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UNIVERSITÄT FREIBURG

Algorithms for Radio Networks

Frequency Assignment

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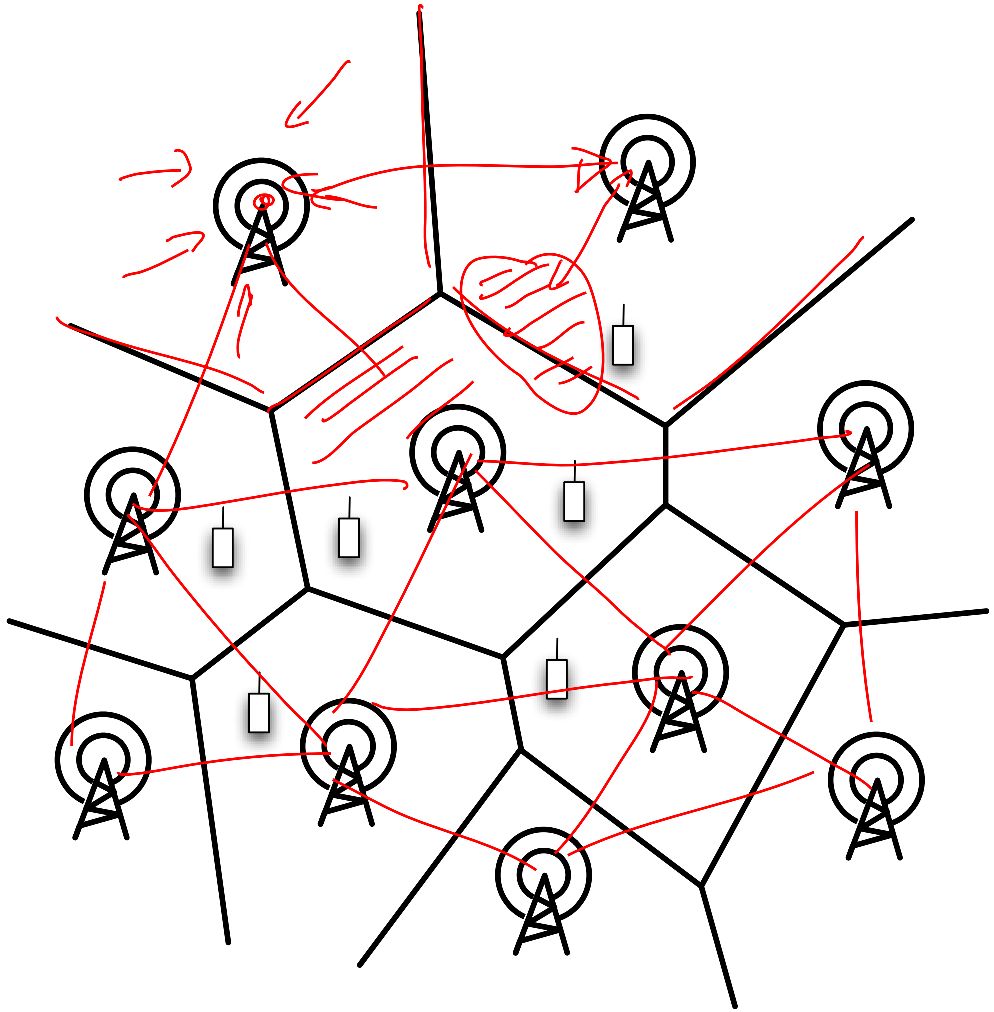
Cellular Networks

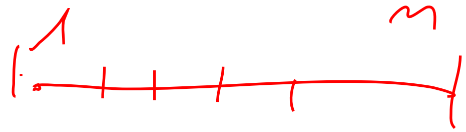
Voronoi-Diagramm

- **Original problem**
  - Rigid frequency multiplexing for a given set of base stations
- **Given**
  - positions of base stations
- **Output**
  - frequency assignment which minimizes the number of interferences
- **How to model acceptable frequency assignments?**



