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Algorithms for Radio Networks

MIMO

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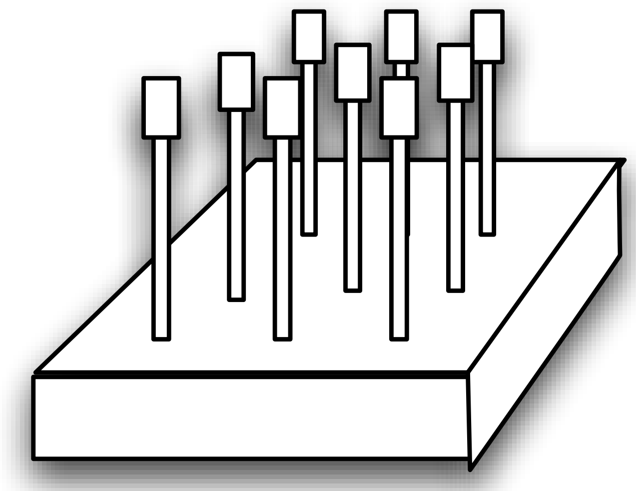
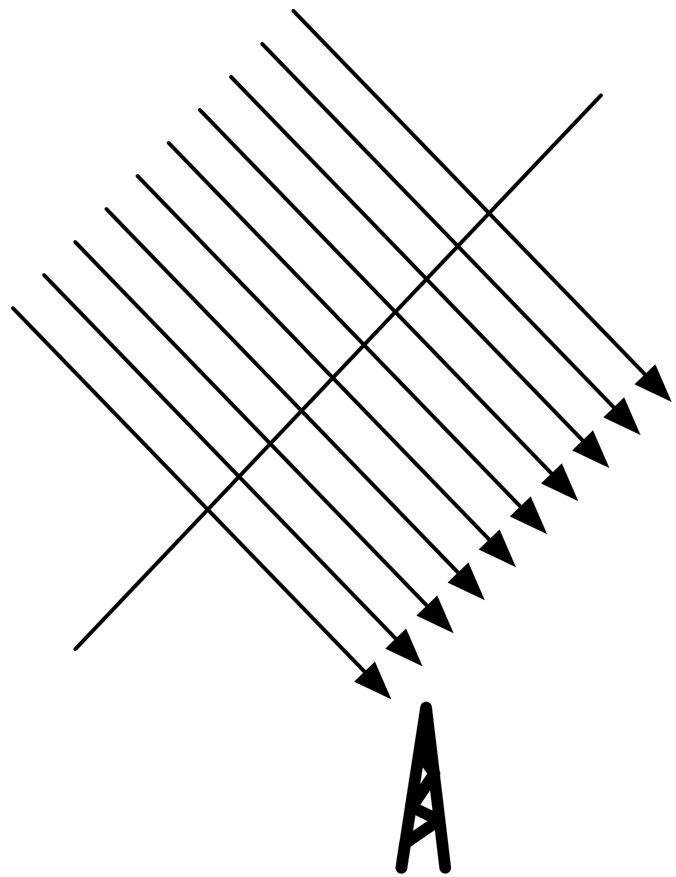
Smart Antennas

- **Alternative terms**
  - Adaptive Array Antennas
  - Multiple Input Multiple Output (MIMO)
- **Prinziple**
  - Multiple antennas are coordinated manner
    - used to improve reception or transmission of behavior
    - to allow additional features
- **Features**
  - Directional receivers
  - Directional senders
    - better path loss exponent
    - spatial multiplexing
    - MIMO communikation



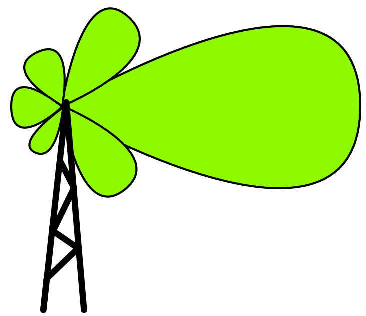
DOA Estimation

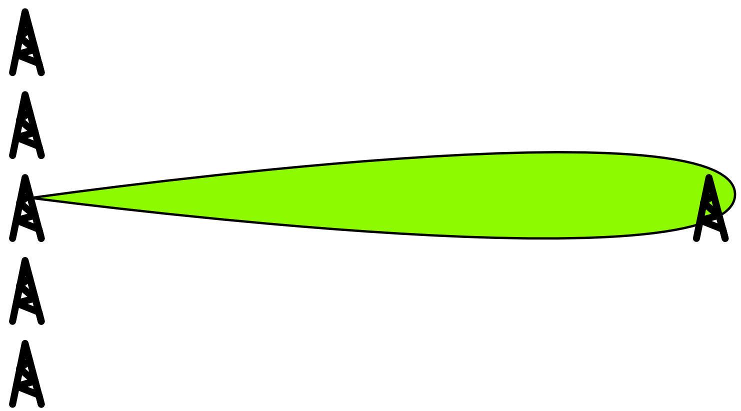
- **With two antennas, one can determine the receive direction (DOA)**
  - Paulraj, Roy, Kailath, Estimation of Signal Parameters via Rotational Invariance Techniques- ESPRIT, Nineteeth Asilomar Conference on Circuits, Systems and Computers, 1985, 83- 89
- **Idea:**
  - The signals arrive at different times to the antennas. By parallel testing of overlays can be candidates for the angle of incidence findenn



Beam forming

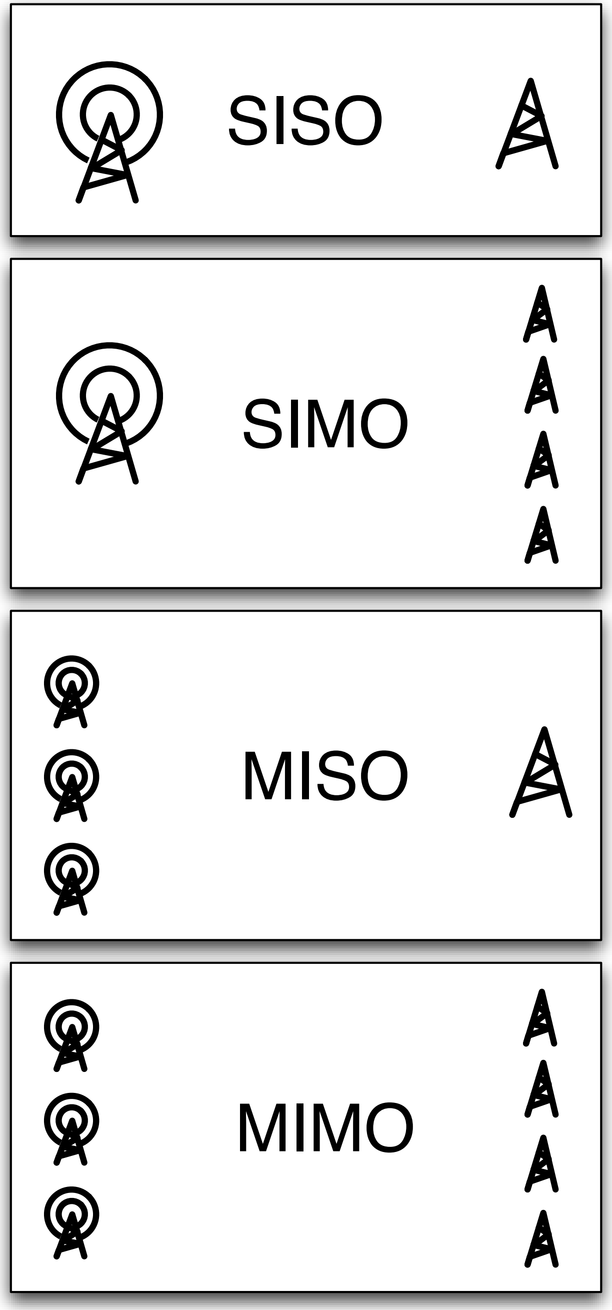
- **Simulation of receiving or transmitting antenna behavior of any of Smart Antennas**
- **Active**
  - By suitably chosen time shift, receipt of signals at the antennas will transmit the desired direction preference
    - Other directions only increase only background noise
  - Applications: radar, mobile communications, MIMO
- **Passive**
  - As with the DOA-detection, the signals are delayed and superimposed
  - Applications: Microphones, MIMO





Smart Antennas Combinations

- **SISO (Single Input Single Output)**
  - Classic radio model
- **SIMO (Single Input Multiple Output)**
  - Classical transmitter with an antenna
  - Antenna array at the receiver
  - Different channels can be received in parallel from different angles
- **MISO (Multiple Input Single Output)**
  - Antenna array as a transmitter
  - Individual recipients (groups) can be individually reached
- **MIMO (Multiple Input Multiple Output)**
  - Directed (and parallel) communication between the transmitter and receiver possible
  - Efficient utilization of the medium



