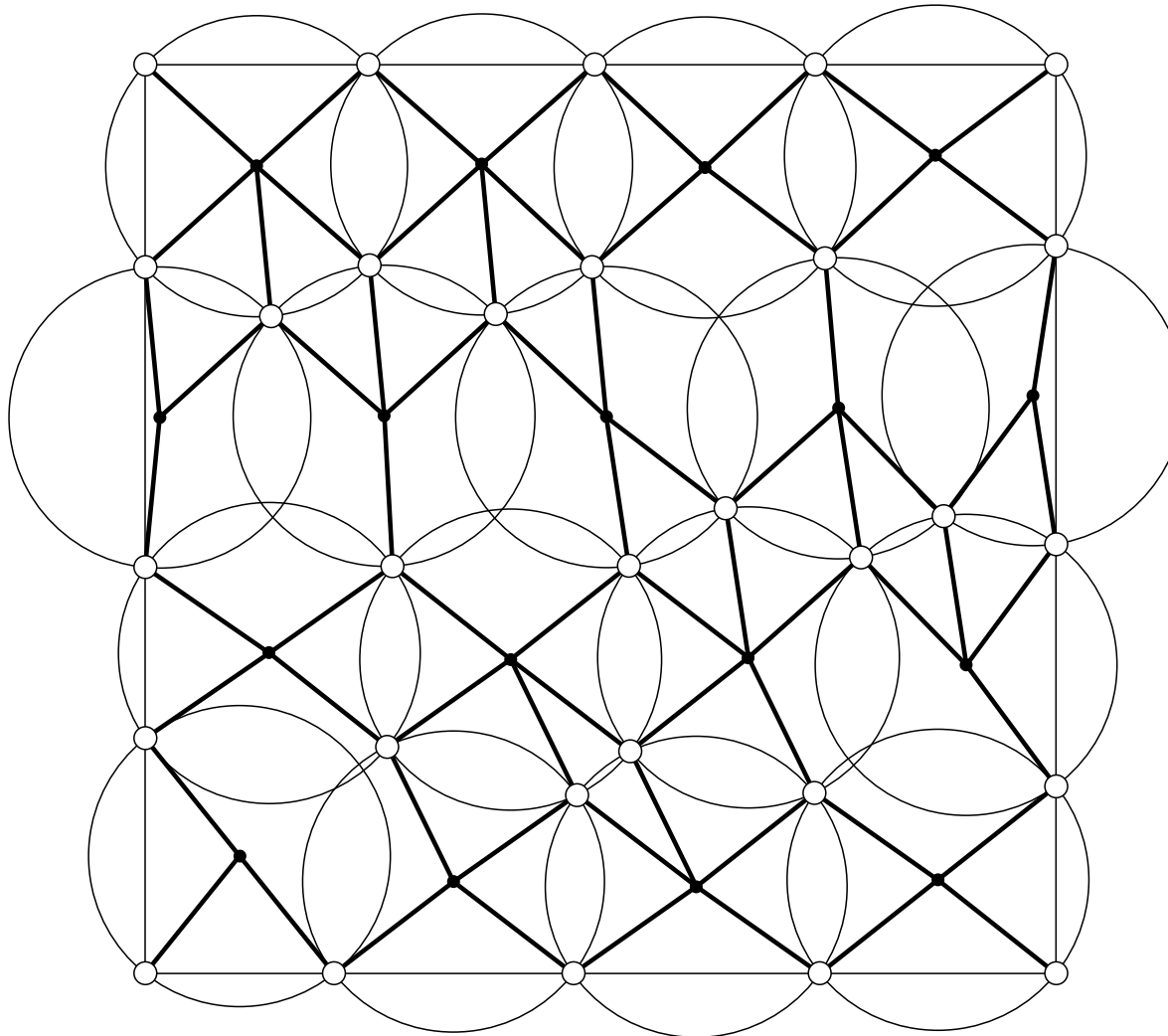
$$n = 16$$

13

► **Nurmela, Östergard**

- Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteoria laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

Disk Coverage of a Square



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Disk Coverage of a Square

n	r_n	G	n	r_n	G
1	0.70710678118654752440...	D_4	16	0.16942705159811602395...	C_2
2	0.55901699437494742410...	D_2	17	0.16568092957077472538...	C_1
3	0.50389110926865935327...	D_1	18	0.16063966359715453523...	D_1
4	0.35355339059327376220...	D_4	19	0.15784198174667375675...	C_1
5	0.32616058400398728086...	D_1	20	0.15224681123338031005...	D_1
6	0.29872706223691915876...	C_2	21	0.14895378955109932188...	C_1
7	0.27429188517743176508...	D_2	22	0.14369317712168800049...	D_2
8	0.26030010588652494367...	D_2	23	0.14124482238793135951...	D_2
9	0.23063692781954790734...	D_1	24	0.13830288328269767697...	C_1
10	0.21823351279308384300...	D_2	25	0.13354870656077049693...	D_1
11	0.21251601649318384587...	D_2	26	0.13176487561482596463...	C_1
12	0.20227588920818008037...	C_2	27	0.12863353450309966807...	D_2
13	0.19431237143171902878...	C_1	28	0.12731755346561372147...	D_2
14	0.18551054726041864107...	D_1	29	0.12555350796411353317...	C_1
15	0.17966175993333219846...	C_1	30	0.12203686881944873607...	C_2

► **Nurmela, Östergard**

- Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteorian laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

Art Gallery Problem

► Given

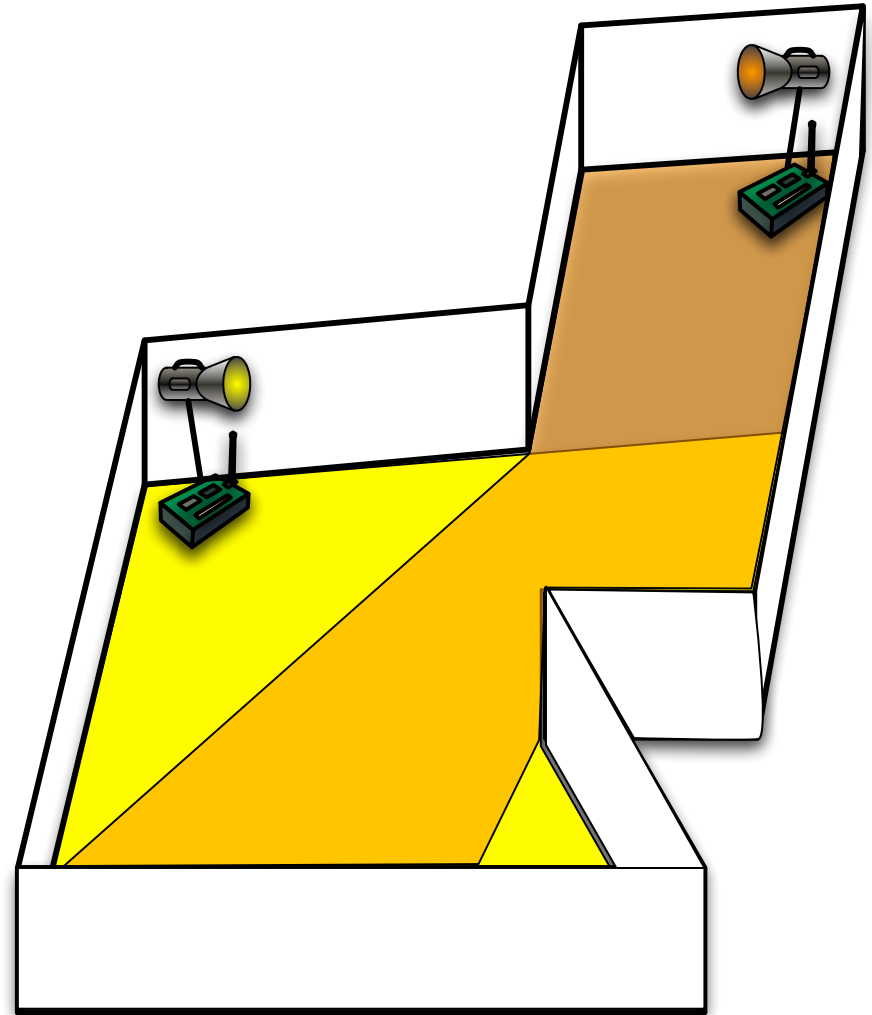
- a room (described by polygon)