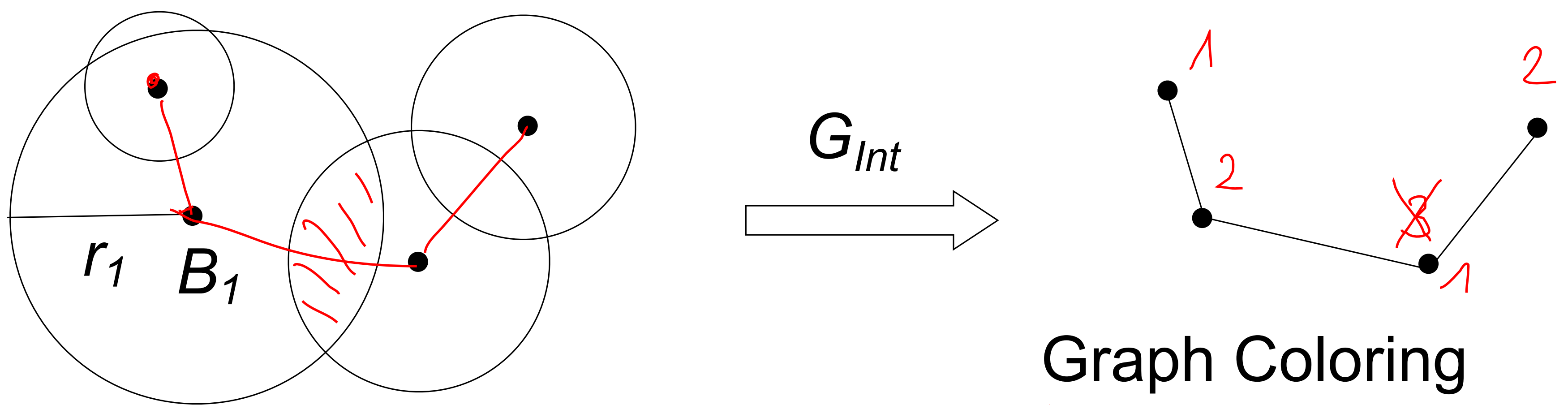
Frequency Assignment

- **Given:**
  - set of points $V \subseteq \mathbb{R}^2$ of n base stations $B_1, \ldots, B_n$
  - each base station covers an area
- **Output:**
  - function $f: V \rightarrow \mathbb{N}$, which maps each base station to a frequency respecting frequency and distance conditions
- **Sample restraints**
  - minimize the number of given frequencies
  - minimize the width of the frequency range
  - minimize the number of interferences



Frequency Assignment: Models

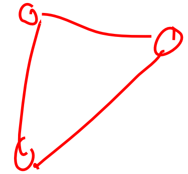
- **Interference graph G_{Int}:**
  - nodes are base stations
  - edges describe possible interferences between base stations



Graph Coloring

Graph Coloring

χ

G:  3

- **node k-coloring**
  - Given undirected graph G=(V,E)
  - A mapping f:V → F is a k-node coloring
    - if f(u) ≠ f(v) for {u,v} ∈ E and |F|=k.
- **chromatic number χ(G)**
  - is the minimum k to color graph G
- **clique number ω(G)**
  - is the largest number of nodes which form a complete sub-graph (clique) in G
- **Relationship of ω(G), χ(G) and the degree of the graph Δ(G)**
  - $\omega(G) \leq \chi(G) \leq \Delta(G) + 1$

Computational Complexity

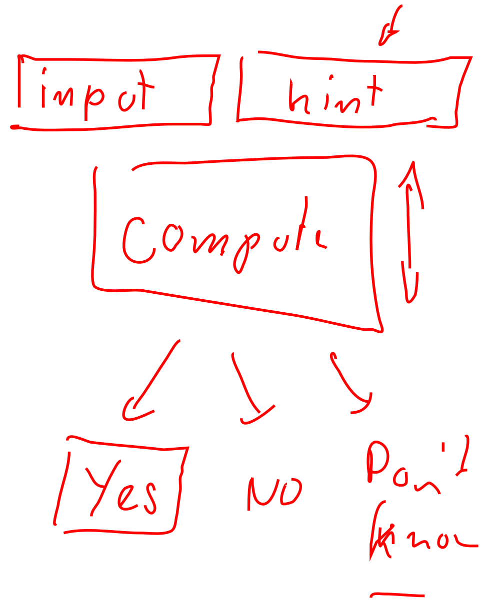
$NP \overset{?}{=} P$

P

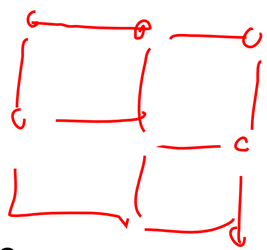
- **The degree can be easily seen from the graph description**
- **Clique number**
  - Computation ω(G) is NP-hard
  - Can be computed in time $O(n^{\omega(G)})$
- **Chromatic Number**
  - k-Coloring of a graph is NP-complete (if k≥3)
  - computation of the chromatic number is NP-hard
  - Can be computed in Zeit $O(\chi(G)^n)$