Motivation for MIMO

- **Increase of SINR by**
  - more sender antennas
  - more receiver antennas
- **Multipaths**
  - are used for increasing the channel capacity
- **Capacity**
  - grows with the complexity of the environment
  - with the number of senders and receivers

MIMO Free Space Model

- **The message m is modulated as x(t) over a carrier**
  - i.e. $s(t) = x(t)\ e^{j2\pi f t}$
- **Electric field is described by the signal**
  - ~ force on charged particles
  - adds up (superposition)
  - decreases proportional to the distance
- **Power is proportional to the square of the electric field**

$$\mathrm{SINR} = \frac{\left| \sum_{\text{sender } i} \sum_{\text{receiver } k} s_i \cdot \frac{\mathrm{e}^{j|u_i - v_k|}}{|u_i - v_k|} \cdot g_k \right|^2}{\sum_{\text{receiver } k} |g_k|^2 \left(N + \sum_{\text{interference } i} \frac{P_i'}{|w_i - v_k|^2} \right)}$$

MIMO Free-Space SINR

amplitude & phase modification by

$$\mathrm{SINR} = \frac{\left| \sum_{\text{sender } i} \sum_{\text{receiver } k} \overset{\text{sender}}{s_i} \cdot \overset{\text{channel}}{\frac{e^{j|u_i - v_k|}}{|u_i - v_k|}} \cdot \overset{\text{receiver}}{g_k} \right|^2}{\sum_{\text{receiver } k} |g_k|^2 \left(N + \sum_{\text{interference } i} \frac{P'_i}{|w_i - v_k|^2} \right)}$$

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$$\mathrm{SINR} = \frac{|s \cdot \overset{\text{channel matrix}}{H} \cdot g|^2}{N' + I}$$

MIMO-SINR = SINR

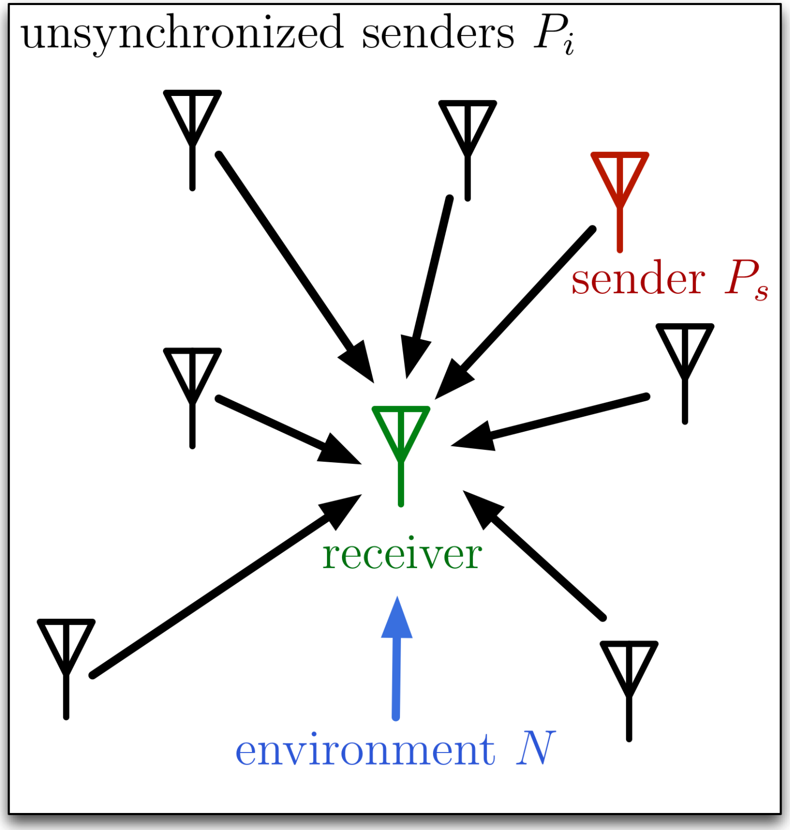
- SINR model adds the power of interferers

$$\frac{P_s}{N + \sum_{i \neq s} P_i} \geq \beta$$

- Superposition of principle (only) for electrical fields

$$P = \left(\sum_i E_i\right)^2$$

- Independent interferences



$$\mathbb{E}\left[\sum_{i \neq s} E_i\right]^2 = \mathbb{E}\left[\sum_{i \neq s} |E_i|^2\right] = \mathbb{E}\left[\sum_{i \neq s} P_i\right]$$

Power Gain

Communication with n sender and m receiver

- transmit power $P = \sum_i P_i = \text{const.}$

Signal power gain

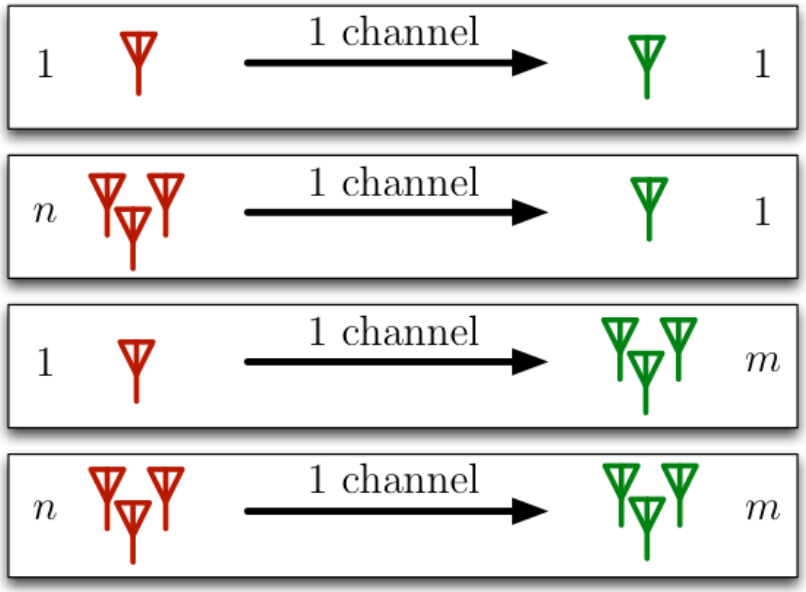
- SISO: $\text{SINR}_{1,1} = \frac{P}{N+I}$
- MISO: $\text{SINR}_{n,1} = n \cdot \text{SINR}_{1,1}$
- SIMO: $\text{SINR}_{1,m} = m \cdot \text{SINR}_{1,1}$
- MIMO: $\text{SINR}_{n,m} \leq n \cdot m \cdot \text{SINR}_{1,1}$
  (equality for $\text{rank}(H) = 1$)