► Compute

- Minimum number of cameras and their placement
 - such that the entire space is covered

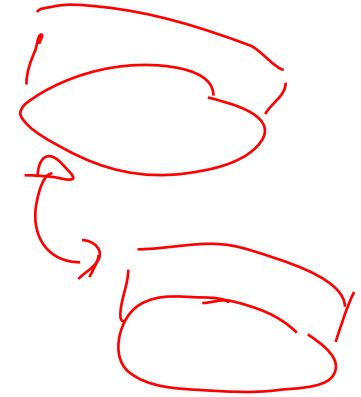
► Results

- Every room with n edges can be monitored by at most $n / 3$
- The exact solution is NP-hard
 - even in the two-dimensional case
- Polynomial time approximation with a factor $O(\log n)$



Energy Saving Methods

oos



- ▶ **Schedule for sleep cycles**
 - MAC, routing protocol, sensing
- ▶ **Optimize transmission routes**
 - many hops of few hops
- ▶ **Selection of nodes depending on the charge battery status**
 - data acquisition
 - change of cluster heads
 - route choice may consider battery status
- ▶ **Reduction of the amount of data**
 - data aggregation
 - compression
 - filtering

Lifetime of a Sensor Network

- ▶ **Wireless Sensor Networks (WSN)**
 - cheap and energy optimized sensors
 - send data to sinks
- ▶ **Lifetime of the network**
 - is hard to analyze
- ▶ **Depends from**
 - network architecture, protocols
 - event or input behavior
 - definition of lifetime
 - hardware, channel characteristics

Lifetime

► On the Lifetime of Wireless Sensor Networks

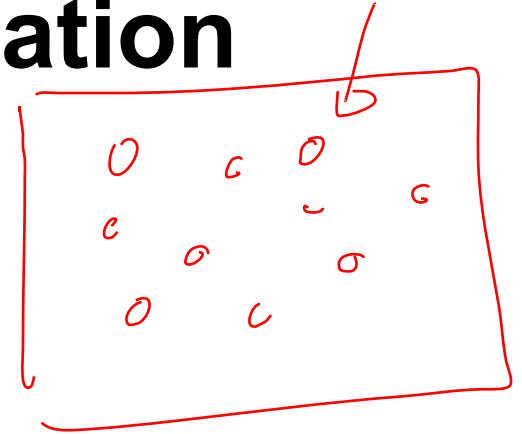
- Yunxia Chen, Qing Zhao, Communication Letters, Vol. 9, No. 11, Nov. 2005

► Theorem

- For a WSN where
 - E_0 : non-rechargable initial energy E_0
 - P_c : constant continuous power consumption in the complete network
 - $E[E_w]$: expected waste of energy
 - λ : average number of reported events
 - $E[E_r]$: expected energy necessary to report an event

$$E[\mathcal{L}] = \frac{E_0 - E[E_w]}{P_c + \lambda E[E_r]}$$

Greedy Lifetime Maximization



► Question

- Which sensors should collect the data

► Greedy Algorithmus

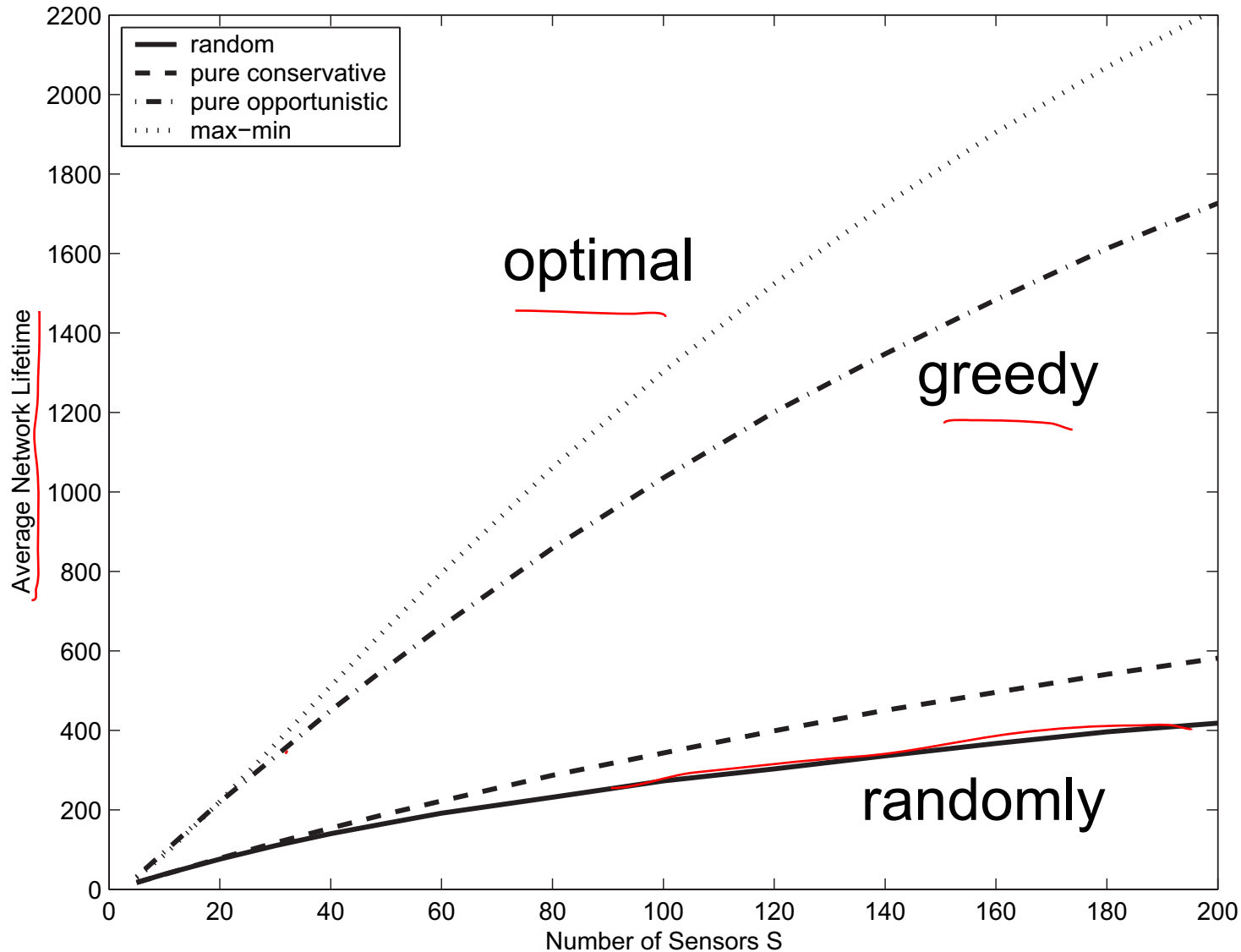
- Choose the sensor with the maximum energy efficiency index γ_i :

-

$$\gamma_i = e_i - E_r(c_i)$$

- $E_r(c_i)$: Energy for the transport of a message for node i
- e_i : Available energy at the node i

Performance Greedy-Algorithm



On the Lifetime of Wireless
Sensor Networks
Yunxia Chen, Qing Zhao,
Communication Letters, Vol. 9,
No. 11, Nov. 2005

Lifetime Maximization by Scheduling

► Cardei, Du

- *Improving Wireless Sensor Network Lifetime through Power Aware Organization*, Wireless Networks 11, 333–340, 2005