NP

n. non deterministic polynomial



3^n



Approximation Algorithms

- **Let P(I) be the solution of an optimization problem for instance I**
  - I=G [given undirected graph]
  - P(I) = χ(G) [chromatic number of G]
- **Definition:**
  - P can be absolutely approximated with additive term f(n), if there is a polynomial time bound bounde algorithm A such that for allinstances I of size n

$$| P(I) - A(I) | \leq f(n)$$

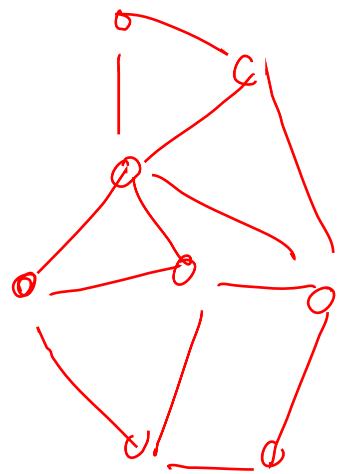
20 20 + n log n

  - P can be relatively approximated with factor g(n), if there is a polynomial time bound bounded algorithm A such that for all instances I of size n

$$\max\left\{\frac{P(I)}{A(I)}, \frac{A(I)}{P(I)}\right\} \leq g(n)$$

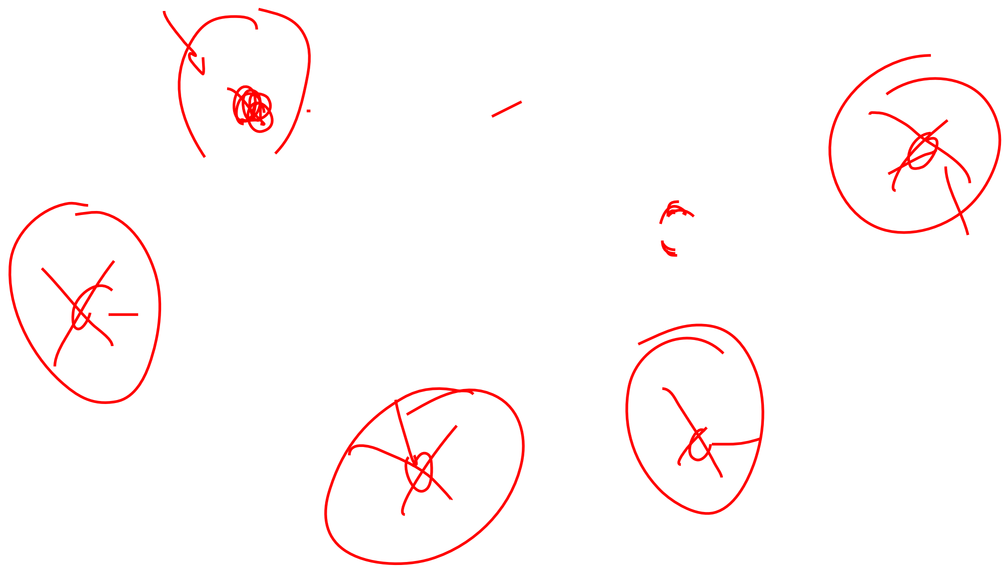
Results for Graph Coloring

- **Graph Coloring is NP-hard**
  - cannot be approximated by a factor of n^ε für $\varepsilon>0$ unless NP≠P.
- **„Can a given planar graph be colored with three colors“**
  - is NP-complete
- **But:**
  - Every planar graph can be colored with four colors in polynomial time
  - Every graph can be colored (if possible) with two colors in polynomial time
  - There is an absolute approximation algorithm with quality O(n/log n) for the general coloring problem



Approximation Algorithm for Node Coloring

- **Independent Set Problem (NP complete):**
  - Let $G=(V,E)$ be a graph and $U \subseteq V$.
    - U is **independent**, if: $\{u,v\} \notin E$ für alle $u,v \in U$
  - Independent set problem
    - compute a maximum set