d

very close

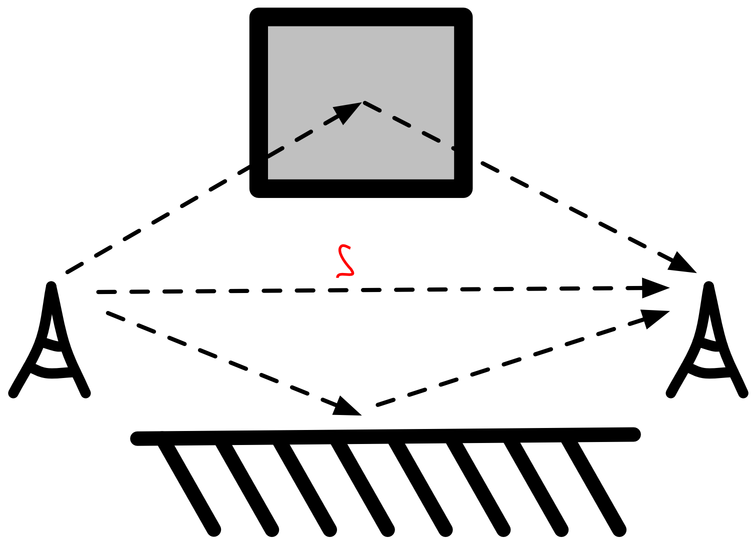
close

far



Multipath Channel

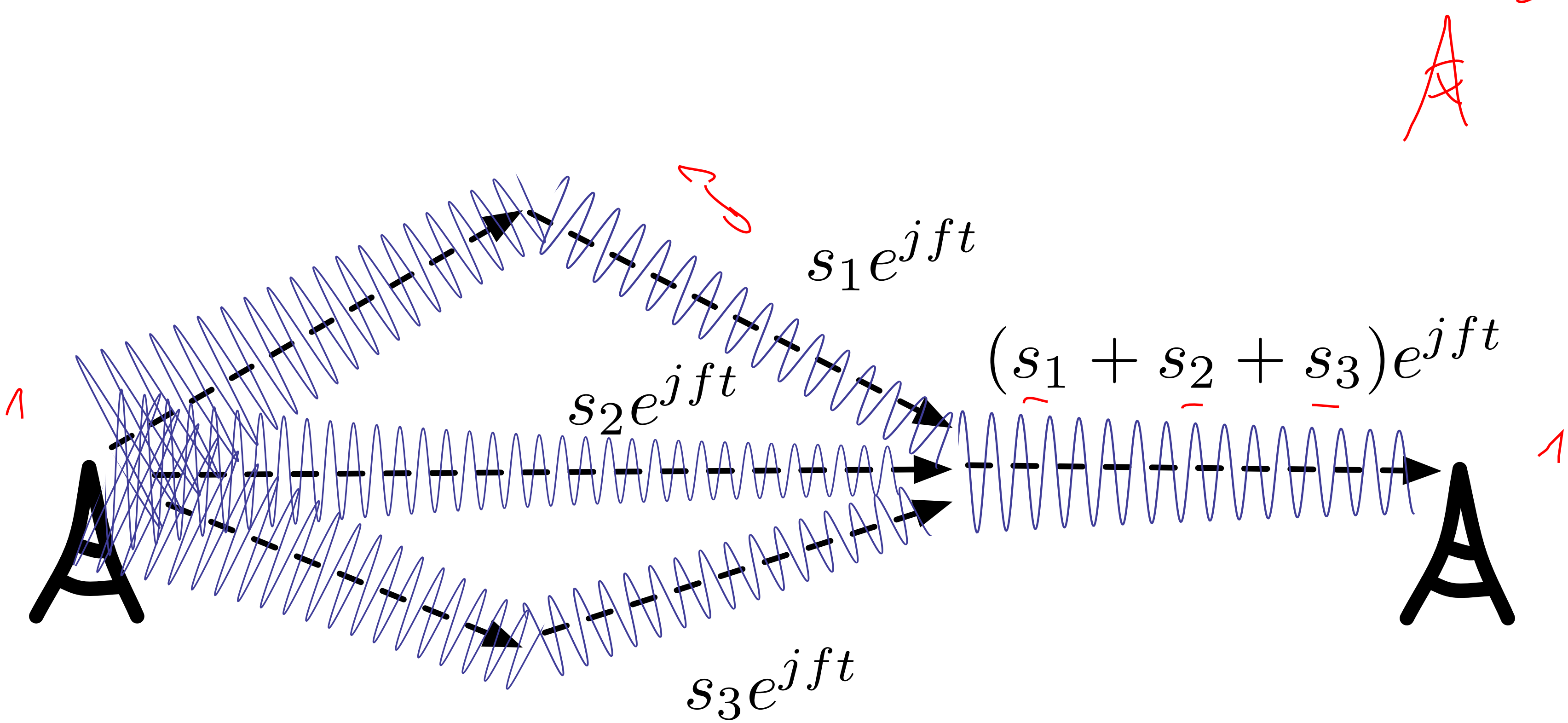
- **Signal is reflected from obstacles**



Multipath Channel

- **Signal is reflected from obstacles**

2

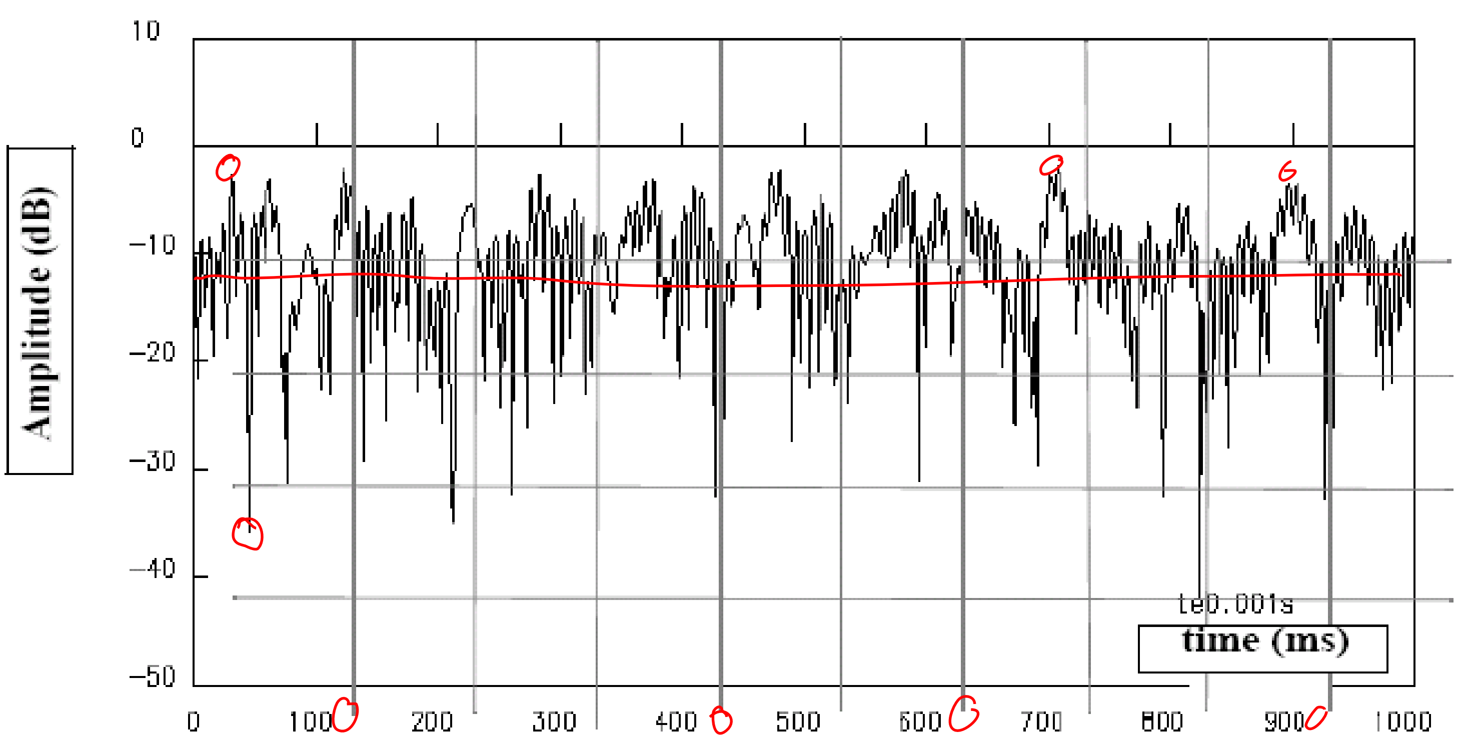


$$h = s_1 + s_2 + s_3$$

Multipath Channels

- **Level is sensitive to the locations**
  - SNR varies a lost

Rayleigh



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Introduction to Wireless MIMO – Theory and Applications
Jacob Sharony IEEE LI 2006

Simple View of MIMO Encoding/Decoding

- **3 x 3 MIMO system**
  - without noise

Sent

Receive

Environment

$$\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

H

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}^{-1} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

H

Diversity Gain

Problem: Noise

noise

- **Channel adds noise**

$$\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}$$

- **Noise will be also decoded**

$$H^{-1}(x + N) = H^{-1}Hb + H^{-1}N$$
$$= b + \boxed{H^{-1}N}$$

- **Noise can be amplified**
  - especially if Det[H] is small

can be large

$j = i$

Example

$$H = M = \begin{pmatrix} 0.9+0.1j & 0.9-0.1j & 0.4+0.2j \\ -0.2+0.3j & 1.+0.4j & 0.-0.2j \\ 1.8+0.25j & 1.9-0.2j & 0.8+0.4j \end{pmatrix}$$

- **|Det[M]| = 0.0142…**

$$H^{-1} = M^{-1} = \begin{pmatrix} 86.0+29.6j & 3.1-0.7j & -42.6-14.3j \\ -5.2-43.0j & -0.3-1.5j & 2.9+21.3j \\ -126.0+70.4j & -3.1+5.7j & 62.6-35.7j \end{pmatrix}$$

- **X = $\boxed{(1,-1,1)^T}$**
- **N = (0.01,-0.01,-0.01)^T**

$$M^{-1}(Mx+N) = \begin{pmatrix} 2.316+0.44i \\ -1.08-0.66i \\ -0.91+1.12i \end{pmatrix} ? \qquad \begin{matrix} 1 \\ -1 \\ 1 \end{matrix}$$

