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Algorithms for Radio Networks

Wireless Sensor Networks: Sensor Coverage & Lifetime

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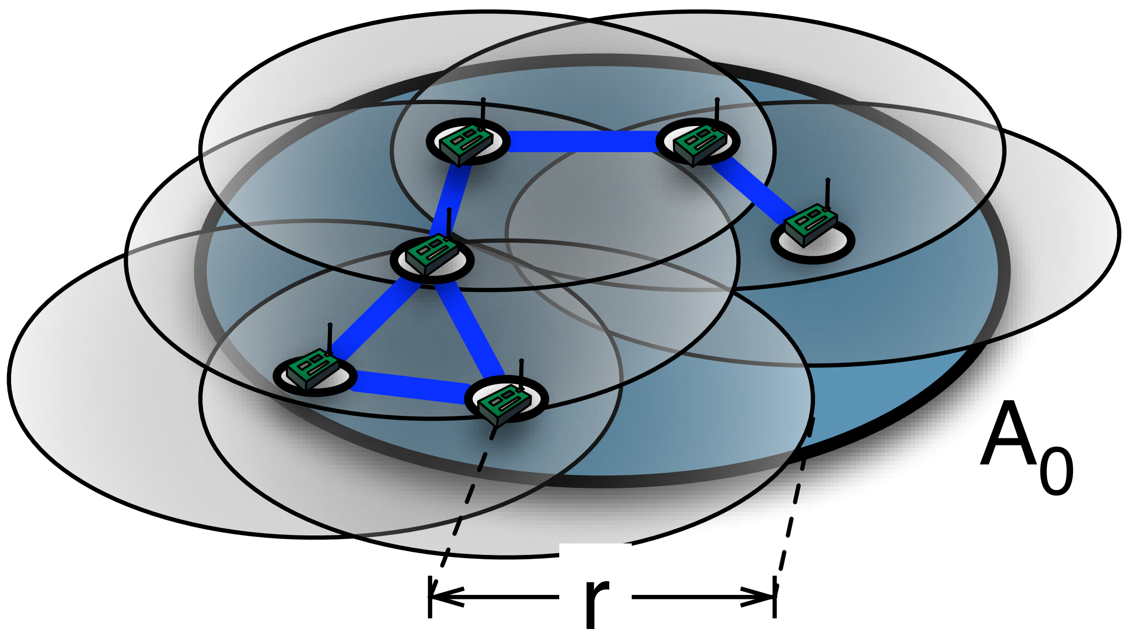
Literature

- **Handbook on Theoretical and Algorithmic Aspects of Sensor, Ad Hoc Wireless and Peer-to-Peer-Networks (Editor: Jie Wu)**
  - Chapter 27: Models and Algorithms for Coverage Problems in Wireless Sensor Networks

Sensor Coverage

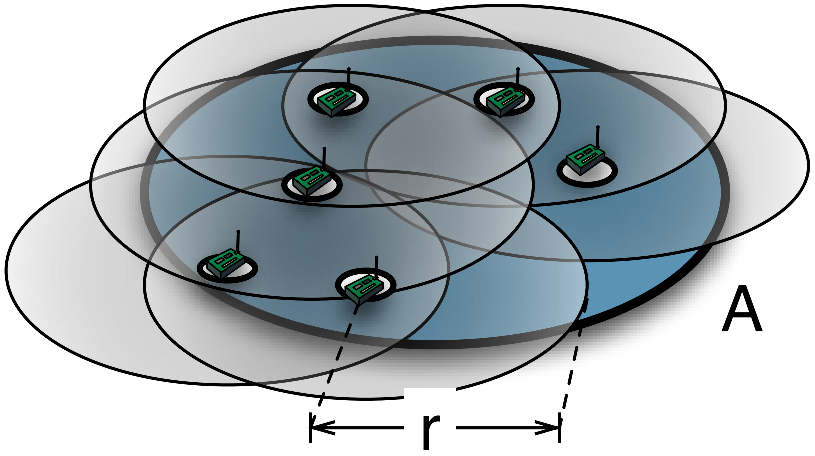


Sensor Coverage

- **Problem**
  - Given an area
  - Cover the area with the smallest possible number of sensor nodes
- **Variants**
  - Circle Covering
    - 2-dimensional surface, sensor coverage is given by circles
  - Art Gallery Problem
    - Angled rooms: Sensor coverage and line of sight angle
      - e.g. camera surveillance
  - Arbitrarily complexer variants conceivable

Random Circle Covering

- **Naive approach**
  - Given a square of area A
  - How many randomly positioned the sensors with unit disk cover the square?
- **Naive calculation**
  - Area of the unit circle: $r^2\pi$
  - Number of sensors required: $n = A / (r^2\pi)$
- **Intuition**
  - $O(A/r^2)$ should be sufficient
- **But: intuition is wrong!**



Random Circle Covering

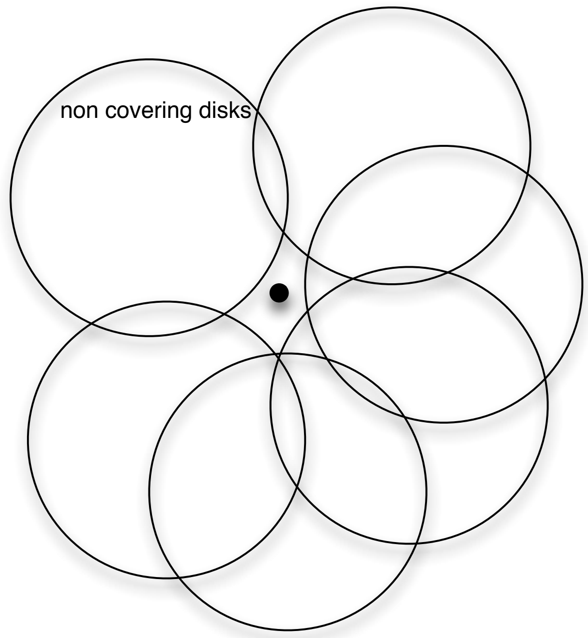
- **Naive approach**
  - Given a square of area A
  - How many randomly positioned sensors with unit disk cover the square?
- **Theorem**
  - Let $n = A / (r^2\pi)$
    - where A denotes the area of the square
    - and r denotes the sensor radius
  - To cover such a square of Θ (n log n) randomly placed sensors are necessary and sufficient

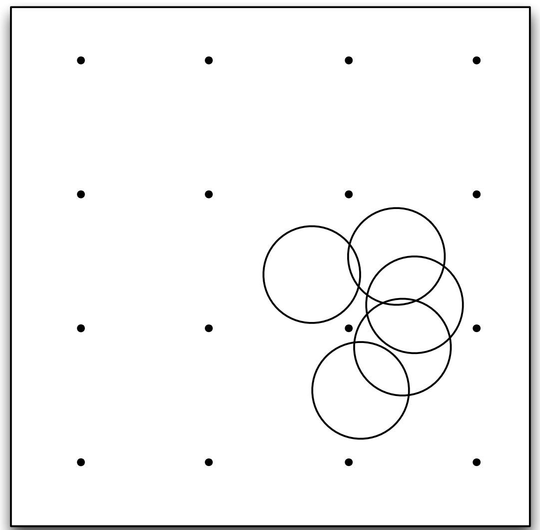
Random Circle Covering

- **Theorem**
  - Let $n = A / (r^2\pi)$
    - where A denotes the area of the square
    - and r denotes the sensor radius
  - To cover such a square of Θ (n log n) randomly placed sensors are necessary and sufficient
- **Proof sketch (lower bound):**
  - The probability that a given point is not covered by a sensor is at least
    $1-r^2\pi/A = 1- 1/n$
  - Consider n such points with distance r
  - The probability that at least 1/ n log n sensors do **not** cover one of these point is therefore

$$\left(1-\frac{1}{n}\right)^{\frac{1}{2}n\log n} \geq \left(\frac{1}{4}\right)^{\frac{1}{2}\log n} = \frac{1}{n}$$

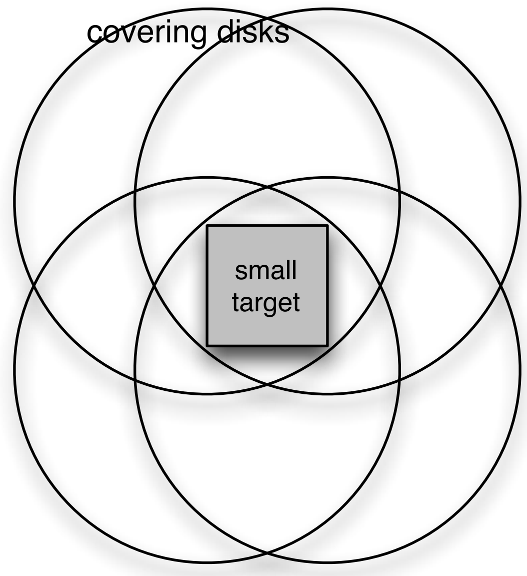
  - Hence, the expected number of uncovered points is 1