~~4 nodes~~

5 nodes

Approximation Algorithm for ~~Node Coloring~~ IS

- **Algorithmus GreedyIS:**

  $U=\emptyset$, $G=(V,E)$

  while V not empty **do**

  Create graph with nodes V

  Choose nodes u with minimal degree

  Erase u and all neighbors of u in G from V

  Insert u into U

  od

  Return U

- **GreedyIS**
  - computes a maximal (non extendable) independent set
  - run-time $O(|V|+|E|)$

Approximation Algorithm for Node Coloring

- **Algorithm GreedyCol:**

  G=(*V*,*E*), Color =1;

  while *V* not empty **do**

  Create *G* from *V* and determine *U* with GreedyIS(*G*)

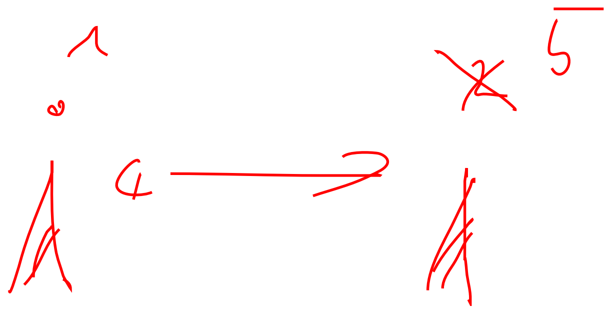
  Color all nodes in *U* with Color

  Remove *U* from *V* and increment Color // Color++

  od

  Return node coloring
- **GreedyCol computes in polynomial time a node coloring with O(n/log n) colors**
  - There are better approximation algorithms

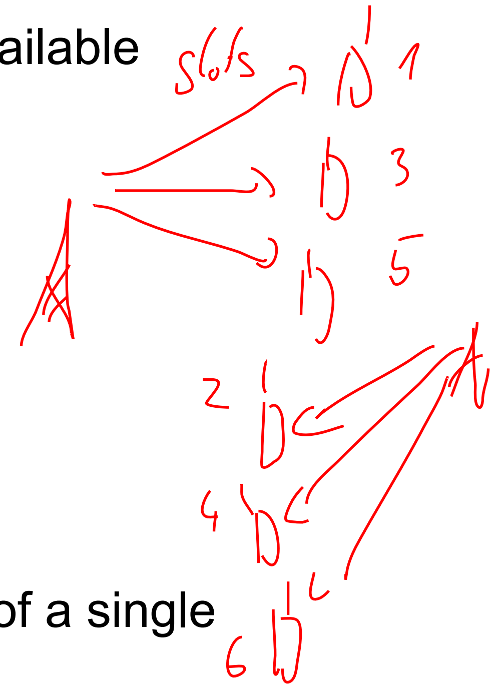
Models



- **Color model**
  - Neighbored cells have different frequencies
  - Leads to node coloring of the interference graph
- **Advantage**
  - Simple model
- **Disadvantage**
  - No efficient algorithms are known for Coloring
  - Not an adequate model
    - relationship of received signal strength and influence of neighbored frequencies is not reflected by the model

Labeling versus Coloring

- **Coloring**
  - Use of reusable frequencies
  - Minimize the total number of colors = frequencies available with minimum frequency distances
- **Labelling**
  - Each frequency is assigned only once
  - Frequency distances must be complied
  - Minimize used spectrum
- **Set-(Coloring/Labeling)**
  - A set of frequencies is assigned to a station instead of a single frequency
- **Distance function d of the interference graph**