- **should be X!**

Solution: Precode the signal

- **Instead of using b as original inputs**

$$\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}$$

- **Start with x and (pre-) code x** $b = H^{-1}x$
  - **then the resulting signal including noise is**

$$Hb + N = H \cdot H^{-1}x + N = x + N$$

Example

$$H = M = \begin{pmatrix} 0.9+0.1j & 0.9-0.1j & 0.4+0.2j \\ -0.2+0.3j & 1.+0.4j & 0.-0.2j \\ 1.8+0.25j & 1.9-0.2j & 0.8+0.4j \end{pmatrix}$$

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- **x = (1,-1,1)^T**
- **N = (0.01,-0.01,-0.01)^T**

$$\boxed{b = \begin{pmatrix} 40.4+16.0i \\ -2.0-20.2i \\ -60.4+29.0i \end{pmatrix}} \leftarrow \cdot \frac{1}{70}$$

- **received signal**

$$Mb+N = \begin{pmatrix} 1.01 \\ -1.01 \\ 0.99 \end{pmatrix} \qquad \frac{x}{70}$$

$Hb + N$

Rectangular Channel Matrices

- **Pre-code the signal** $$b = H^{-1} x$$
  - What to do if H is not a square matrix?
  - i.e. more sender than receiver antennas
- **Use pseudo-inverse H+**

$$H^{+} = (H^* H)^{-1} H^*$$

  - where H* is the transposed complex conjugate of H
  - i.e. $H^* = \overline{H}^T$

$H^T = I$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

$$\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Channel Capacity by Diversity Gain

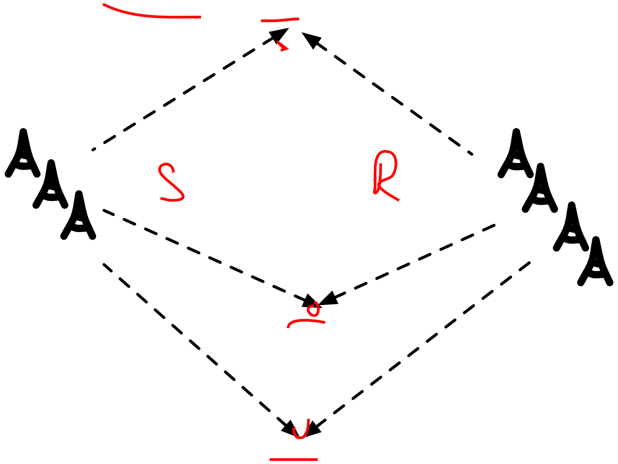
- **For maximum capacity it is necessary to know the Channel State Information (CSI)**
  - for this the receiver feedback is necessary
- **H^+ may have large entries**
  - this results in large amplification
- **Use singular value decomposition of H**
  - the maximum capacity can be computed by solving an optimization problem

→ water-filling method

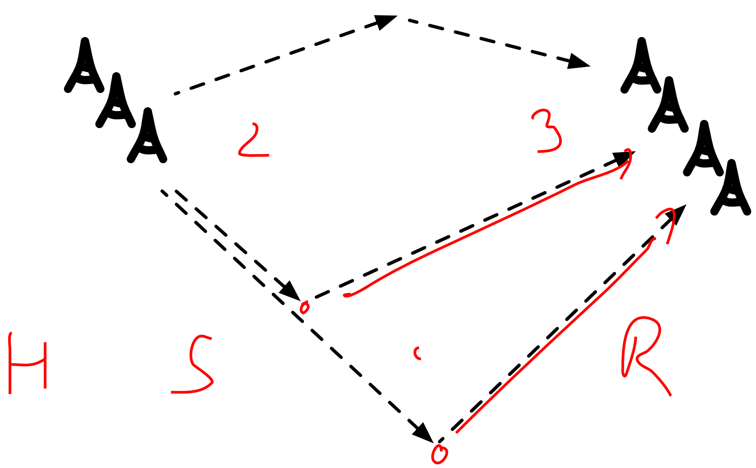
Maximum Diversity Gain

- **Given n sender antennas and m receiver antennas**
  - the maximum diversity gain is ~~max~~ min{n,m}
  - only if ~~max~~ min{n,m} reflections are in different angles from senders and receivers

three channels



two channels



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MIMO

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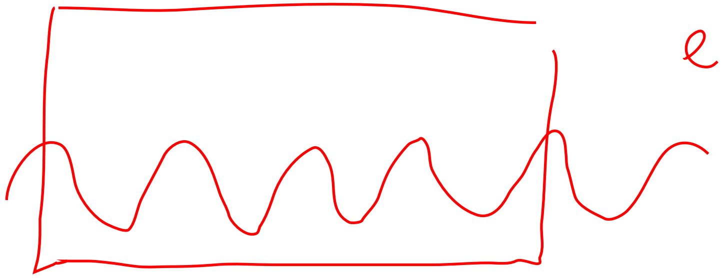
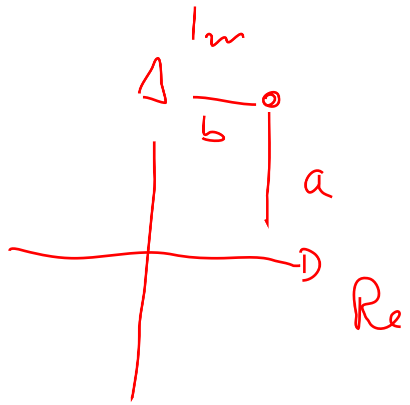
Signal



$e^{i \cdot f \cdot t}$

+ modulation

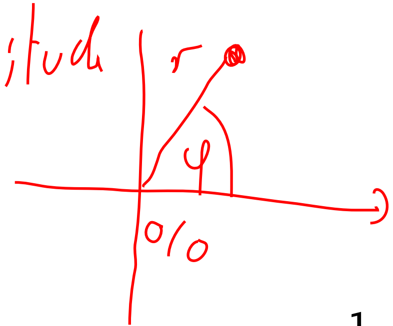
$S \in \mathbb{C}$



$S = a + bi$

$S = r \cdot e^{i \cdot \varphi}$

$r = \sqrt{a^2 + b^2}$:= amplitude



$\tan \varphi = \frac{a}{b}$;