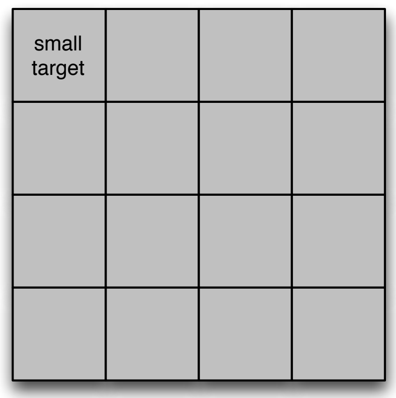




Random Circle Covering

- **Theorem**
  - Let $n = A / (r^2\pi)$
    - where A denotes the area of the square
    - and r denotes the sensor radius
  - To cover such a square of $\Theta(n \log n)$ randomly placed sensors are necessary and sufficient
- **Proof sketch (upper bound):**
  - By $c\, n \log n$ random sensors every square of size $r/3 \times r/3$ is covered with probability $1-n^{-k}$
    - where k grows linear with c
  - Then the whole square is covered with probability $1-n^{-k-1}$

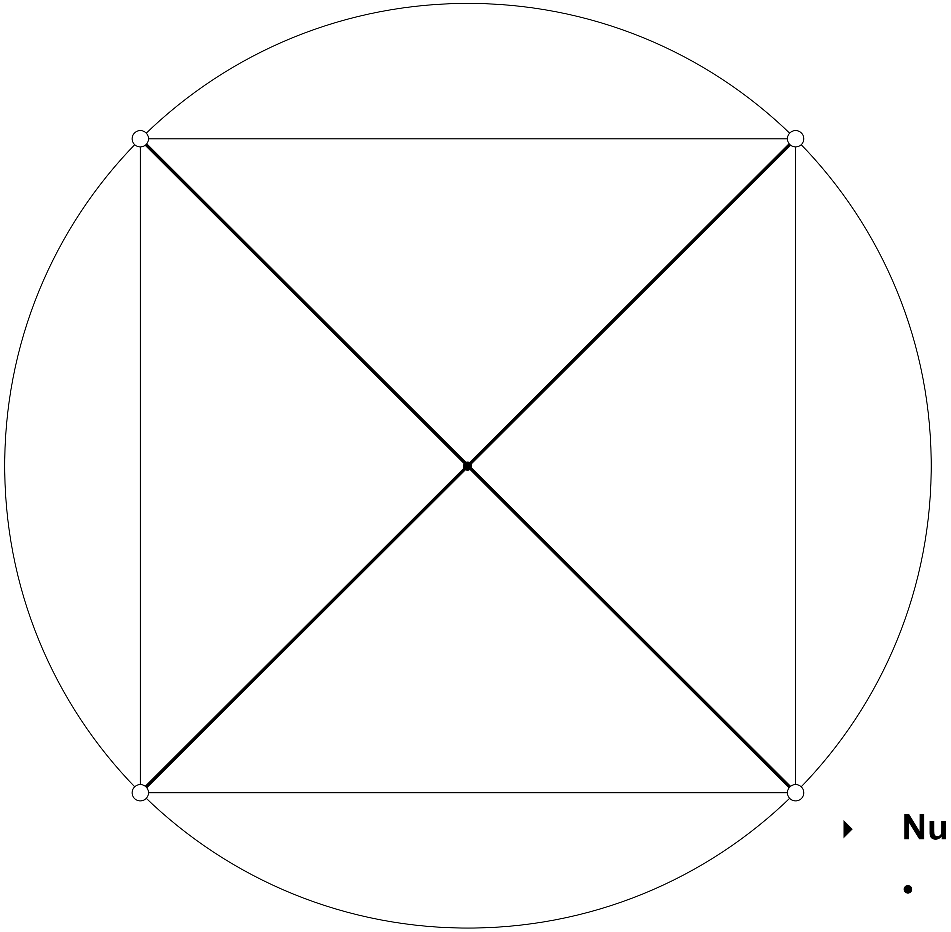




Optimal Deterministic Bound

- **Nurmela, Östergard**
  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteorian laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)
- **How many circles can cover a square?**
  - A closed form solution is unknown
  - However for a small number of circles the problem can be solved by exhaustive search

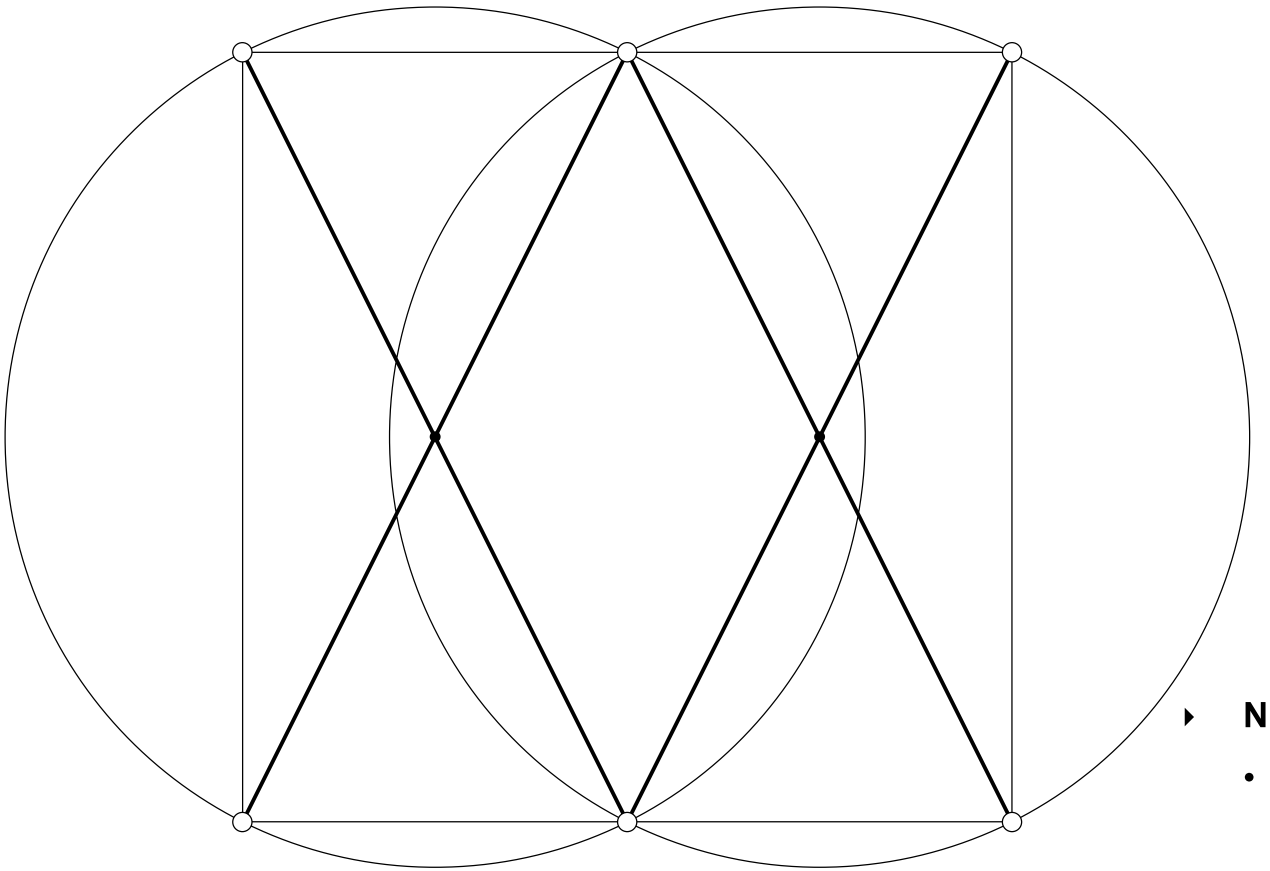
Disk Coverage of a Square



$$n = 1$$

- **Nurmela, Östergard**
  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteoria laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

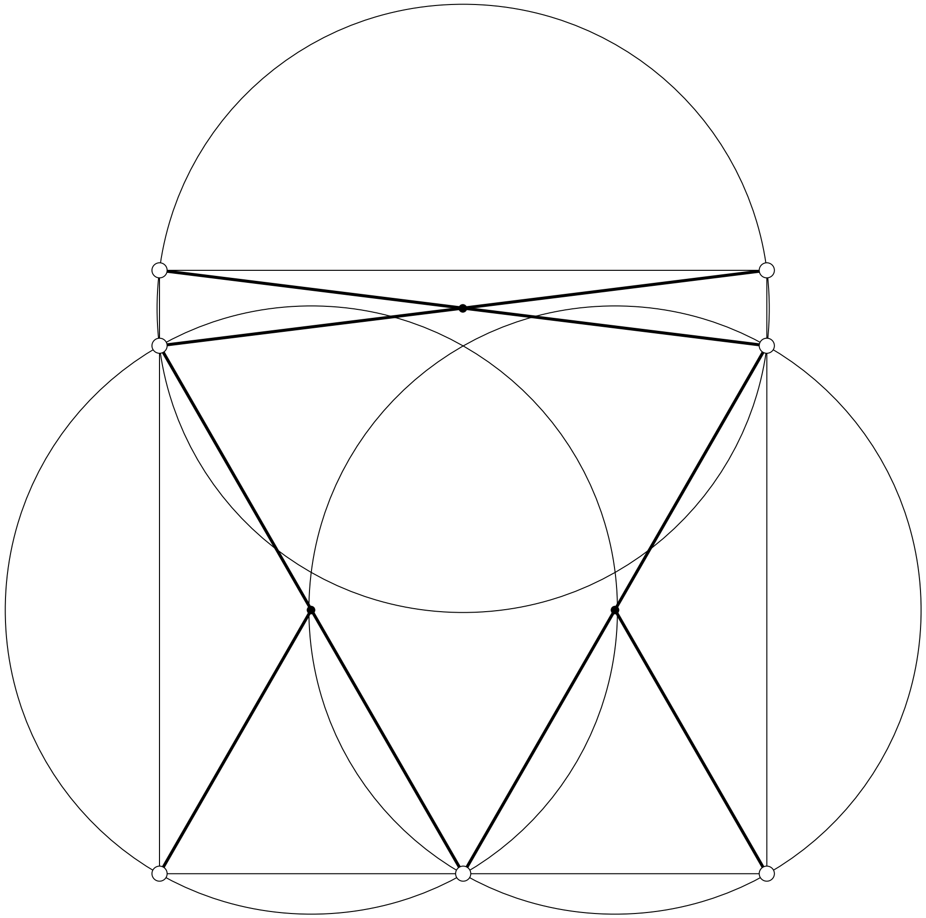
Disk Coverage of a Square



$$n = 2$$

- **Nurmela, Östergard**
  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteoria laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