► Problem

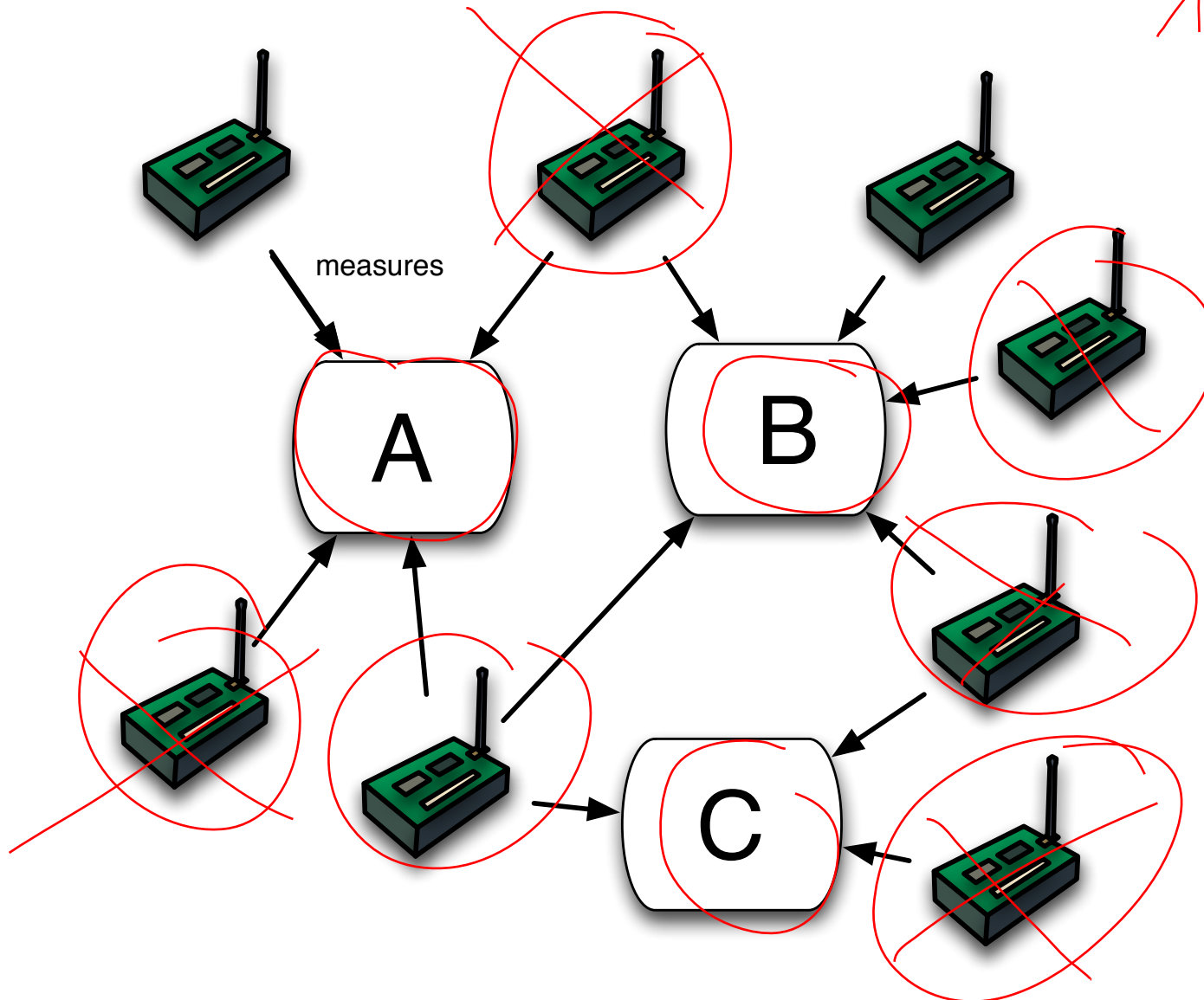
- Measurement points are covered by more than one sensors
- Multiple measurements waste energy

► Solution

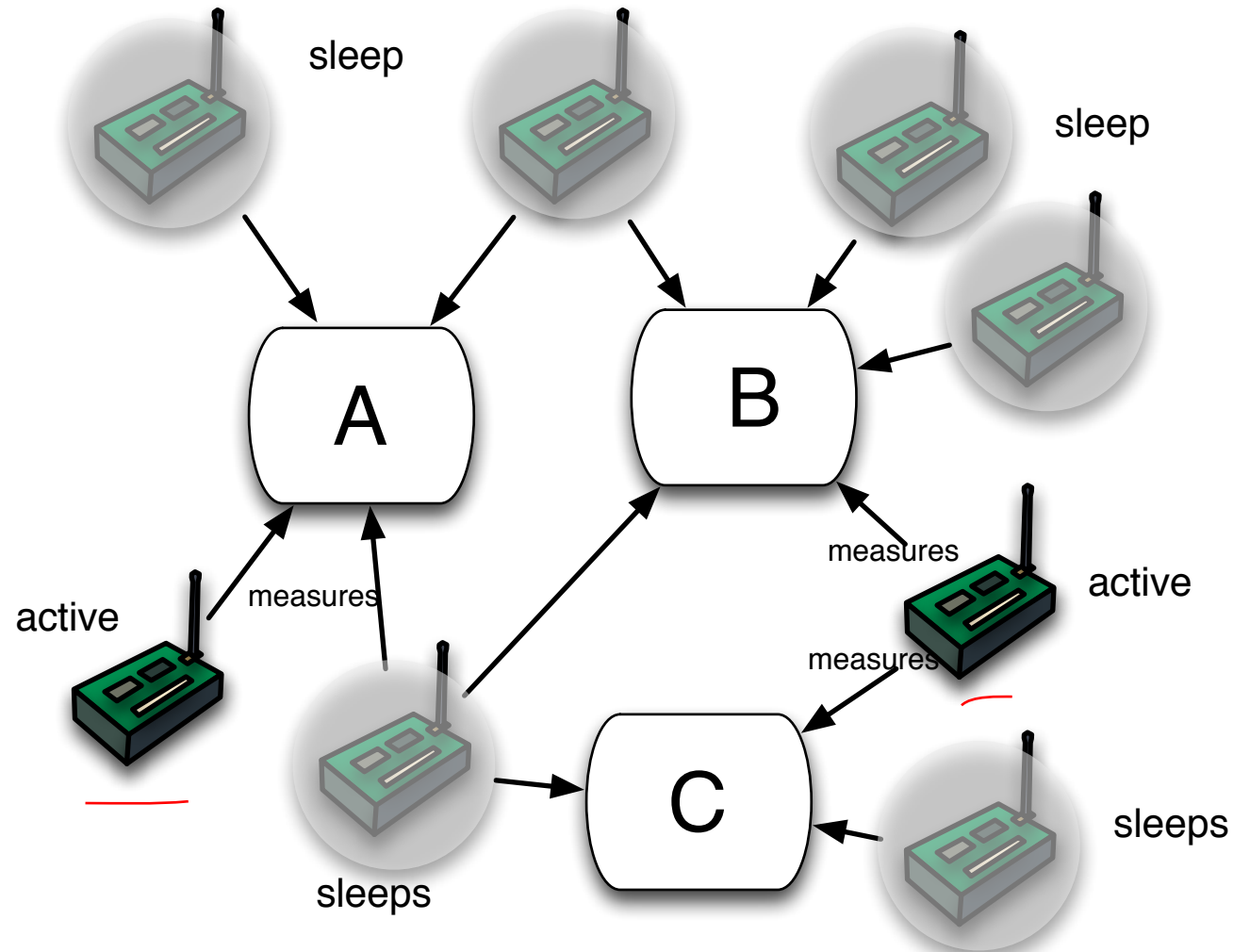
- Activate only the nodes with minimum set-cover