φ : phaseshift

Power

$P \in \mathbb{R}_0^+$

→ noise

→ signal strength

Power

$P \sim r^2$

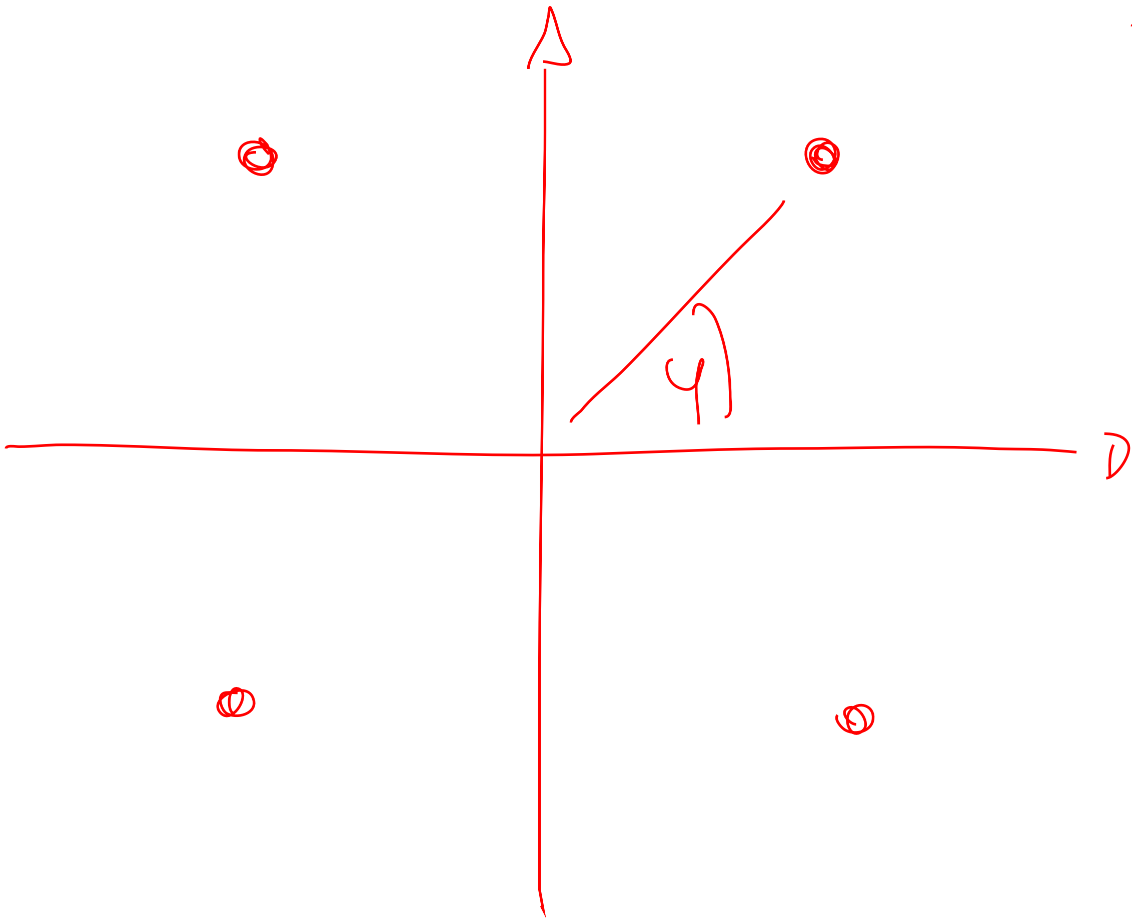
$= |S|^2$

$= a^2 + b^2$

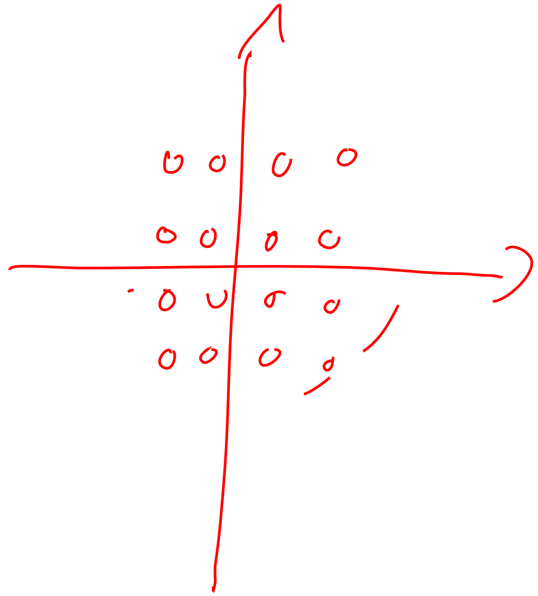
$= S \cdot \overline{S}$

S^*

QAM



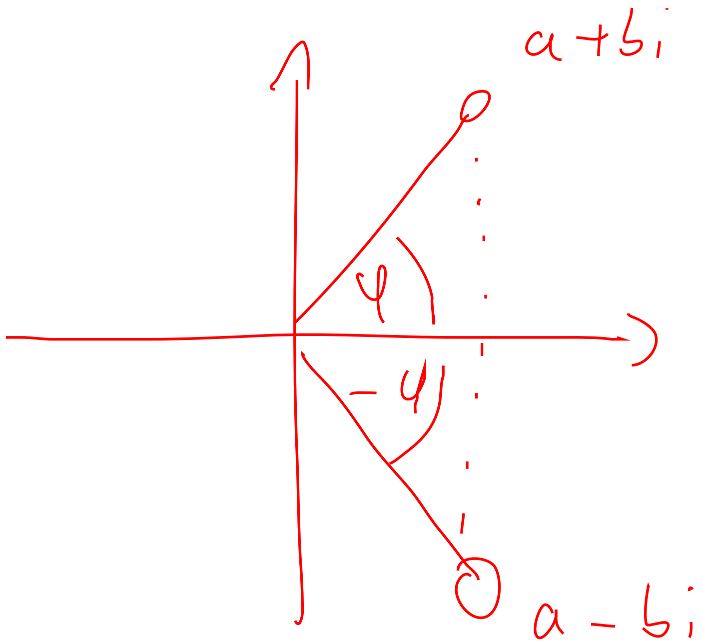
16-QAM



Complex Conjugate

$$s = a + bi \qquad s^* = \bar{s} = a - bi$$

$$s \cdot \bar{s} = (a+bi)\cdot(a-bi) = a^2 + b^2 = r^2$$



$$s = r \cdot e^{i\varphi} \qquad \bar{s} = s^* = r \cdot e^{-i\varphi}$$

$$s \cdot \bar{s} = r \cdot e^{i\varphi} \cdot r \cdot e^{-i\varphi}$$

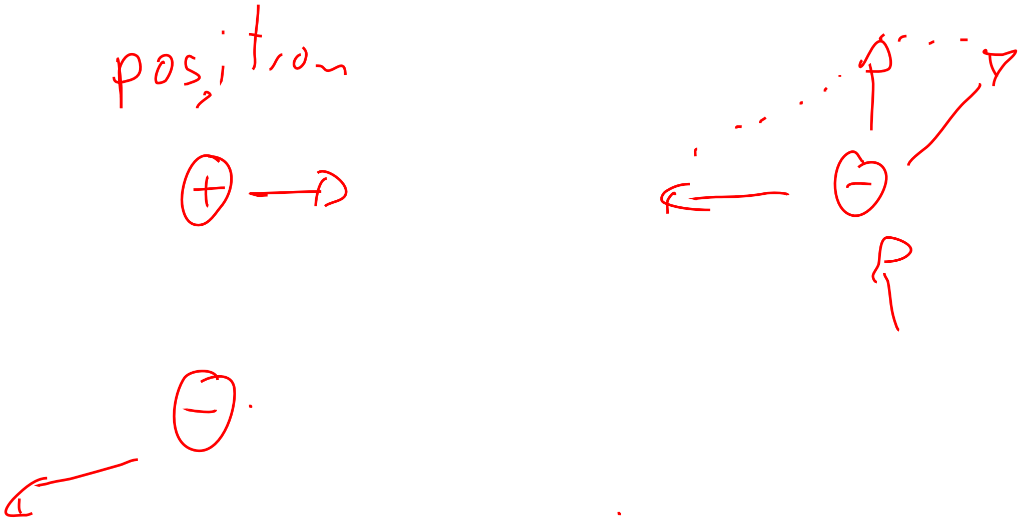
$$= r^2 \cdot e^{\underbrace{i\varphi - i\varphi}_{0}}$$

$$= r^2$$

Superposition

$\boxed{\text{Signal}}$ ~ electric field

↳ possibility to perform a force.