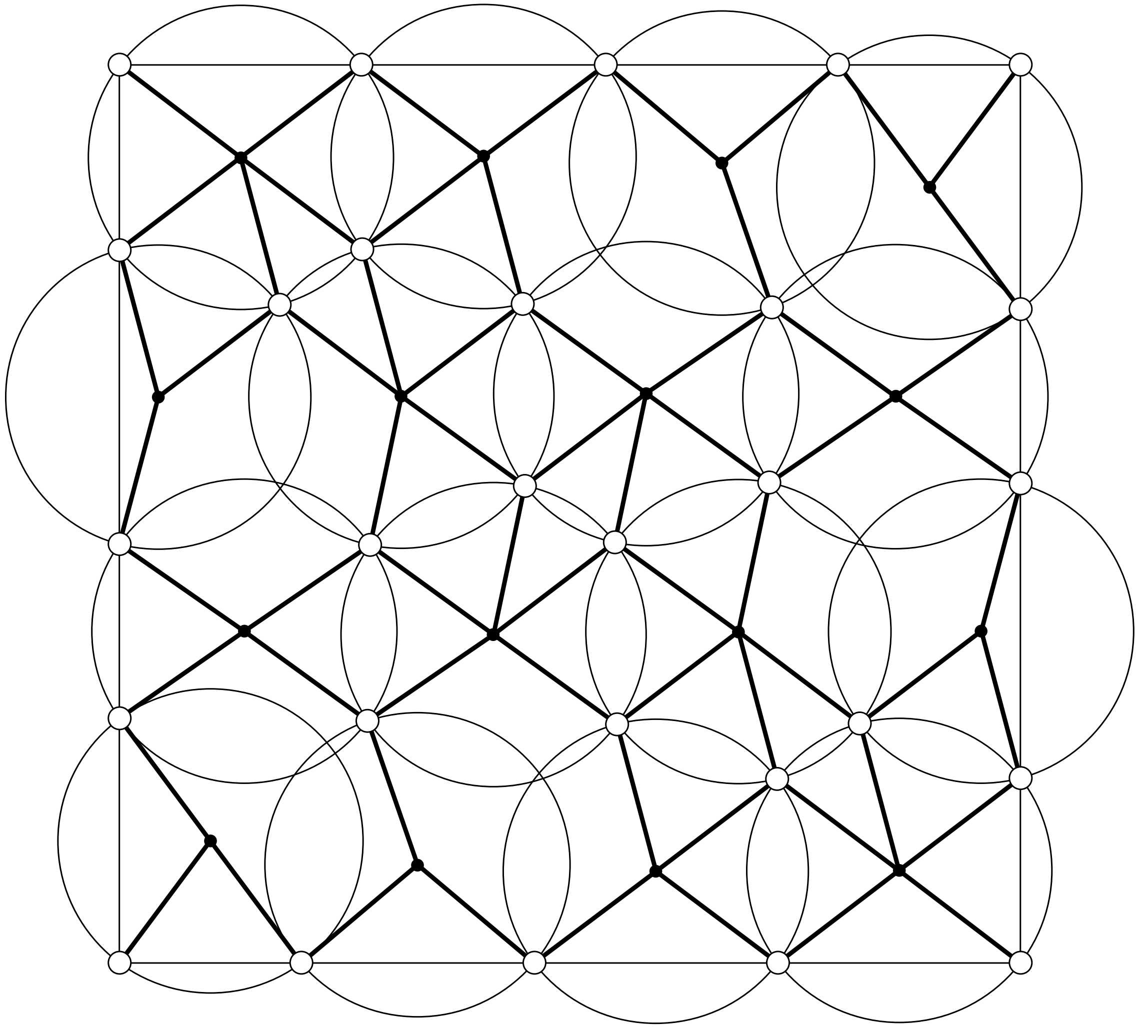
Disk Coverage of a Square



$n = 3$

- **Nurmela, Östergard**
  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteoria laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

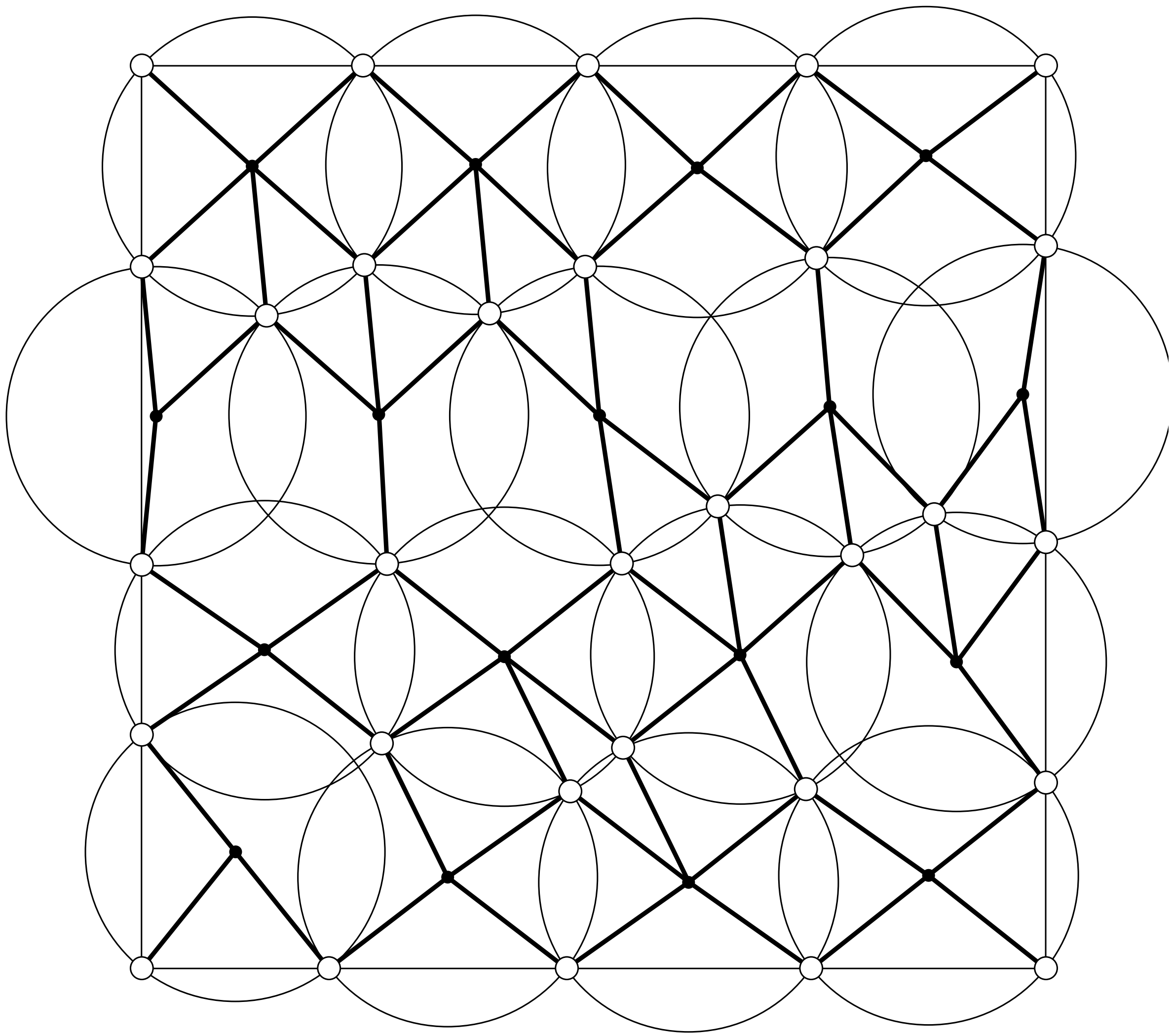
Disk Coverage of a Square



$n = 16$

- **Nurmela, Östergard**
  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteoria laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

Disk Coverage of a Square



$$n = 17$$

- **Nurmela, Östergard**
  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteoria laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