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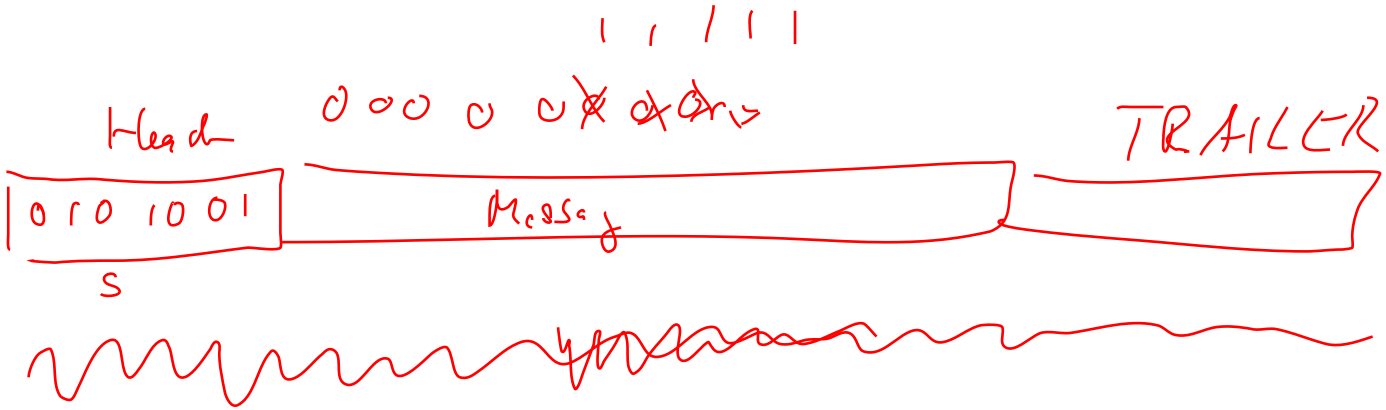
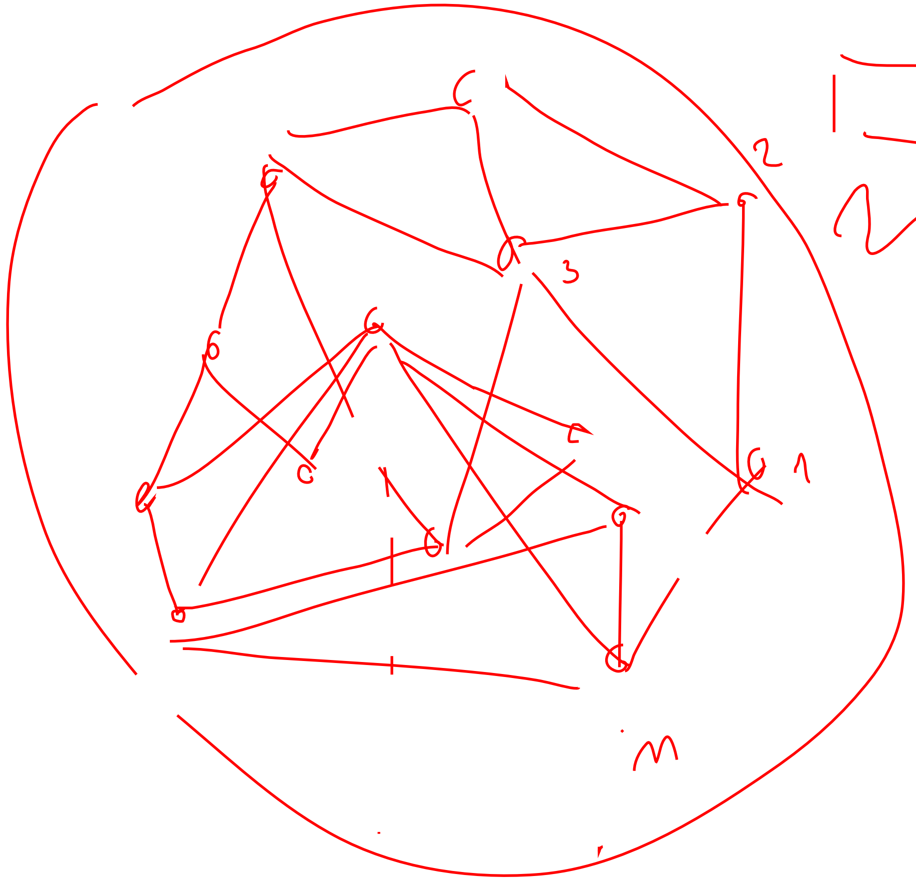
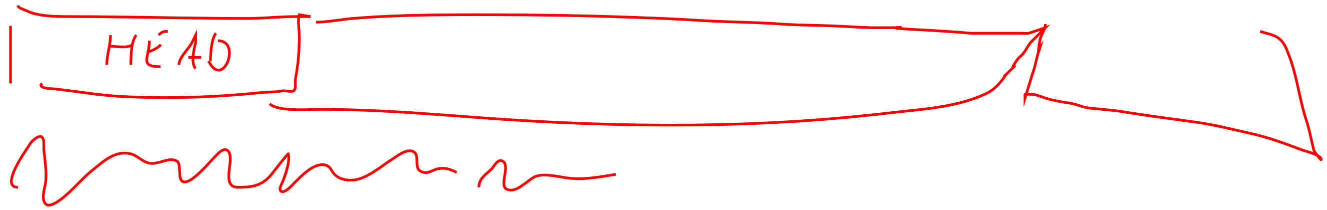
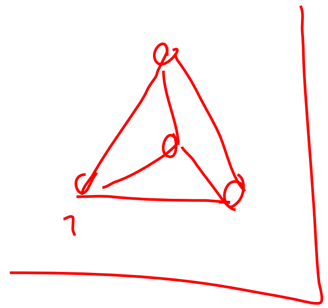