but noise ~ Power

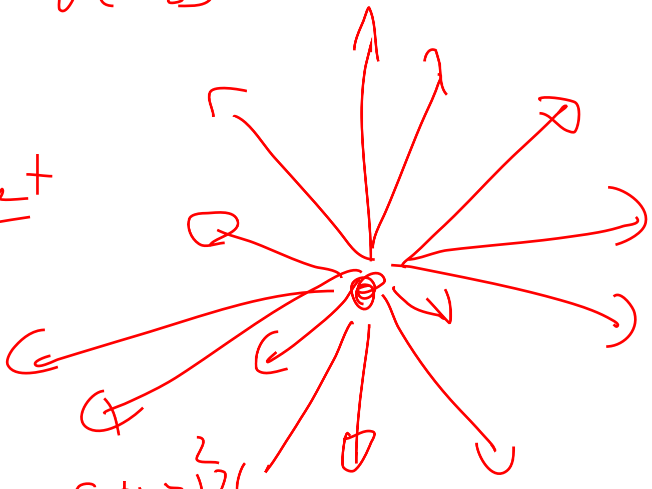
also adds up

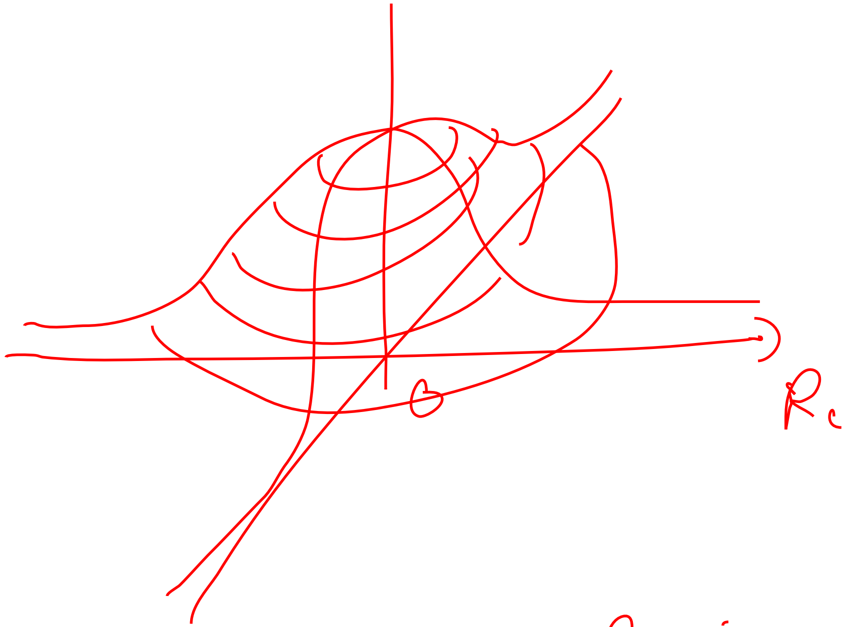
Noise

$$V[X_1 + X_2]$$

$$= V[X_1] + V[X_2]$$

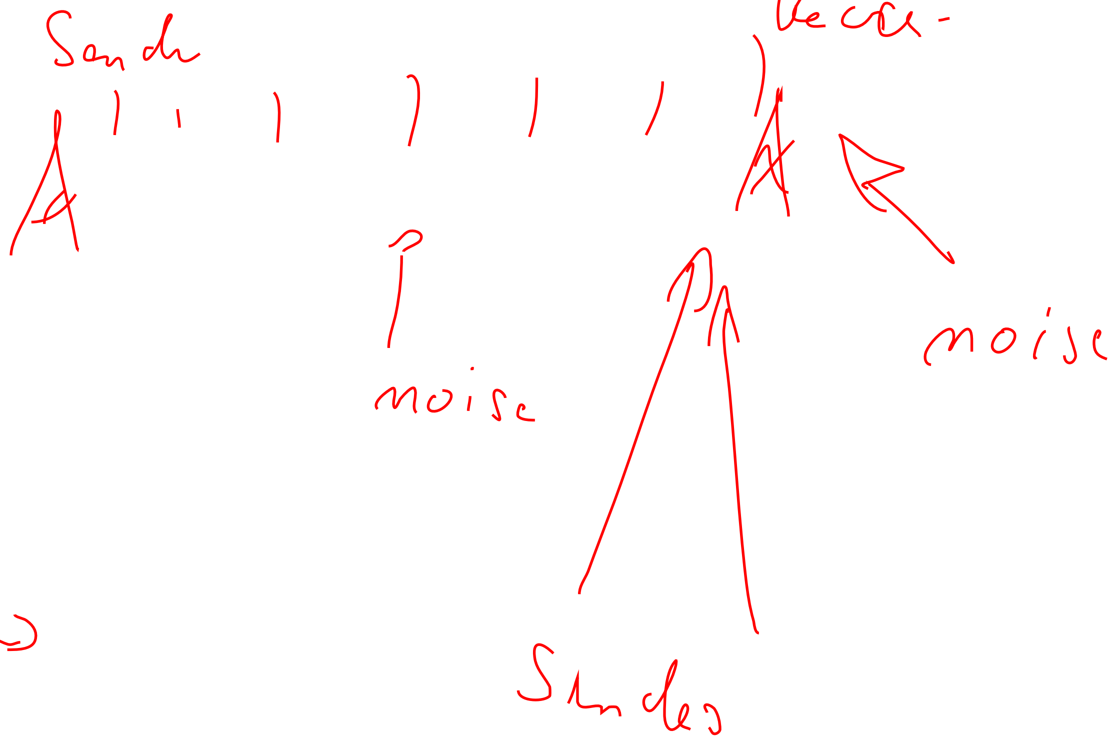
if X_1, X_2 are independent



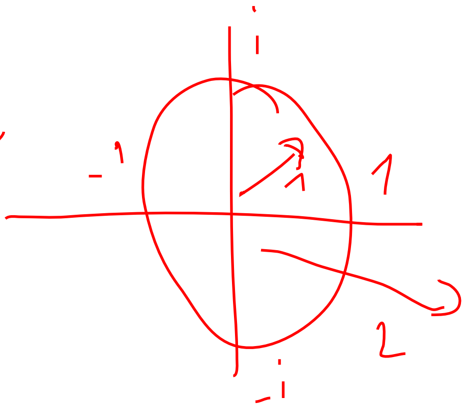


$$V[X] = E[(X - E[X])^2]$$

expected signal: 0



Variance ≈ Energy



n intervals $V[N]$

$\rightsquigarrow n \cdot V[N]$

1 Power: P

1 Sender

1 Receiver

d

$\rightsquigarrow$ Signal $\sim \sqrt{P}$

Signal: $\dfrac{\sqrt{P}}{d}$: Receive Power $\dfrac{P}{d^2} / N \geq \ldots$

$P \to p_i = \dfrac{P}{100}$

100 Senders

$\boxed{\text{in phase}}$

$\rightsquigarrow |\text{Signal}| \sim \dfrac{\sqrt{P}}{\sqrt{100}}$

1 Receiver

$|\text{Signal}| \sim \dfrac{\sqrt{P}}{d} \cdot \dfrac{100}{\sqrt{100}}$

$= \dfrac{\sqrt{P} \cdot \sqrt{100}}{d} \to \dfrac{P \cdot 100}{d^2}$

1 send Power P

1 receiver

d

Signal $\sqrt{P}$

received signal $\cdot \dfrac{\sqrt{P}}{d}$

100 Receivers

1 send

d

$\dfrac{\sqrt{P}}{d} + n_i$

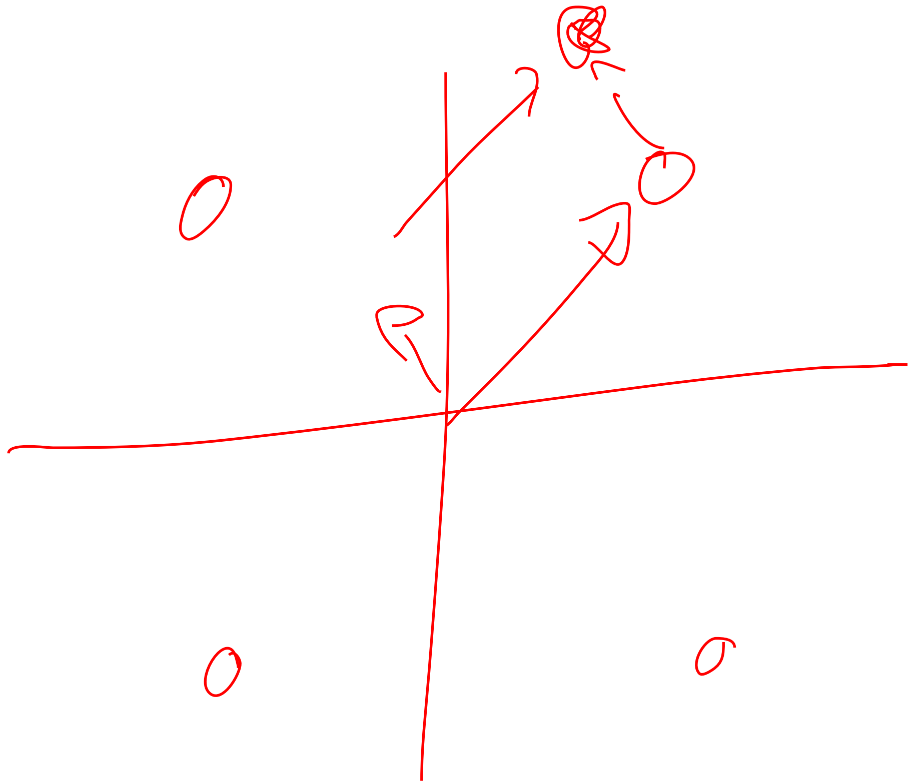
$+$

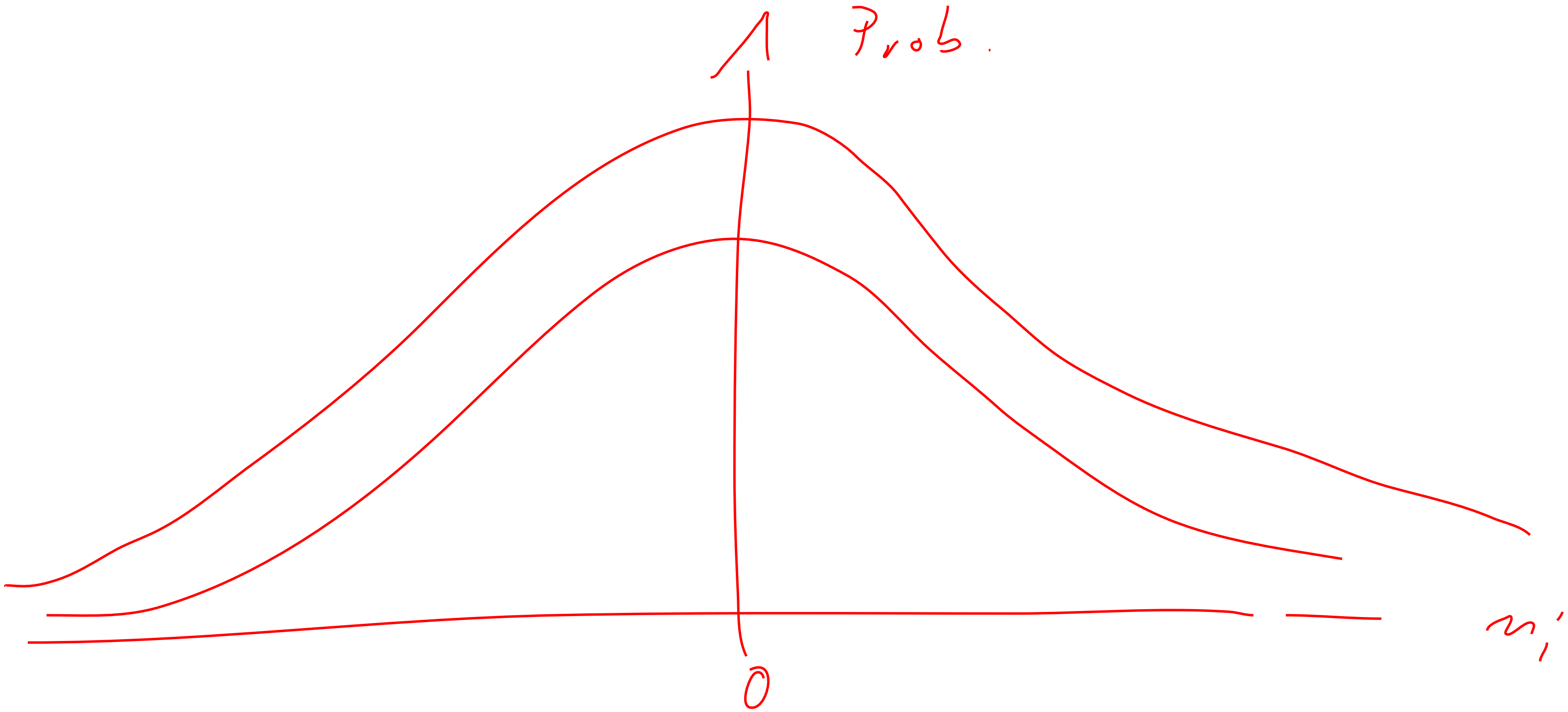
Sum of signals : $100 \dfrac{\sqrt{P}}{d} + \sum n_i$