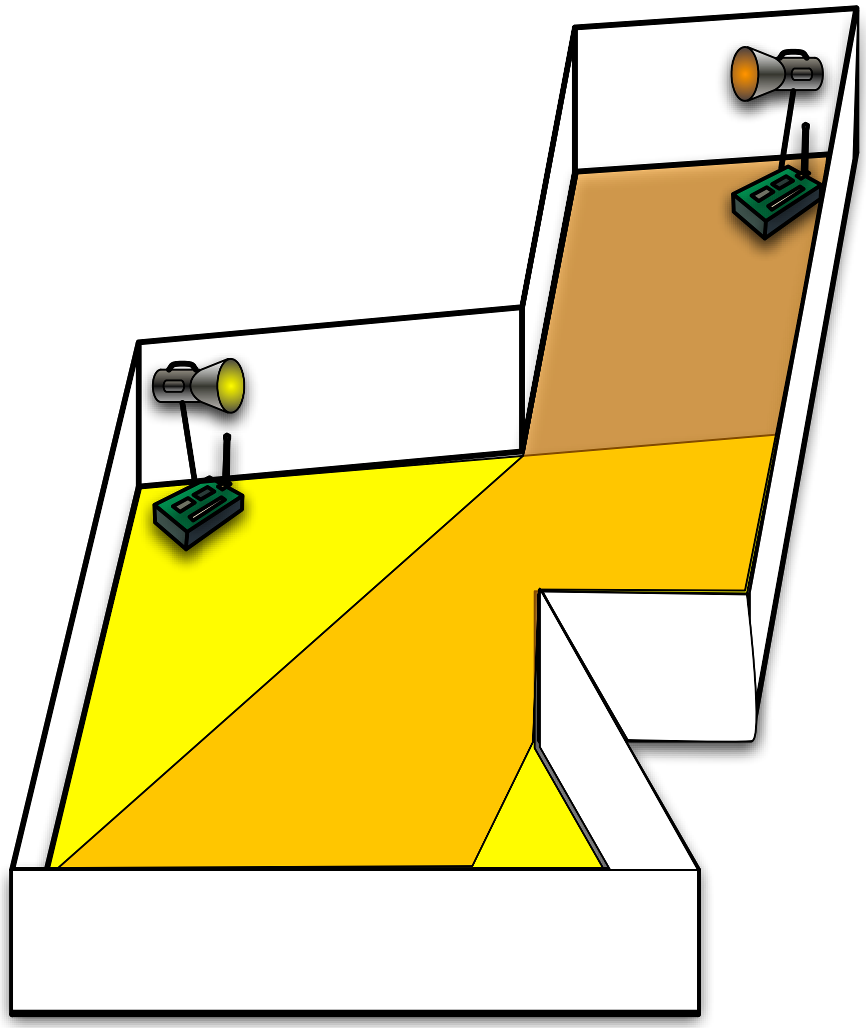
Disk Coverage of a Square

n	r_n	G	n	r_n	G
1	0.70710678118654752440...	D_4	16	0.16942705159811602395...	C_2
2	0.55901699437494742410...	D_2	17	0.16568092957077472538...	C_1
3	0.50389110926865935327...	D_1	18	0.16063966359715453523...	D_1
4	0.35355339059327376220...	D_4	19	0.15784198174667375675...	C_1
5	0.32616058400398728086...	D_1	20	0.15224681123338031005...	D_1
6	0.29872706223691915876...	C_2	21	0.14895378955109932188...	C_1
7	0.27429188517743176508...	D_2	22	0.14369317712168800049...	D_2
8	0.26030010588652494367...	D_2	23	0.14124482238793135951...	D_2
9	0.23063692781954790734...	D_1	24	0.13830288328269767697...	C_1
10	0.21823351279308384300...	D_2	25	0.13354870656077049693...	D_1
11	0.21251601649318384587...	D_2	26	0.13176487561482596463...	C_1
12	0.20227588920818008037...	C_2	27	0.12863353450309966807...	D_2
13	0.19431237143171902878...	C_1	28	0.12731755346561372147...	D_2
14	0.18551054726041864107...	D_1	29	0.12555350796411353317...	C_1
15	0.17966175993333219846...	C_1	30	0.12203686881944873607...	C_2

- **Nurmela, Östergard**
  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteorian laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

Art Gallery Problem

- **Given**
  - a room (described by polygon)
- **Compute**
  - Minimum number of cameras and their placement
    - such that the entire space is covered
- **Results**
  - Every room with n edges can be monitored by at most n / 3
  - The exact solution is NP-hard
    - even in the two-dimensional case
  - Polynomial time approximation with a factor O (log n)



Energy Saving Methdos

- **Schedule for sleep cycles**
  - MAC, routing protocol, sensoring
- **Optimize transmission routes**
  - many hops of few hops
- **Selection of nodes depending on the charge battery status**
  - data acquisition
  - change of cluster heads
  - route choice may consider battery status
- **Reduction of the amount of data**
  - data aggregation
  - compression
  - filtering

Lifetime of a Sensor Network

- **Wireless Sensor Networks (WSN)**
  - cheap and energy optimized sensors
  - send data to sinks
- **Lifetime of the network**
  - is hard to analyze
- **Depends from**
  - network architecture, protocols
  - event or input behavior
  - definition of lifetime
  - hardware, channel characteristics

Lifetime

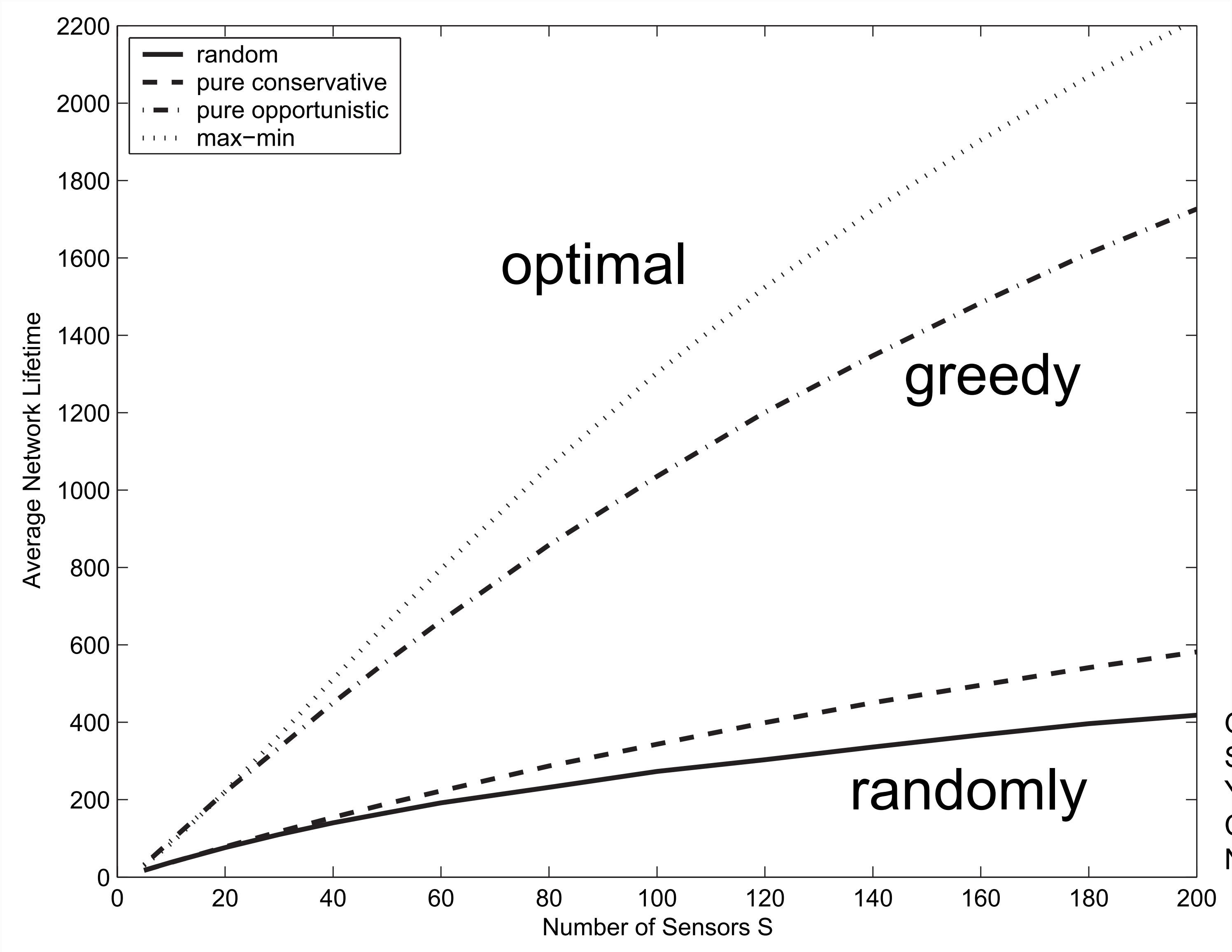
- **On the Lifetime of Wireless Sensor Networks**
  - Yunxia Chen, Qing Zhao, Communication Letters, Vol. 9, No. 11, Nov. 2005
- **Theorem**
  - For a WSN where
    - E_0: non-rechargable inital energy E_0
    - P_c: constant continuous power consumption in the complete network
    - **E**[E_w]: expected waste of energy
    - λ: average number of reported events
    - **E**[E_r]: expected energy necessary to report an event

$$E[\mathcal{L}] = \frac{\mathcal{E}_0 - E[E_w]}{P_c + \lambda E[E_r]}$$

Greedy Lifetime Maximization

- **Question**
  - Which sensors should collect the data
- **Greedy Algorithmus**
  - Choose the sensor with the maximum energy efficiency index γ_i:
  - $$\gamma_i = e_i - E_r(c_i)$$
  - $E_r(c_i)$: Energy for the transport of a message for node i
  - e_i: Available energy at the node i