Multiple Coverage of Sensors

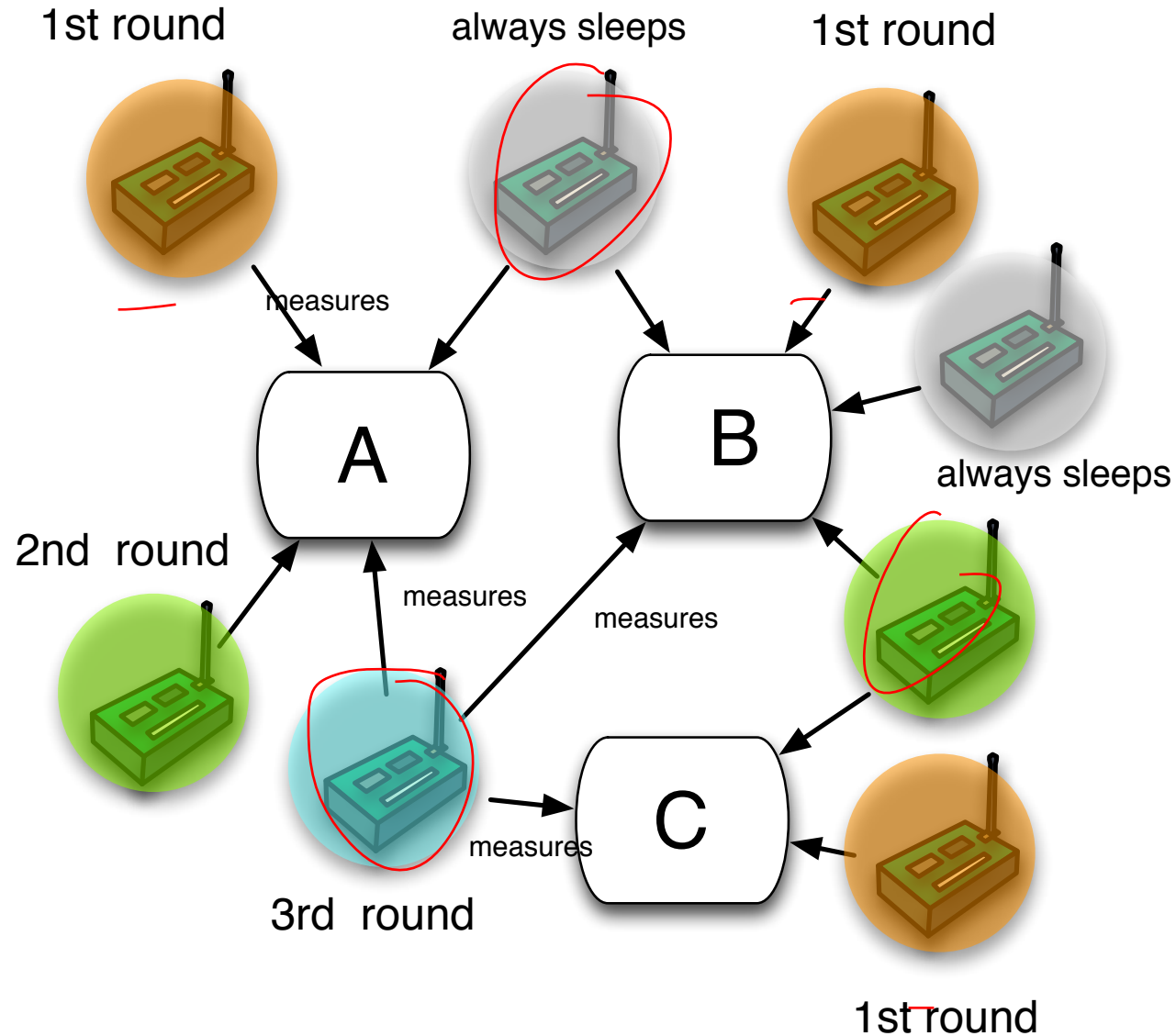
1 + 1 + 1



Covering Set



Disjoint Set-Cover



Definition Disjoint Set-Cover (DSC)

► Given

- n sensors $S = \{S_1, S_2, \dots, S_n\}$
- m measurement points $T = \{T_1, T_2, \dots, T_m\}$
- Sensor coverage $S_i \subseteq T$

► Compute

- Maximal number of disjoint coverings, i.e.
 - disjoint sets M_1, \dots, M_k from S , such that each set covers the set T

► Motivation

- The network lifetime increases by a factor of k

Complexity von Disjoint Set-Cover (DSC)

► Theorem

- DSC is NP-hard for two sets
- DSC is in general NP-hard
- DSC can not be approximated by a factor of 2 without solving an NP-hard problem

► Several heuristics are known

Heuristiks for DSC

► Slijepcevic Potkonjak 2001

- *Power Efficient Organization of Wireless Sensor Networks*, IEEE International Conference on Communications
- Greedy algorithm
 - Greedily selects a mimal covering set
 - Removed this one and repeated until no more covering set is found

► Cardei, Du 2006

- Problem is represented as flow problem
- This is solved as linear problem
- The solution gives an approximation of the disjoint set-cover problem