Energy

$$\mathrm{Exp}\left(100 \cdot \frac{\sqrt{P}}{d} + \sum n_i\right)^2$$

$$100^2 \cdot \frac{P}{d^2} + 100N + \cancel{\sum_{i,j} n_i \cdot s_j}$$

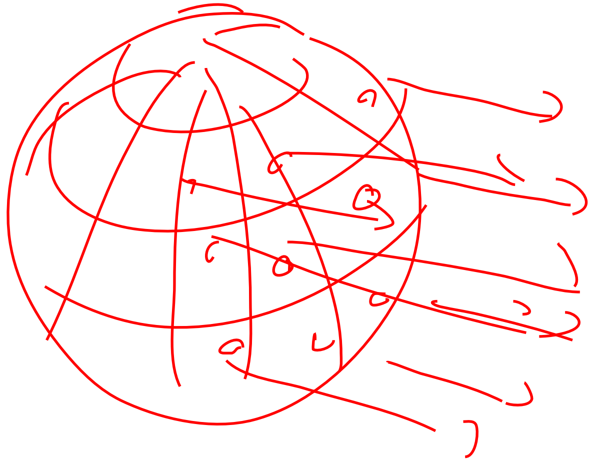


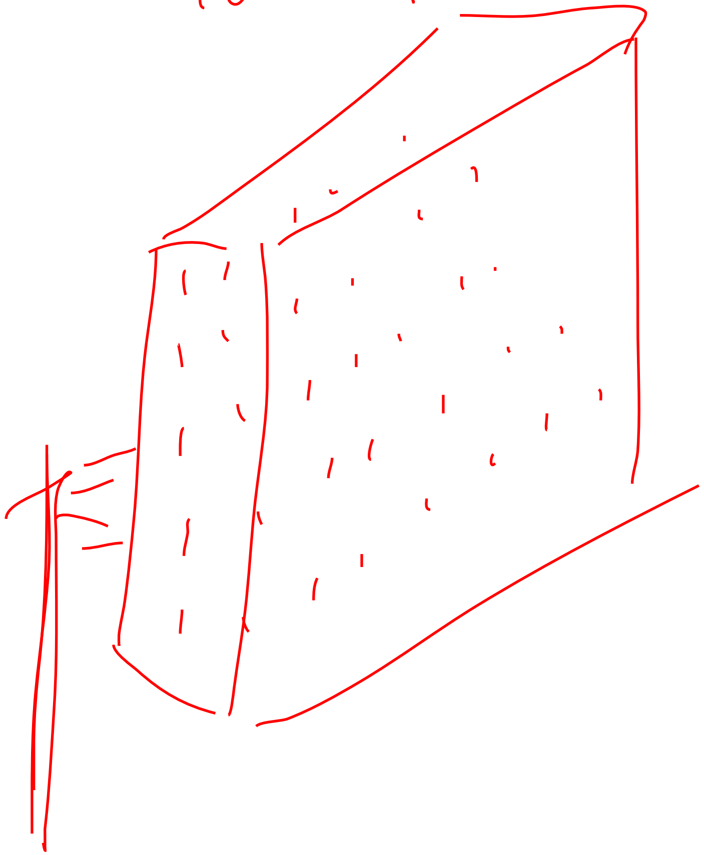


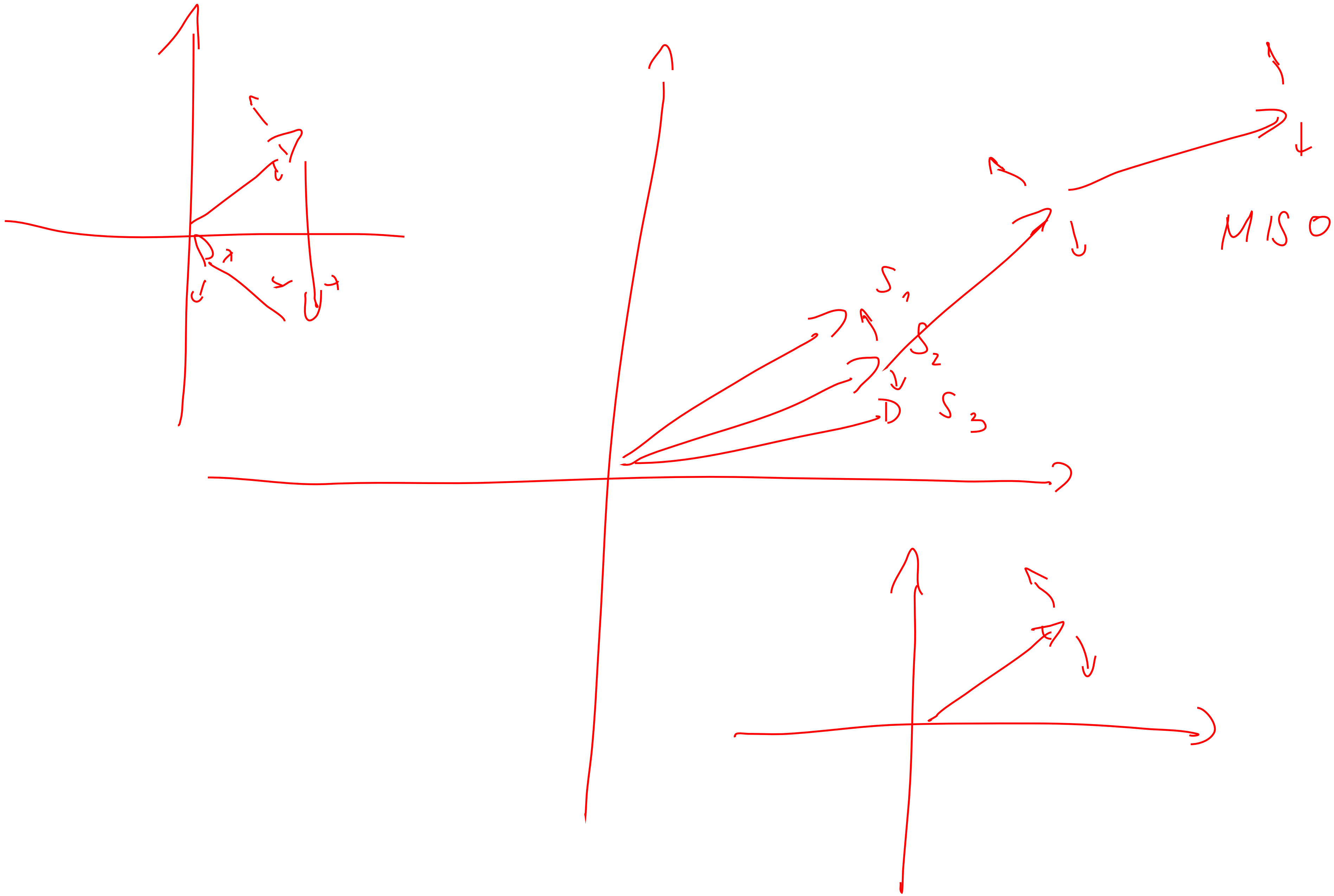
$$E[n_i] = 0$$

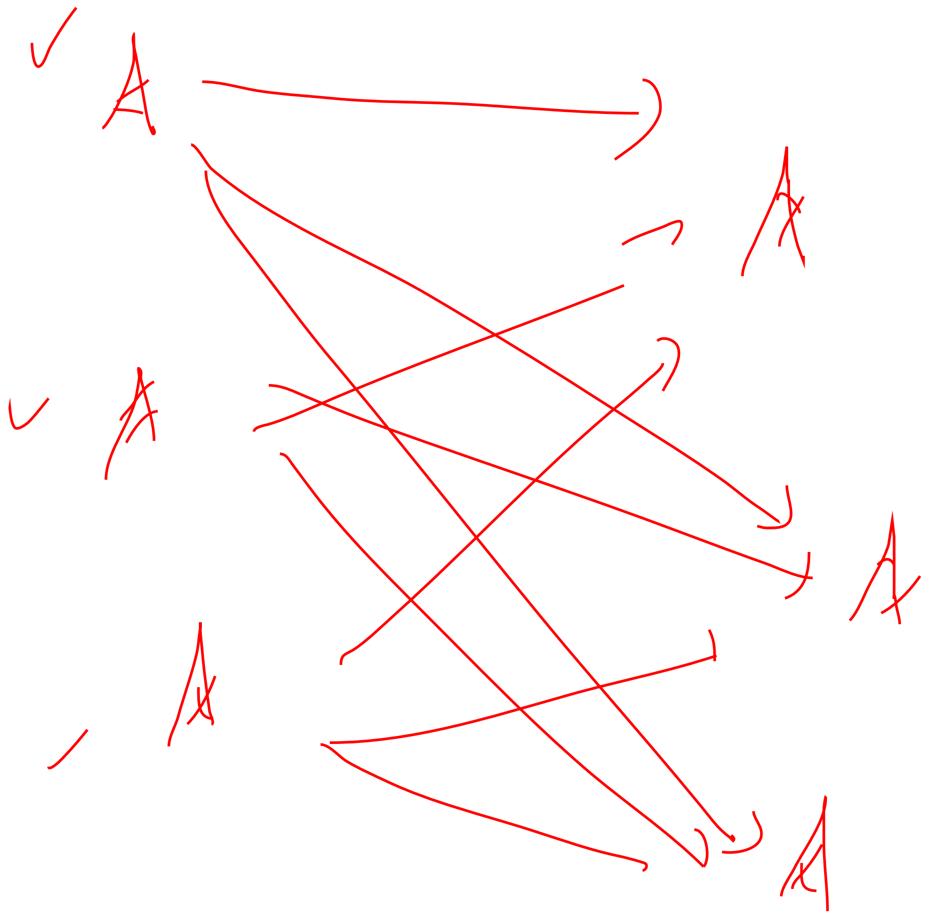
$$E[n_i \cdot s_j] = 0$$