Performance Greedy-Algorithm

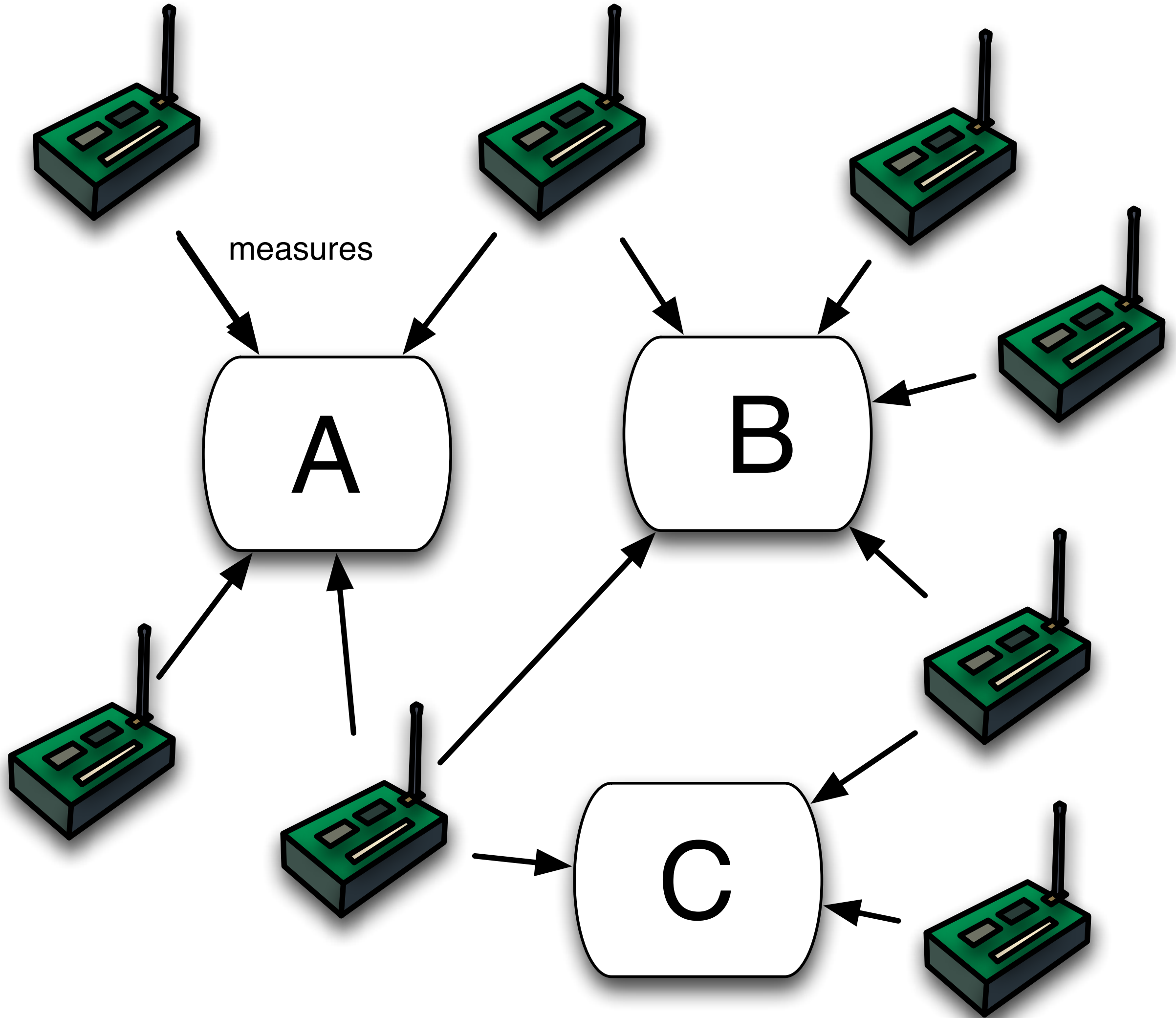


On the Lifetime of Wireless Sensor Networks
Yunxia Chen, Qing Zhao, Communication Letters, Vol. 9, No. 11, Nov. 2005

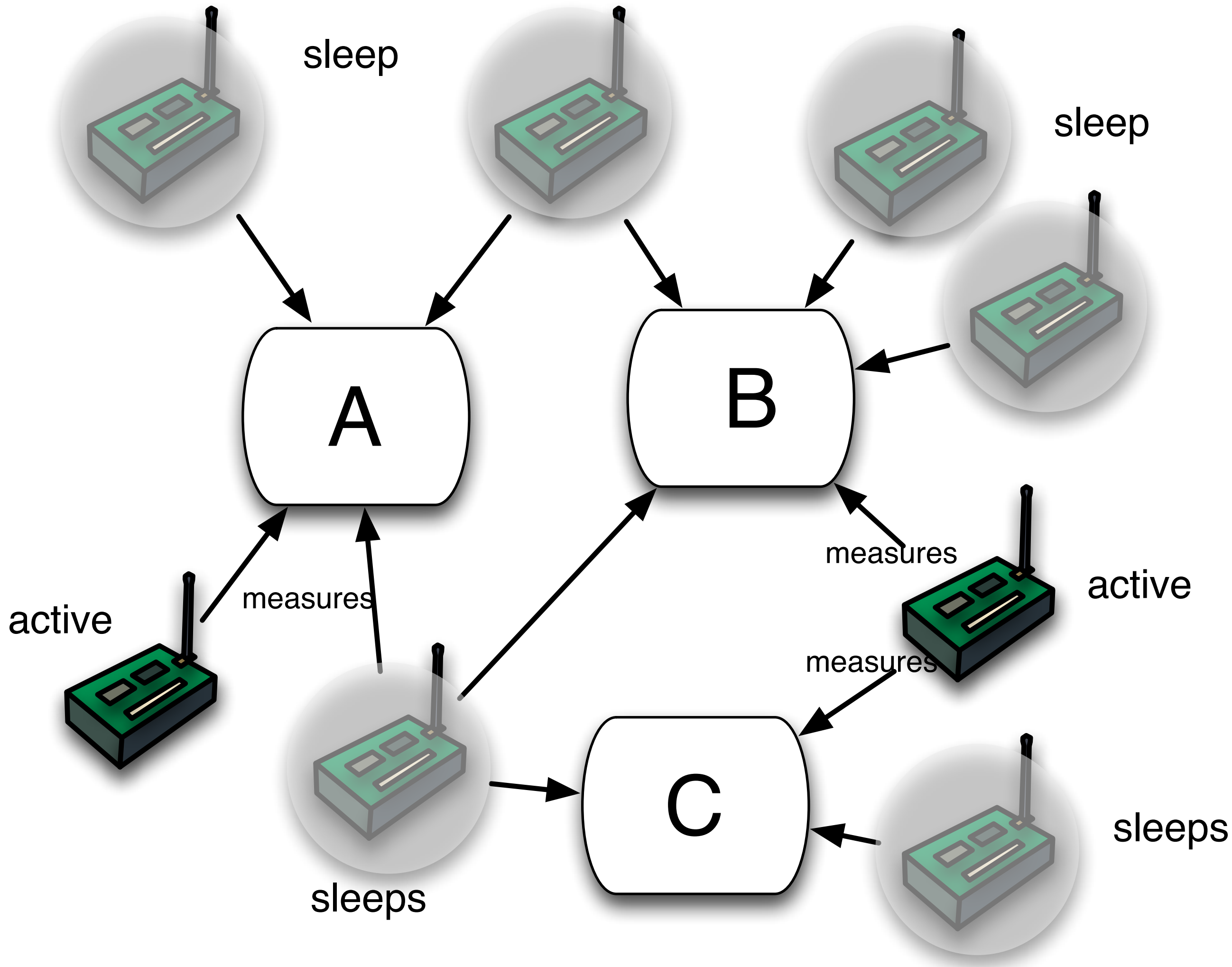
Lifetime Maximization by Scheduling

- **Cardei, Du**
  - *Improving Wireless Sensor Network Lifetime through Power Aware Organization*, Wireless Networks 11, 333–340, 2005
- **Problem**
  - Measurement points are covered by more than one sensors
  - Multiple measurements waste energy
- **Solution**
  - Activate only the nodes with minimum set-cover