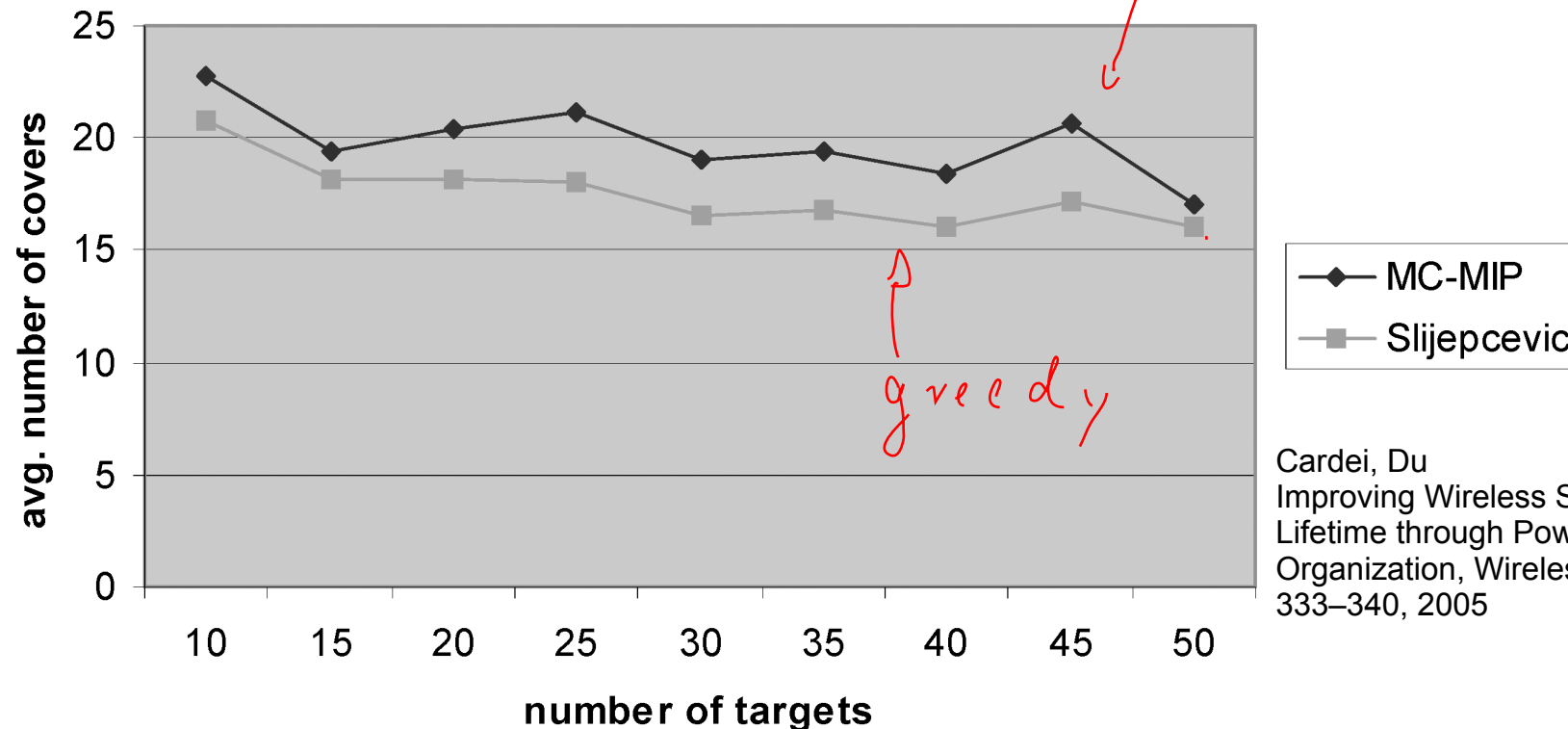
Comparison

► Slijepcevic Potkonjak 2001

- simple distributed greedy solution

► Cardei, Du 2006

- MC-MIP complex central algorithm

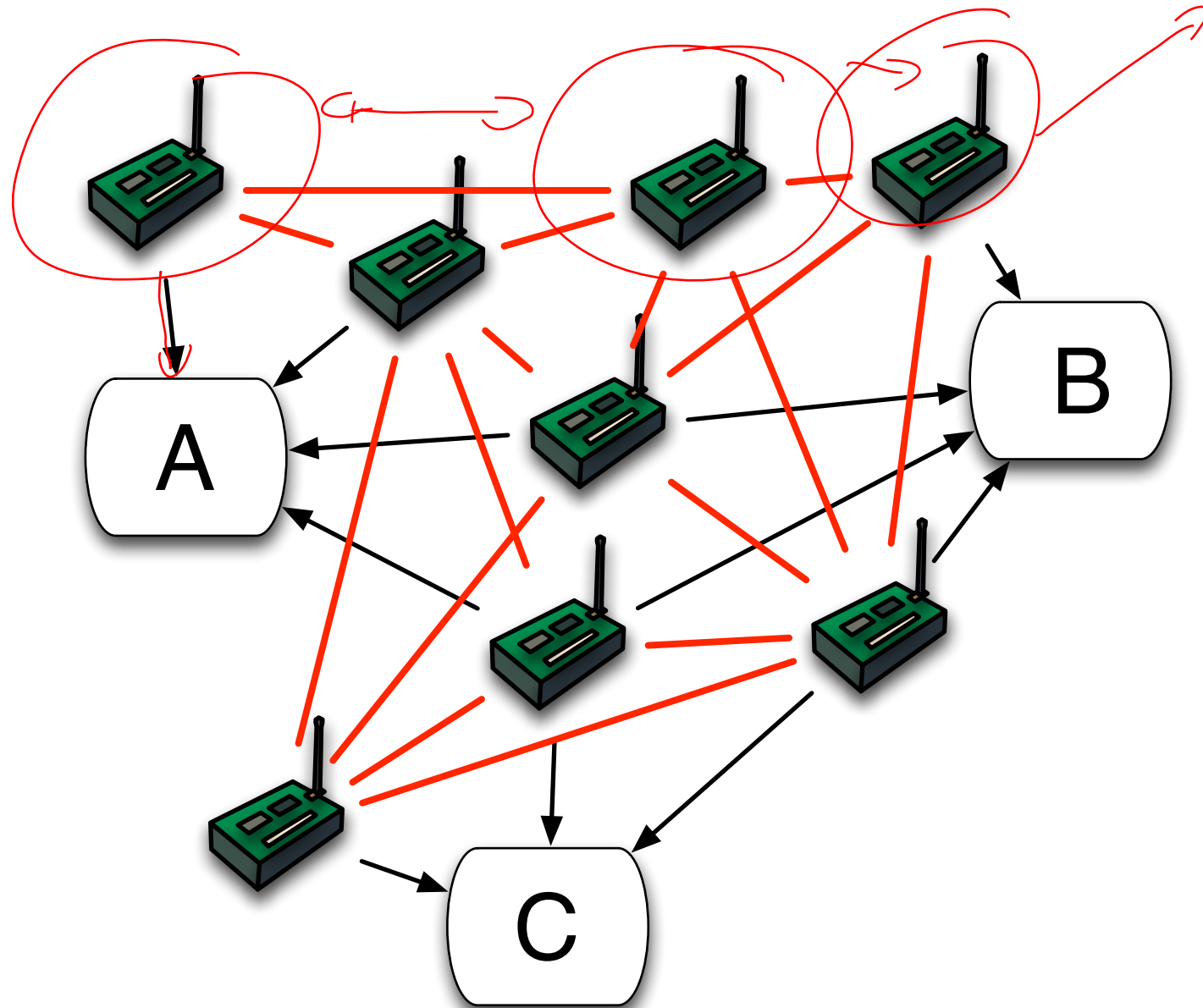


Cardei, Du
Improving Wireless Sensor Network
Lifetime through Power Aware
Organization, Wireless Networks 11,
333–340, 2005

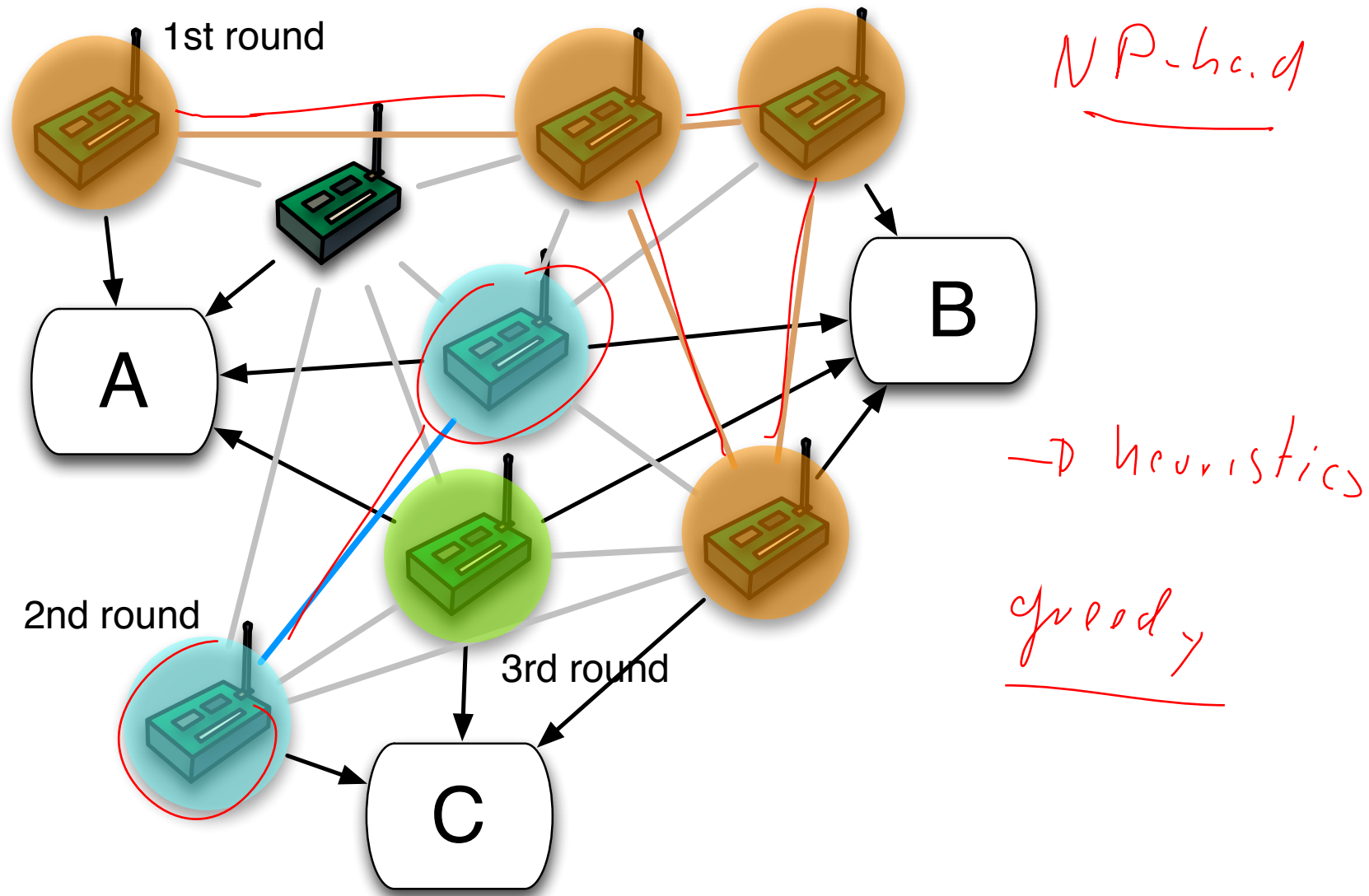
Outlook

- ▶ **Disjoint sets of network nodes may not be useful**
 - might be too far away from each other
 - important relay nodes are not activated
- ▶ **Extension**
 - Disjoint Connected Set Problem::
 - Find vertex-connected subgraph
 - Also NP-hard
- ▶ **Similar heuristics exist**