$$SINR = \frac{100^2 \cdot P}{100 \cdot N} = 100 \cdot \frac{P}{N}$$

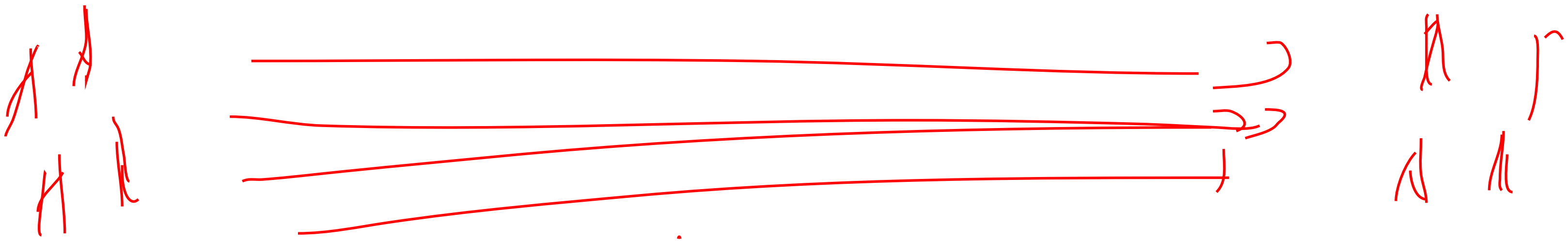




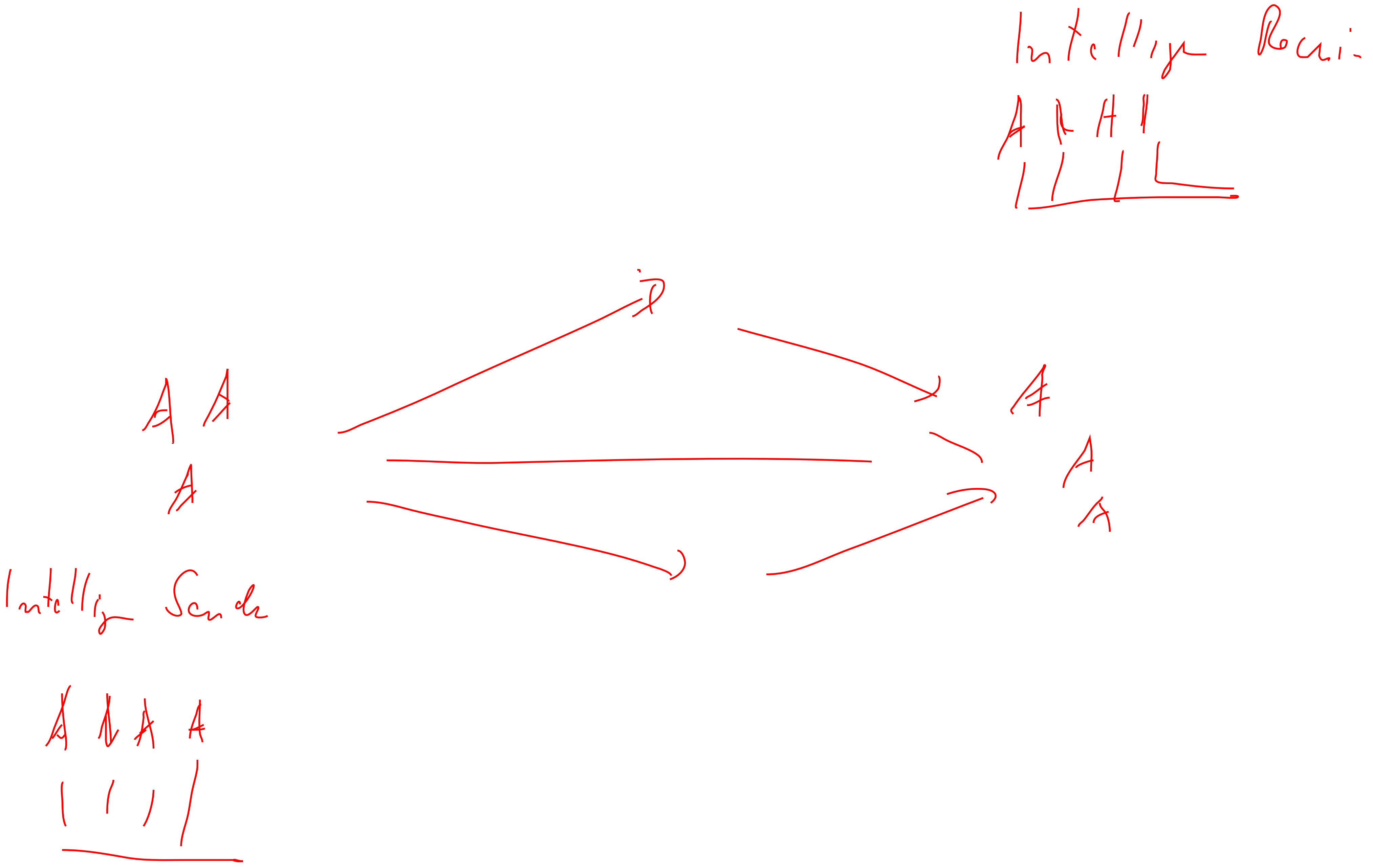

S1
S2
S3
D
MISO



channel matrix H



$$\begin{pmatrix} \frac{1}{d}e^{i\varphi} & \frac{1}{d}e^{i\varphi} & - & \cdot \\ \cdot & - & & \cdot \\ - & - & - & - \end{pmatrix}$$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