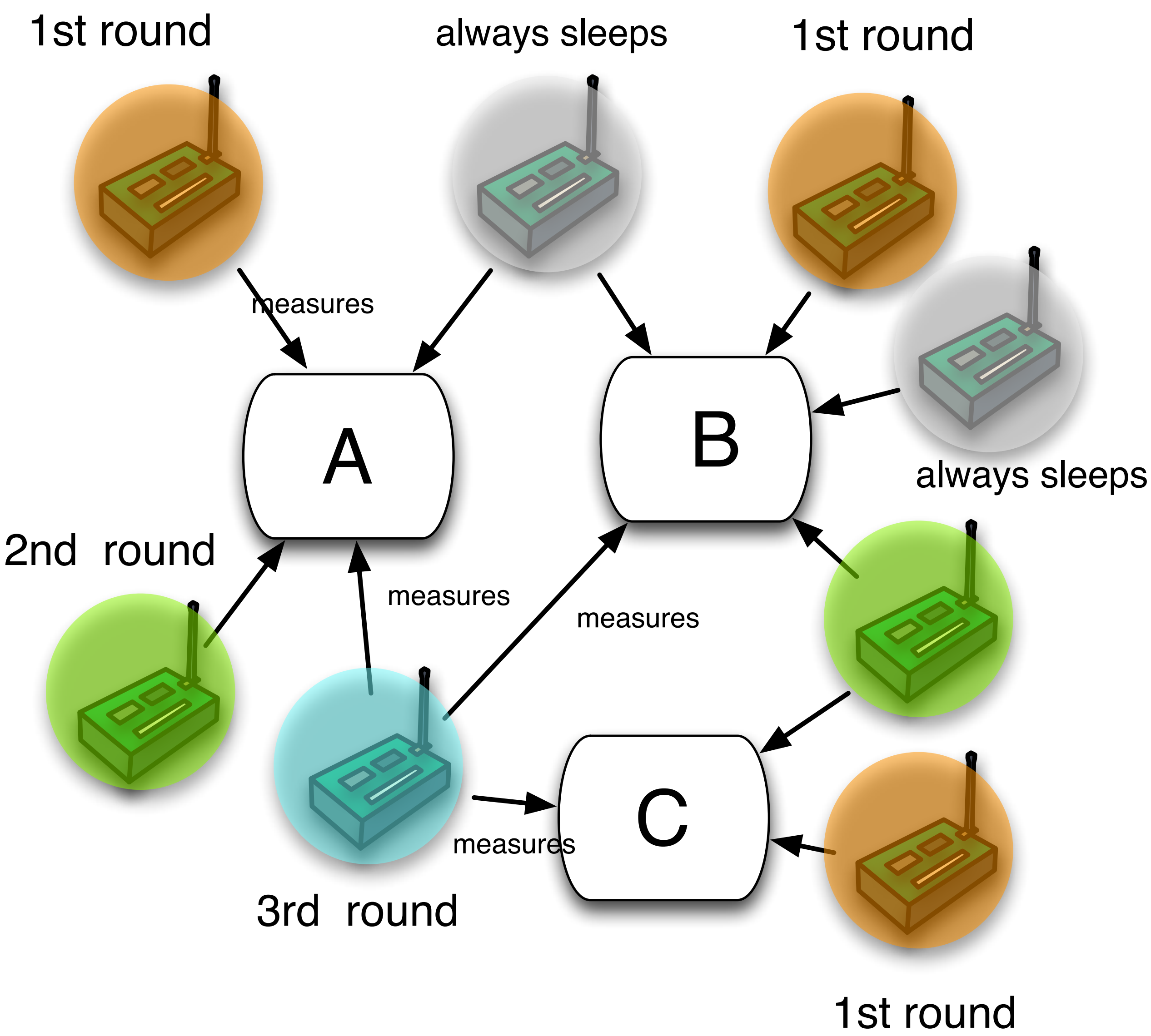
Multiple Coverage of Sensors



Covering Set



Disjoint Set-Cover



Definition Disjoint Set-Cover (DSC)

- **Given**
  - n sensors $S=\{S_1, S_2, ..., S_n\}$
  - m measurement points $T=\{T_1, T_2, ..., T_m\}$
  - Sensor coverage $S_i \subseteq T$
- **Compute**
  - Maximal number of disjoint coverings, i.e.
    - disjoint sets $M_1, .., M_k$ from S, such that each set covers the set T
- **Motivation**
  - The network lifetime increases by a factor of k

Complexity von Disjoint Set-Cover (DSC)

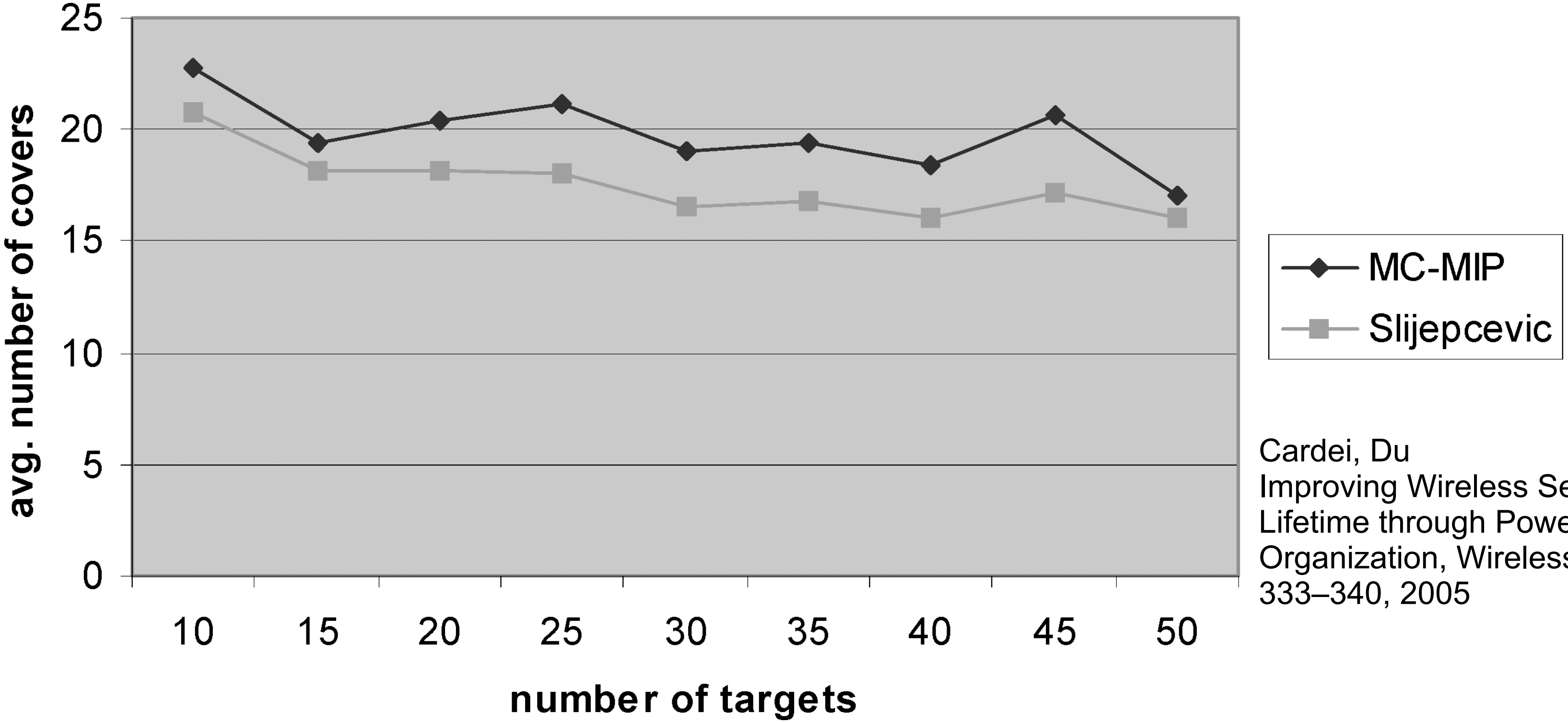
- **Theorem**
  - DSC is NP-hard for two sets
  - DSC is in general NP-hard
  - DSC can not be approximated by a factor of 2 without solving an NP-hard problem
- **Several heuristics are known**

Heuristiks for DSC

- **Slijepcevic Potkonjak 2001**
  - *Power Efficient Organization of Wireless Sensor Networks*, IEEE International Conference on Communications
  - Greedy algorithm
    - Greedily selects a mimal covering set
    - Removed this one and repeated until no more covering set is found
- **Cardei, Du 2006**
  - Problem is represented as flow problem
  - This is solved as linear problem
  - The solution gives an approximation of the disjoint set-cover problem

Comparison

- **Slijepcevic Potkonjak 2001**
  - simple distributed greedy solution
- **Cardei, Du 2006**
  - MC-MIP complex central algorithm