$$n(n-1)(n-2)(n-3) \le n^4$$

$$\sum n^1 + n^2 + \dots + n^{n/2}$$

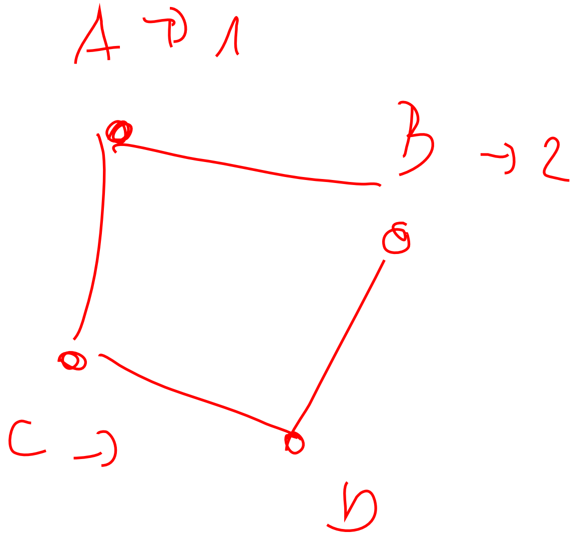
$$n^{n/2} = 2^{n \cdot \frac{\log n}{2}} \geq 2^n$$

undirected

graph: $G = (V, E)$

vertices   edges

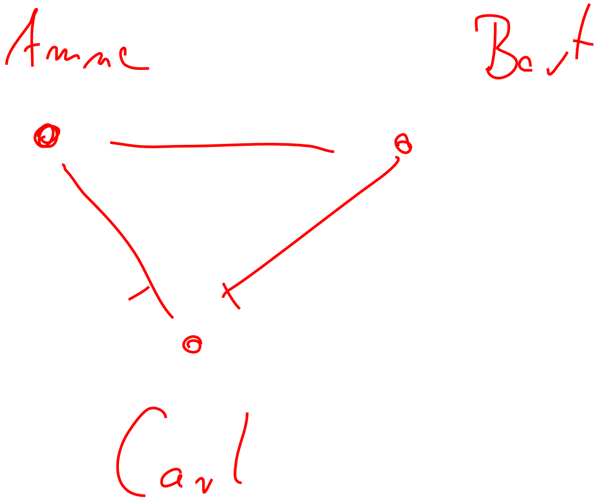
$E := \{\{u,v\} \mid u,v \in V\}$



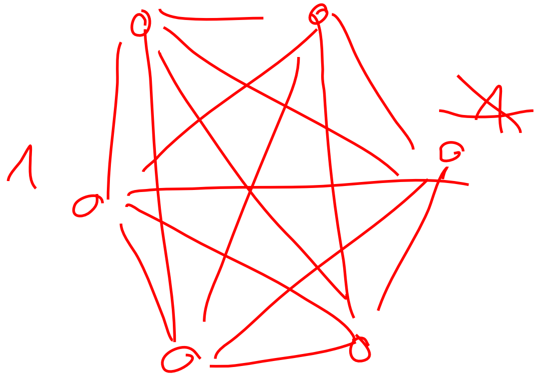
$V = \{A, B, C, D\}$

$E = \{\{A,B\}, \{A,C\}, \{B,D\}, \{C,D\}\}$

Clique



$\boxed{K_6}$



$\omega(K_6) = 6$

$\chi(K_6) = 6$

$$|E(K_6)| = \overset{15}{\cancel{30}} = \frac{6 \cdot 5}{2}$$

$$\delta(v) := |\{\{v,w\} ; \{v,w\} \in E\}|$$

$$\Delta(G) = \max_{v \in V} \delta(v)$$

$4 = 2$

$\chi(G) \leq \Delta(G) + 1$