Disjoint Connected Set Problem

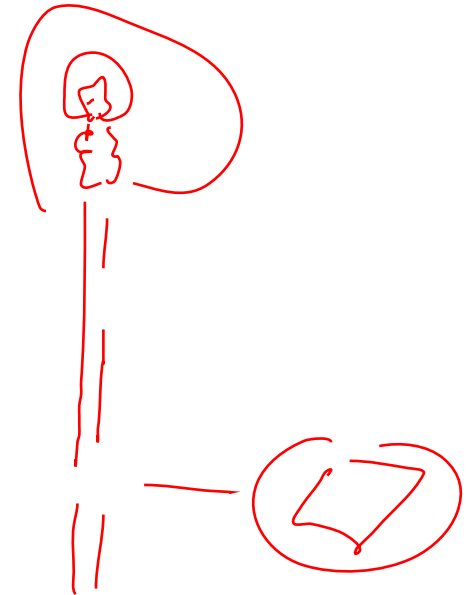
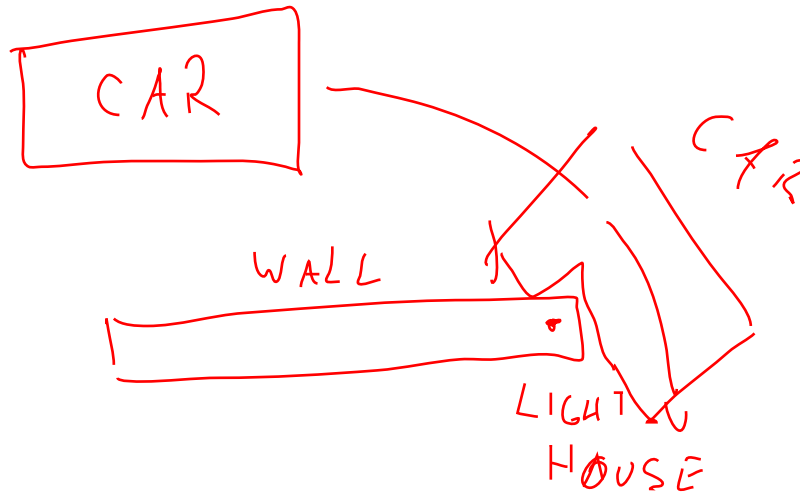


Disjoint Connected Set Problem



Literature Energy Harvesting

- ▶ **Kansal, Hsu, Zahedi, Srivastava**
 - *Power management in energy harvesting sensor networks*. ACM Trans. Embed. Comput. Syst. 6, 4, Sep. 2007



Motivation

► Energy harvesting

- can remove batteries from WSNs
- potentially infinite lifetime
- active time can be increased (or reduced)

► Example

- solar energy only available at daylight



► Energy concept

- necessary for the entire period
- regulates interplay of sleep phase, data rate and short term energy source

Harvesting Paradigm

- ▶ **Typical task in battery operated WSN**
 - minimize energy consumption
 - maximize lifetime
- ▶ **Task in harvesting-WSN**
 - continuous operation ~~⌘~~
 - i.e. infinite lifetime
 - term: energy-neutral operation



Possible Sources

- ▶ **Piezoelectric effect**
 - mechanical pressures produces voltage
- ▶ **Thermoelectric effect**
 - temperature difference of conductors with different thermal coefficient
- ▶ **Kinetic energy**
 - • e.g. self-rewinding watches
- ▶ **Micro wind turbines**
- ▶ **Antennas**
- ▶ **Chemical sources,...**

1 W

80 W

400 W



Differences Compared to Batteries

▶ Time dependent

- form of operation has to be adapted over time
- sometimes not predictable

▶ Location dependent

- different nodes have have different energy
 - load balancing necessary

▶ Never ending supply

→ ▶ New efficiency paradigm

- utilization of energy for maximum performance
- energy saving may result in unnecessary opportunity costs

Solutions without Power Management

▶ Without energy buffer

- harvesting hardware has to supply maximal necessary energy level at minimum energy input
- only in special situation possible
 - e.g. light switch

▶ With energy buffer

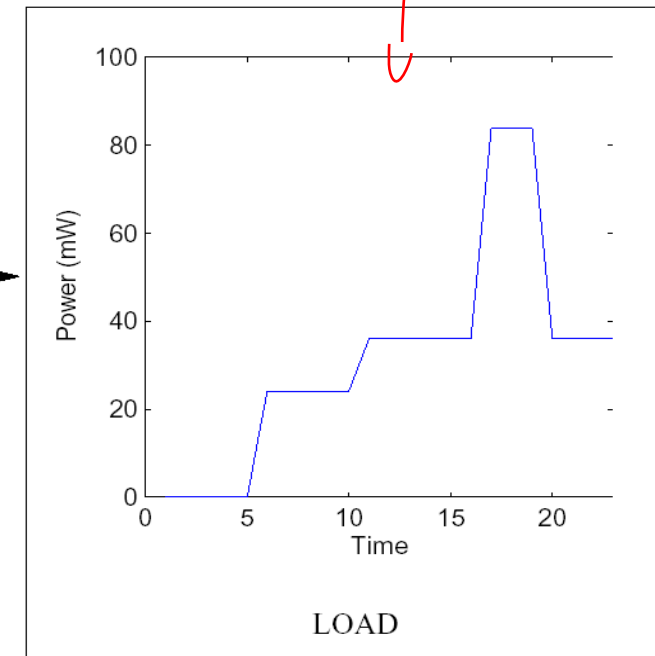
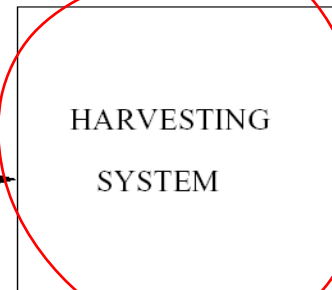
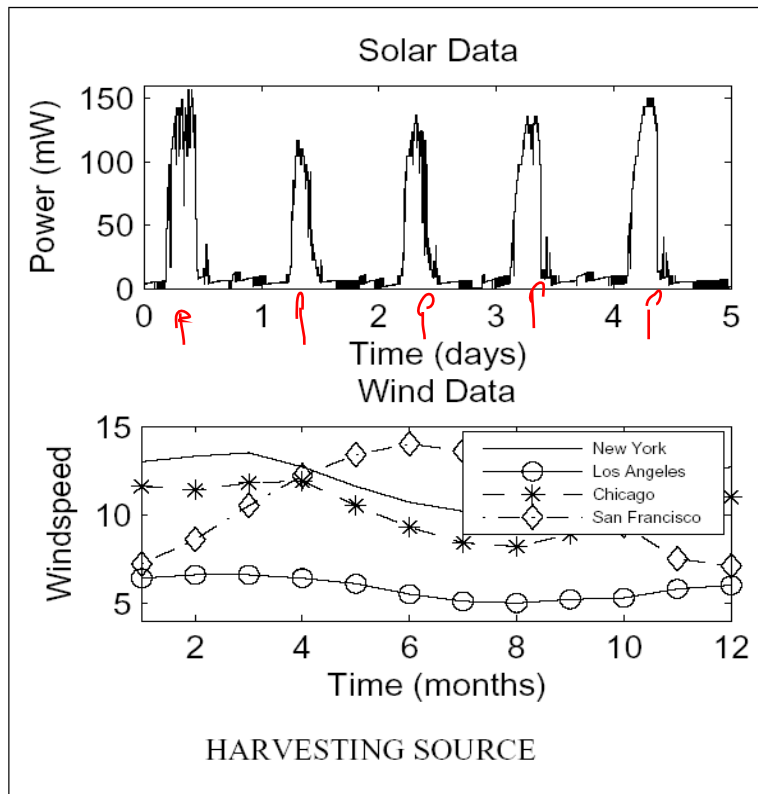
- power management system necessary