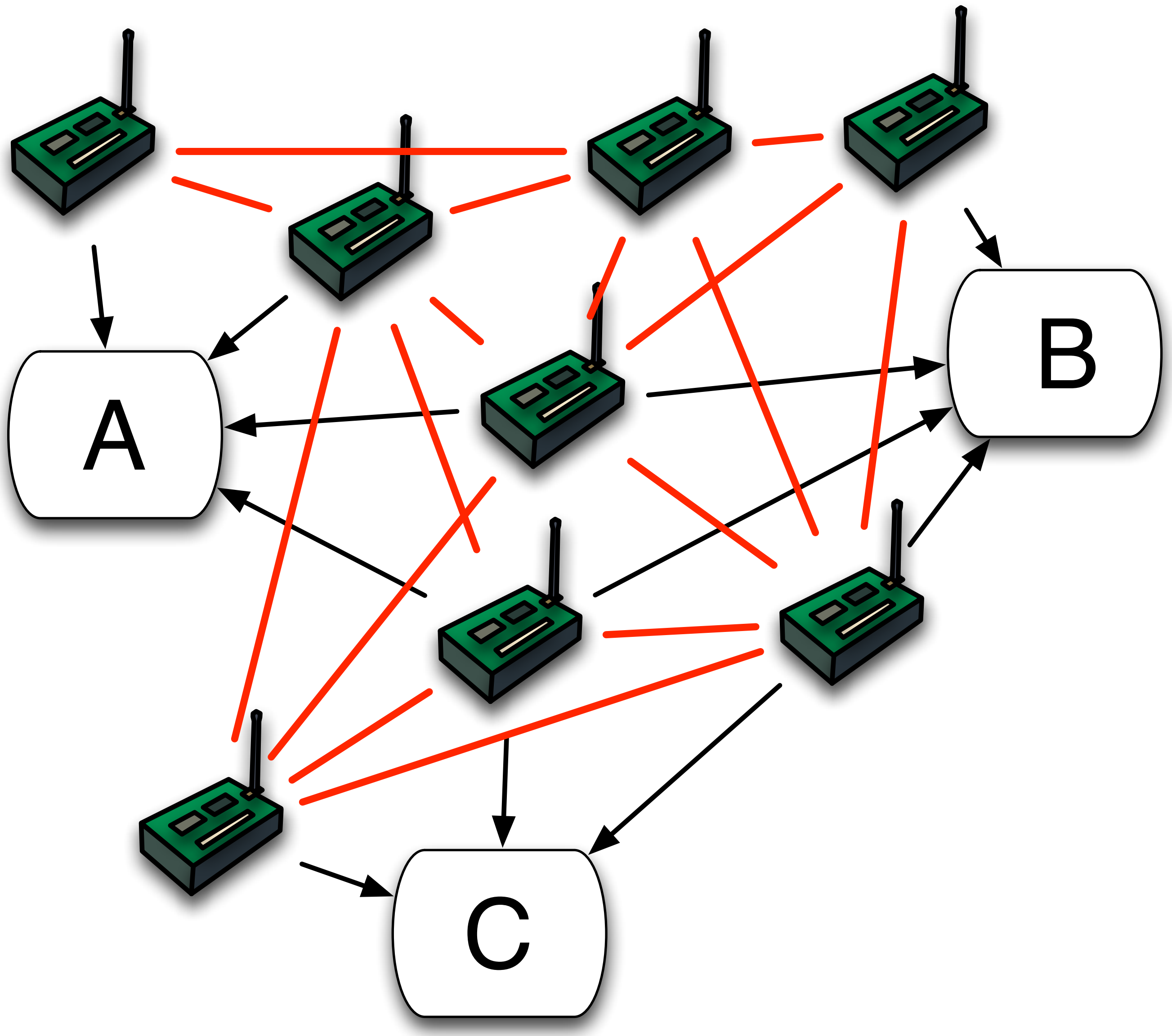


Cardei, Du
Improving Wireless Sensor Network Lifetime through Power Aware Organization, Wireless Networks 11, 333–340, 2005

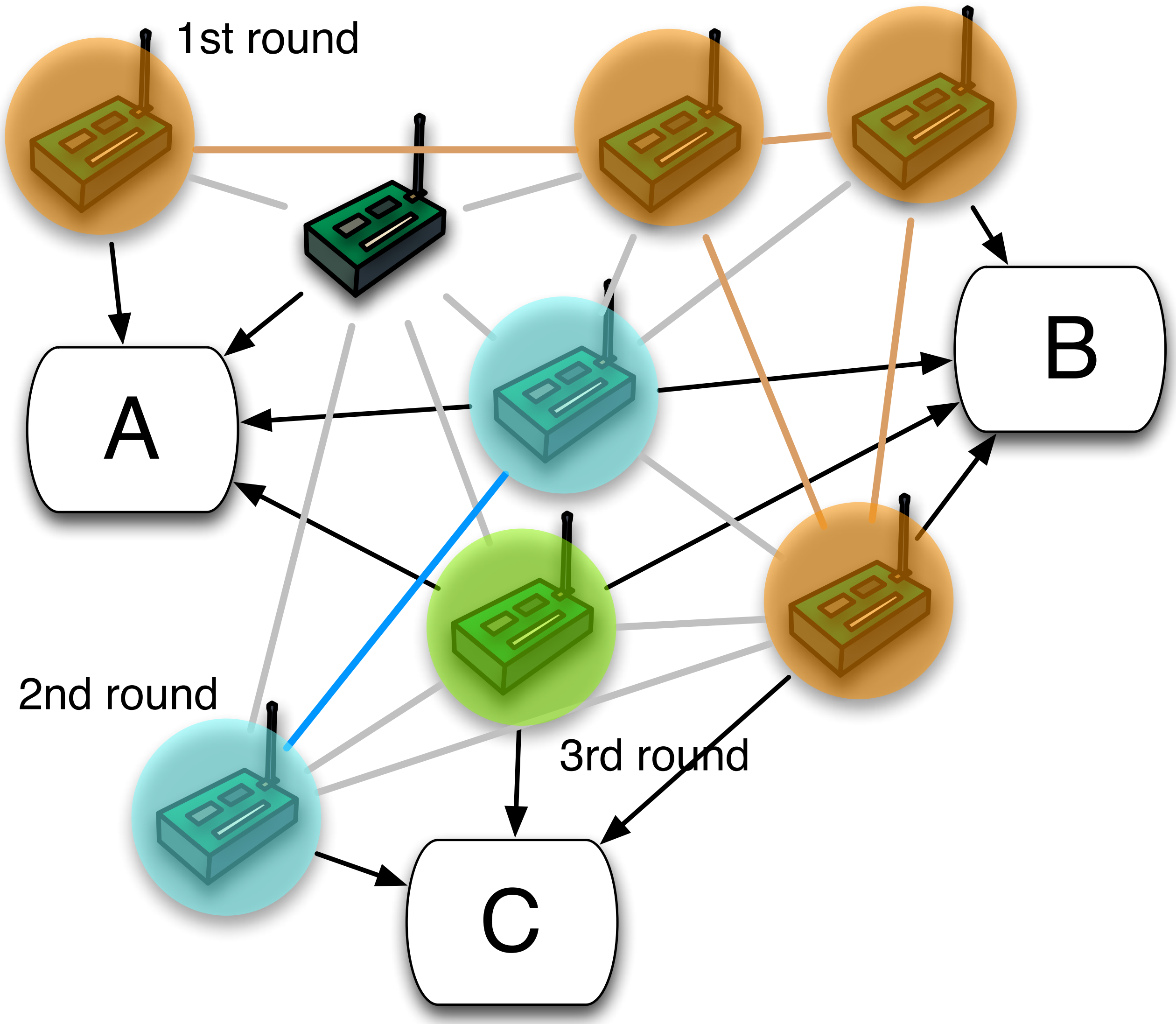
Outlook

- **Disjoint sets of network nodes may not be useful**
  - might be too far away from each other
  - important relay nodes are not activated
- **Extension**
  - Disjoint Connected Set Problem::
  - Find vertex-connected subgraph
    - Also NP-hard
- **Similar heuristics exist**

Disjoint Connected Set Problem



Disjoint Connected Set Problem



Literature Energy Harvesting

- **Kansal, Hsu, Zahedi, Srivastava**
  - *Power management in energy harvesting sensor networks*. ACM Trans. Embed. Comput. Syst. 6, 4, Sep. 2007

Motivation

- **Energy harvesting**
  - can remove batteries from WSNs
  - potentially infinite lifetime
  - active time can be increased (or reduced)
- **Example**
  - solar energy only available at daylight
- **Energy concept**
  - necessary for the entire period
  - regulates interplay of sleep phase, data rate and short term energy source

Harvesting Paradigma

- **Typical task in battery operated WSN**
  - minimize energy consumption
  - maximize lifetime
- **Task in harvesting-WSN**
  - continuous operation
    - i.e. infinite lifetime
  - term: energy-neutral operation

Possible Sources

- **Piezoelectric effect**
  - mechanical pressures produces voltage
- **Thermoelectric effect**
  - temperature difference of conductors with differen thermal coefficient
- **Kinetic energy**
  - e.g. self-rewinding watches
- **Micro wind turbines**
- **Antennas**
- **Chemical sources,...**

Differences Compared to Batteries

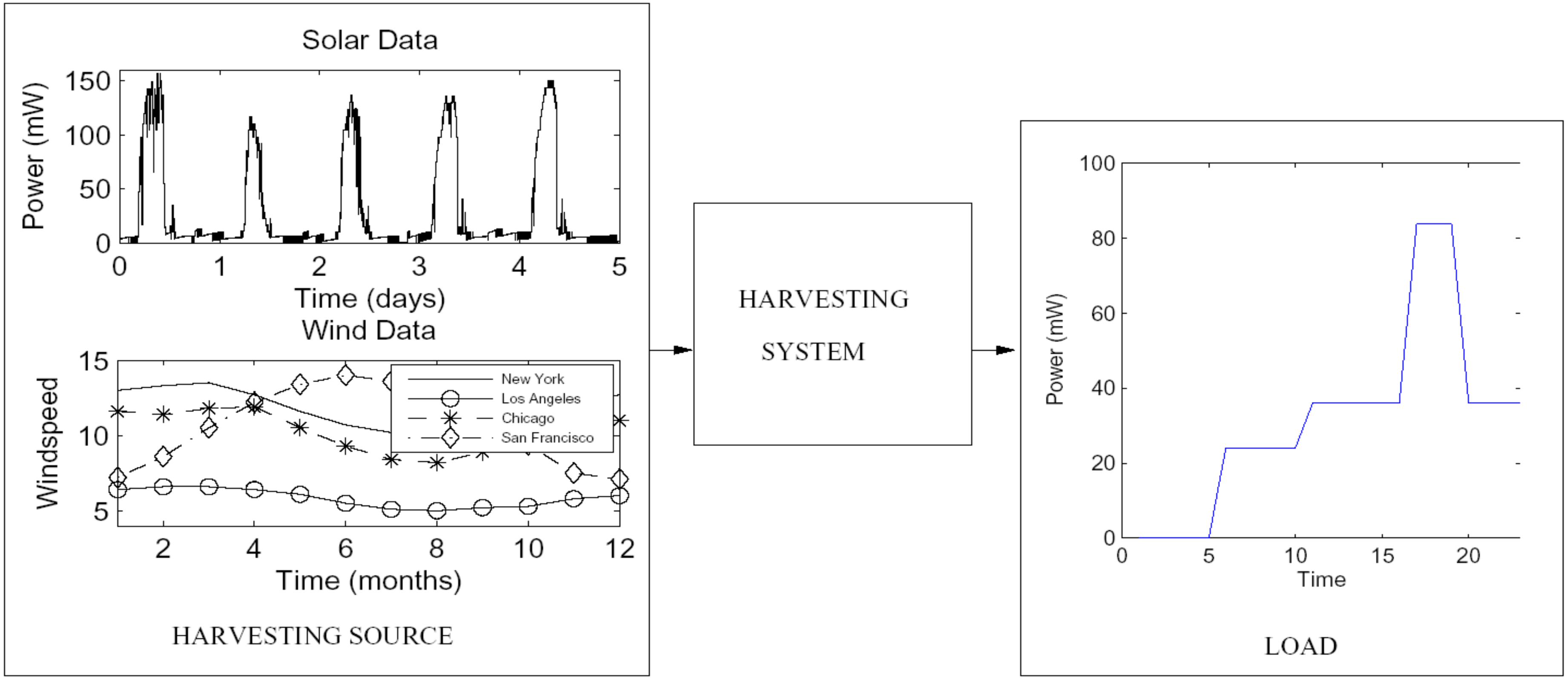
- **Time dependent**
  - form of operation has to be adapted over time
  - sometimes not predictable
- **Location dependent**
  - different nodes have have different energy
    - load balancing necessary
- **Never ending supply**
- **New efficiency paradigm**
  - utilization of energy for maximum performance
  - energy saving may result in unnecessary opportunity costs