Power Management System

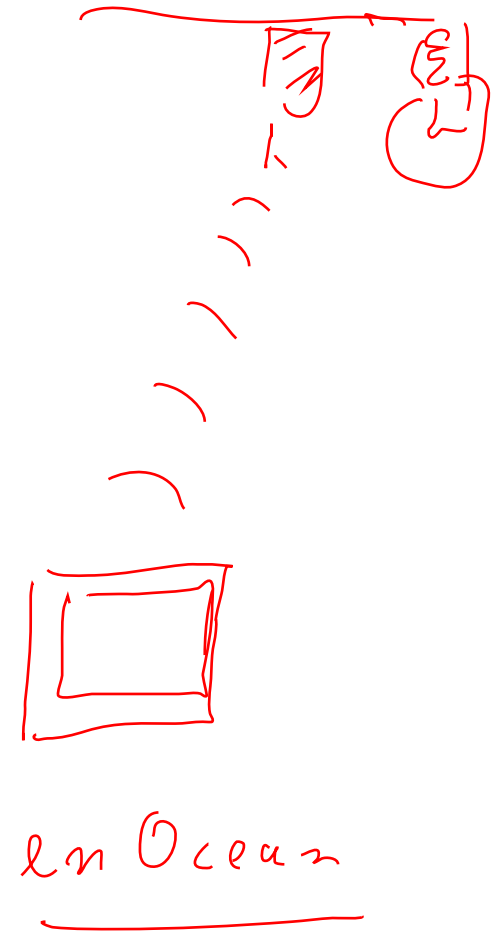
► Target

- Providing the necessary energy from external energy source and energy buffer



Energy Sources

- ⑥ **Uncontrolled but predictable**
 - e.g. daylight
- ⑥ **Uncontrolled and unpredictable**
 - e.g. wind
- ⑦ **Controllable**
 - energy is produced if necessary
 - e.g. light switch, dynamo on bike
- ⑧ **Partially controllable**
 - energy is not always available
 - e.g. radio source in the room with changing reception



Harvesting Theory

- ▶ $P_s(t)$: Power output from energy source a time t
- ▶ $P_c(t)$: Energy demand at time t
- ▶ Without energy buffer

- $P_s(t) \geq P_c(t)$: node is active

- ▶ Ideal energy buffer

- Continuous operation if

$$\int_0^T P_c(t) dt \leq \int_0^T P_s(t) dt + B_0 \quad \forall \quad T \in [0, \infty)$$

- where B_0 is the initial energy
- energy buffer is lossless, store any amount of energy

Harvesting Theory

- ▶ $P_s(t)$: Power output from energy source a time t
- ▶ $P_c(t)$: Energy consumed at time t

▶ Let
$$\underline{[x]^+} = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

▶ Non-ideal energy buffer

- Continuous operation if

$$\underline{B_0} + \eta \int_0^T \underline{[P_s(t) - P_c(t)]^+} dt - \int_0^T \underline{[P_c(t) - P_s(t)]^+} dt - \int_0^T \underline{P_{leak}(t)} dt \geq 0$$

Handwritten notes: "using a copy" with an arrow pointing to the second integral, and a red arrow pointing to the first integral.

- B_0 is the initial energy
- η : efficiency of energy buffer
- $\overline{P_{leak}(t)}$: energy loss of the memory

Harvesting Theory

‣ $P_s(t)$: Power output from energy source a time t

‣ $P_c(t)$: Energy consumed at time t

‣ Let
$$[x]^+ = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

‣ **Non-ideal energy buffer with limited reception B**

- Continuous operation if

$$\rightarrow B_0 + \eta \int_0^T [P_s(t) - P_c(t)]^+ dt - \int_0^T [P_c(t) - P_s(t)]^+ dt - \int_0^T P_{leak}(t) dt \geq 0$$

- B_0 is the initial energy of the buffer
- η : efficiency of energy buffer
- $P_{leak}(t)$: leakage power of the energy buffer

$$B_0 + \eta \int_0^T [P_s(t) - P_c(t)]^+ dt - \int_0^T [P_c(t) - P_s(t)]^+ dt - \int_0^T P_{leak}(t) dt \leq B$$

Model of Benign Energy Behavior

- ▶ If the power source $P_s(t)$ occurs regularly, then it satisfies the following equations

$$\int_{\tau}^{\tau+T} P_s(t) dt \leq \rho_1 T + \sigma_1$$

$$\int_{\tau}^{\tau+T} P_s(t) dt \geq \rho_1 T - \sigma_2$$

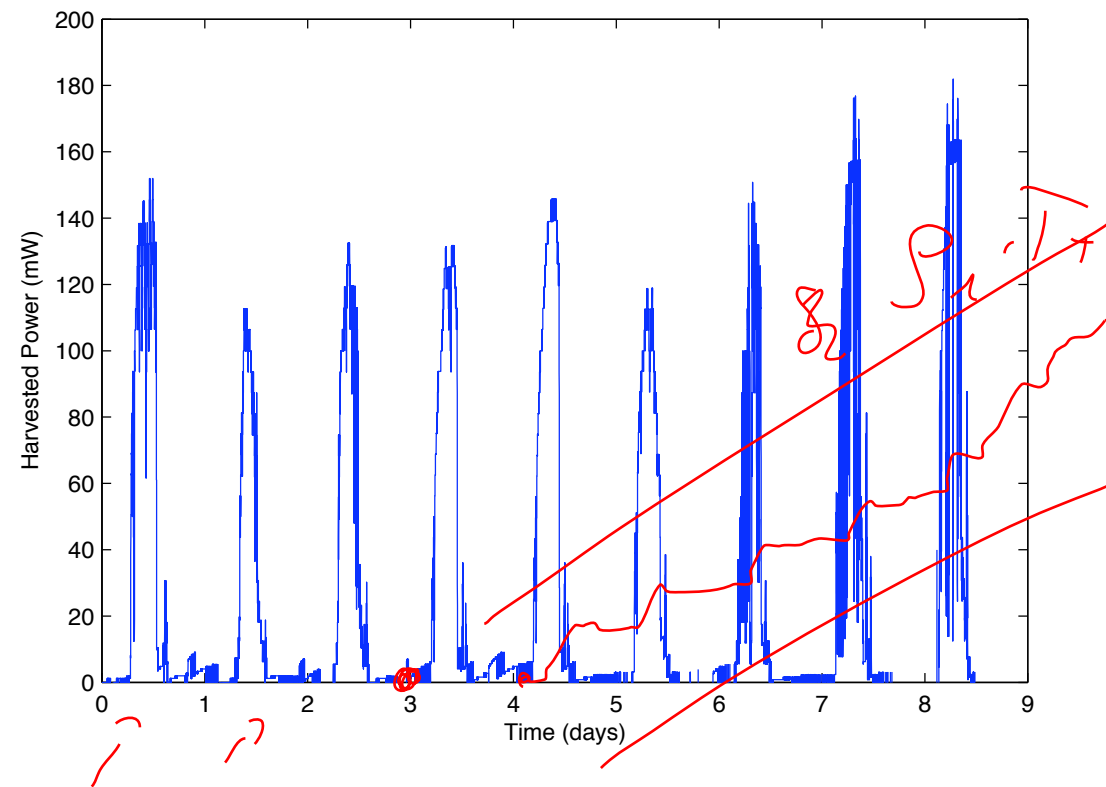


Fig. 2. Solar energy based charging power recorded for 9 days

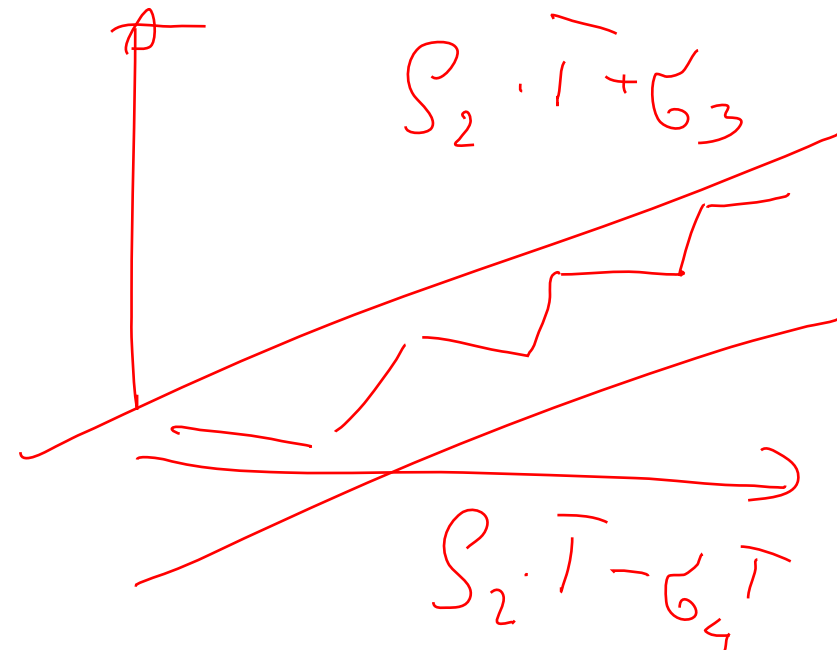
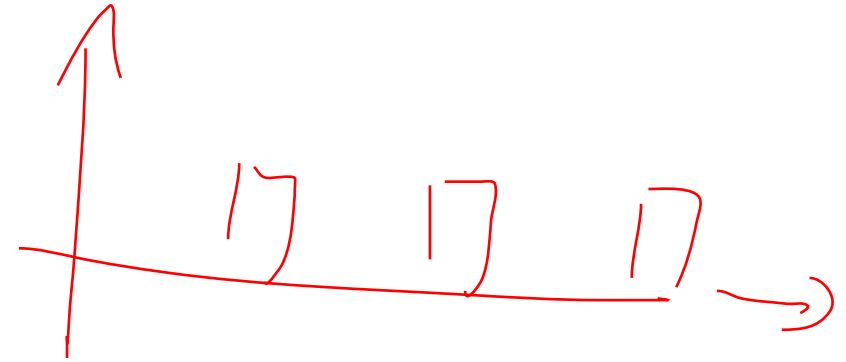
Model of Benign Energy Behavior

► Benign energy consumption:

- $P_c(t)$ satisfies the following

$$\int_{\tau}^{\tau+T} P_c(t) dt \leq \rho_2 T + \sigma_3$$

$$\int_{\tau}^{\tau+T} P_c(t) dt \geq \rho_2 T - \sigma_4$$



Energy Neutrality for Benign Sources

- ▶ Substitution into the non-ideal energy source inequality:

$$B_0 + \eta \cdot \min\left\{\int_T P_s(t)dt\right\} - \max\left\{\int_T P_c(t)dt\right\} - \int_T P_{leak}(t)dt \geq 0$$
$$\Rightarrow B_0 + \eta(\rho_1 T - \sigma_2) - (\rho_2 T + \sigma_3) - \rho_{leak} T \geq 0$$

- ▶ This inequality must hold for $T=0$

$$B_0 \geq \eta\sigma_2 + \sigma_3$$

- ▶ This condition must hold for all T

$$\eta\rho_1 - \rho_{leak} \geq \rho_2$$

- ▶ If these inequalities hold then continuous operation can be guaranteed

Necessary Energy Buffer for Benign Energy Sources

- ▶ Substituting in the second equation

$$B_0 + \eta \cdot \max\left\{\int_T P_s(t)dt\right\} - \min\left\{\int_T P_c(t)dt\right\} - \int_T P_{leak}(t)dt \leq B$$

$$\Rightarrow B_0 + \eta(\rho_1 T + \sigma_1) - (\rho_2 T - \sigma_4) - \rho_{leak} T \leq B$$

- ▶ For $T=0$ we need

$$B_0 + \eta(\sigma_1 - \sigma_4) \leq B$$

- ▶ Substitution of $B_0 \geq \eta\sigma_2 + \sigma_3$ yields

$$B \geq \eta(\sigma_1 + \sigma_2) + \sigma_3 - \sigma_4$$

- ▶ For $T \rightarrow \infty$ we have

$$\eta\rho_1 - \rho_{leak} \leq \rho_2$$

- This condition may be violated without problems

$$\eta\rho_1 - \rho_{leak} = \rho_2$$

Energy Neutral Operation

► Theorem

- For benign energy sources the energy neutrality can be satisfied if the following conditions apply

- $\rho_2 \leq \eta\rho_1 - \rho_{\text{leak}}$
- $B \geq \eta\sigma_1 + \eta\sigma_2 + \sigma_3$
- $B_0 \geq \eta\sigma_2 + \sigma_3$

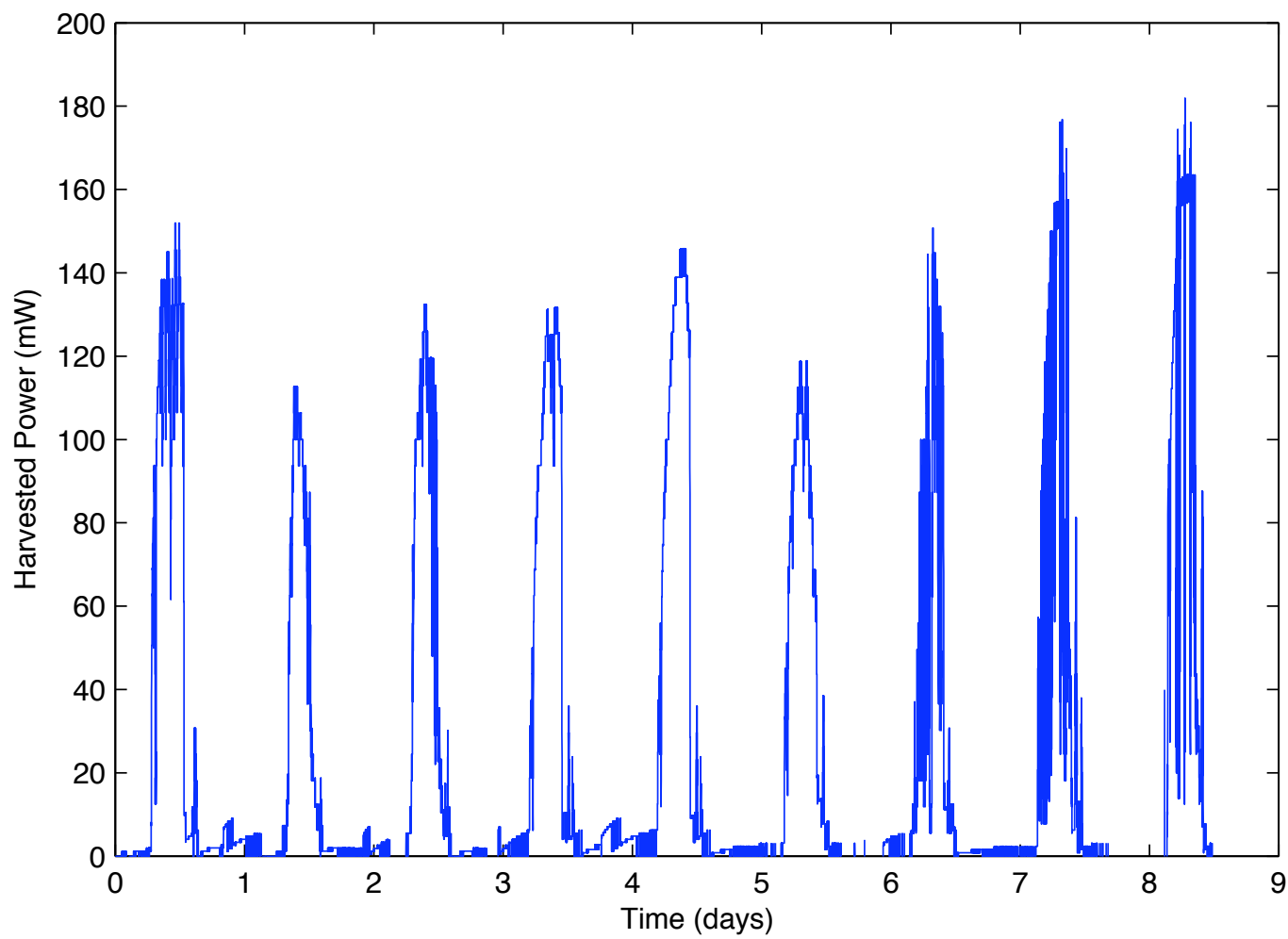


Fig. 2. Solar energy based charging power recorded for 9 days

Parameter	Value	Units
ρ_1	23.6	mW
σ_1	1.4639×10^3	J
σ_2	1.8566×10^3	J

Further Considerations

- ▶ **The behavior of energy sources can be learned**
 - As a result, the available energy can be calculated
 - The task can be adapted to the energy supply
- ▶ **Thereby**
 - Nodes with better energy situation can take over routing
 - Measurements can occur seldomer, but will never stop



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