Solutions without Power Management

- **Without energy buffer**
  - harvesting hardware has to supply maximal necessary energy level at minimum energy input
  - only in special situation possible
    - e.g. light switch
- **With energy buffer**
  - power management system necessary

Power Management System

- **Target**
  - Providing the necessary energy from external energy source and energy buffer



Energy Sources

- **Uncontrolled but predictable**
  - e.g. daylight
- **Uncontrolled and unpredictable**
  - e.g. wind
- **Controllable**
  - energy is produced if necessary
  - e.g. light switch, dynamo on bike
- **Partially controllable**
  - energy is not always available
  - e.g. radio source in the room with changing reception

Harvesting Theory

- **$P_s(t)$: Power output from energy source a time t**
- **$P_c(t)$: Energy demand at time t**
- **Without energy buffer**
  - $P_s(t) \geq P_c(t)$: node is active
- **Ideal energy buffer**
  - Continuous operation if

$$\int_0^T P_c(t)dt \leq \int_0^T P_s(t)dt + B_0 \quad \forall \quad T \in [0, \infty)$$

  - where B_0 is the initial energy
  - energy buffer is lossless, store any amount of energy

Harvesting Theory

- **$P_s(t)$: Power output from energy source a time t**
- **$P_c(t)$: Energy consumed at time t**
- **Let**

$$[x]^+ = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

- **Non-ideal energy buffer**
  - Continuous operation if

$$B_0 + \eta \int_0^T [P_s(t) - P_c(t)]^+ dt - \int_0^T [P_c(t) - P_s(t)]^+ dt - \int_0^T P_{leak}(t) dt \geq 0$$

  - B_0 is the initial energy
  - η: efficiency of energy buffer
  - $P_{leak}(t)$: energy loss of the memory

Harvesting Theory

- **$P_s(t)$: Power output from energy source a time t**
- **$P_c(t)$: Energy consumed at time t**
- **Let**

$$[x]^+ = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

- **Non-ideal energy buffer with limited reception B**
  - Continuous operation if

$$B_0 + \eta \int_0^T [P_s(t) - P_c(t)]^+ dt - \int_0^T [P_c(t) - P_s(t)]^+ dt - \int_0^T P_{leak}(t) dt \geq 0$$

  - B_0 is the initial energy of the buffer
  - η: efficiency of energy buffer
  - $P_{leak}(t)$: leakage power of the energy buffer

$$B_0 + \eta \int_0^T [P_s(t) - P_c(t)]^+ dt - \int_0^T [P_c(t) - P_s(t)]^+ dt - \int_0^T P_{leak}(t) dt \leq B$$

Model of Benign Energy Behavior

- **If the power source $P_s(t)$ occurs regularly, then it satisfies the following equations**

$$\int_{\tau}^{\tau+T} P_s(t)\,dt \;\leq\; \rho_1 T + \sigma_1$$

$$\int_{\tau}^{\tau+T} P_s(t)\,dt \;\geq\; \rho_1 T - \sigma_2$$

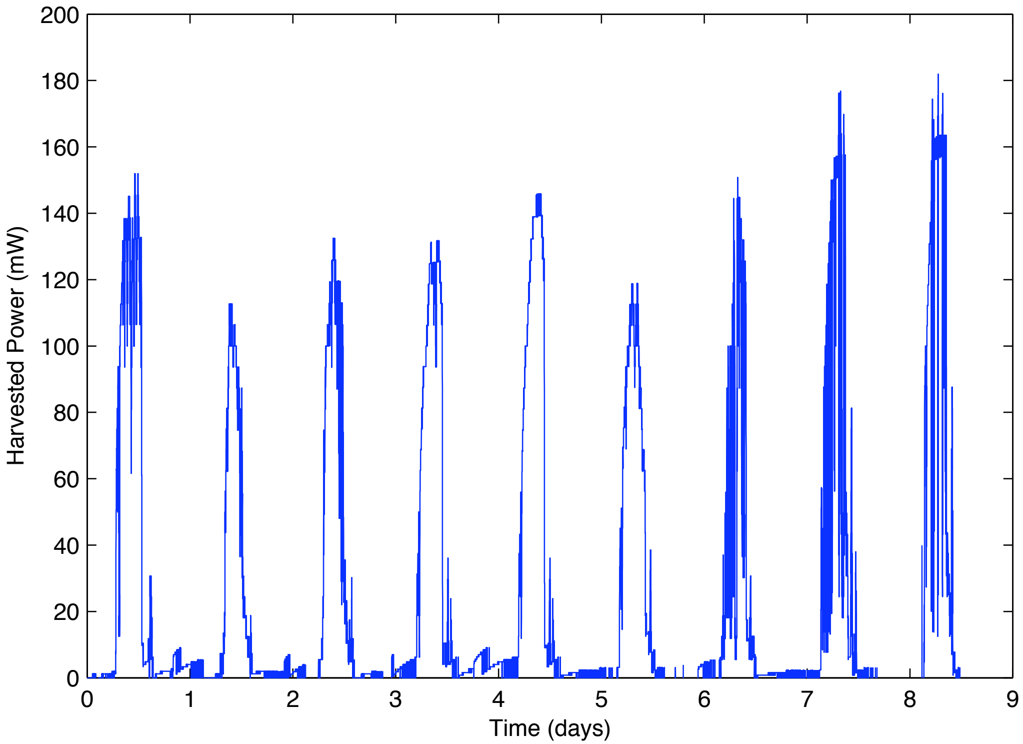


Fig. 2. Solar energy based charging power recorded for 9 days

Model of Benign Energy Behavior

- **Benign energy consumption:**
  - $P_c(t)$ satisfies the following

$$\int_{\tau}^{\tau+T} P_c(t)dt \;\leq\; \rho_2 T + \sigma_3$$

$$\int_{\tau}^{\tau+T} P_c(t)dt \;\geq\; \rho_2 T - \sigma_4$$

Energy Neutrality for Benign Sources

- **Substitution into the non-ideal energy source inequality:**

$$B_0 + \eta \cdot \min\{\int_T P_s(t)dt\} - \max\{\int_T P_c(t)dt\} - \int_T P_{leak}(t)dt \ \geq \ 0$$

$$\Rightarrow B_0 + \eta(\rho_1 T - \sigma_2) - (\rho_2 T + \sigma_3) - \rho_{leak} T \ \geq \ 0$$

- **This inequality must hold for T=0**

$$B_0 \geq \eta\sigma_2 + \sigma_3$$

- **This condition must hold for all T**

$$\eta\rho_1 - \rho_{leak} \geq \rho_2$$

- **If these inequalities hold then continuous operation can be guaranteed**

Necessary Energy Buffer for Benign Energy Sources

- **Substituting in the second equation**

$$B_0 + \eta \cdot \max\{\int_T P_s(t)dt\} - \min\{\int_T P_c(t)dt\} - \int_T P_{leak}(t)dt \ \leq \ B$$

$$\Rightarrow B_0 + \eta(\rho_1 T + \sigma_1) - (\rho_2 T - \sigma_4) - \rho_{leak} T \ \leq \ B$$

- **For T=0 we need**

$$B_0 + \eta(\sigma_1 - \sigma_4) \leq B$$

- **Substitution of** $B_0 \geq \eta\sigma_2 + \sigma_3$ **yields**

$$B \geq \eta(\sigma_1 + \sigma_2) + \sigma_3 - \sigma_4$$

- **For T $\rightarrow \infty$ we have**

$$\eta\rho_1 - \rho_{leak} \leq \rho_2$$

  - This condition may be violated without problems

Energy Neutral Operation

- **Theorem**
  - For benign energy sources the energy neutrality can be satisfied if the following conditions apply
    - $\rho_2 \leq \eta\rho_1 - \rho_{leak}$
    - $B \geq \eta\sigma_1 + \eta\sigma_2 + \sigma_3$
    - $B_0 \geq \eta\sigma_2 + \sigma_3$

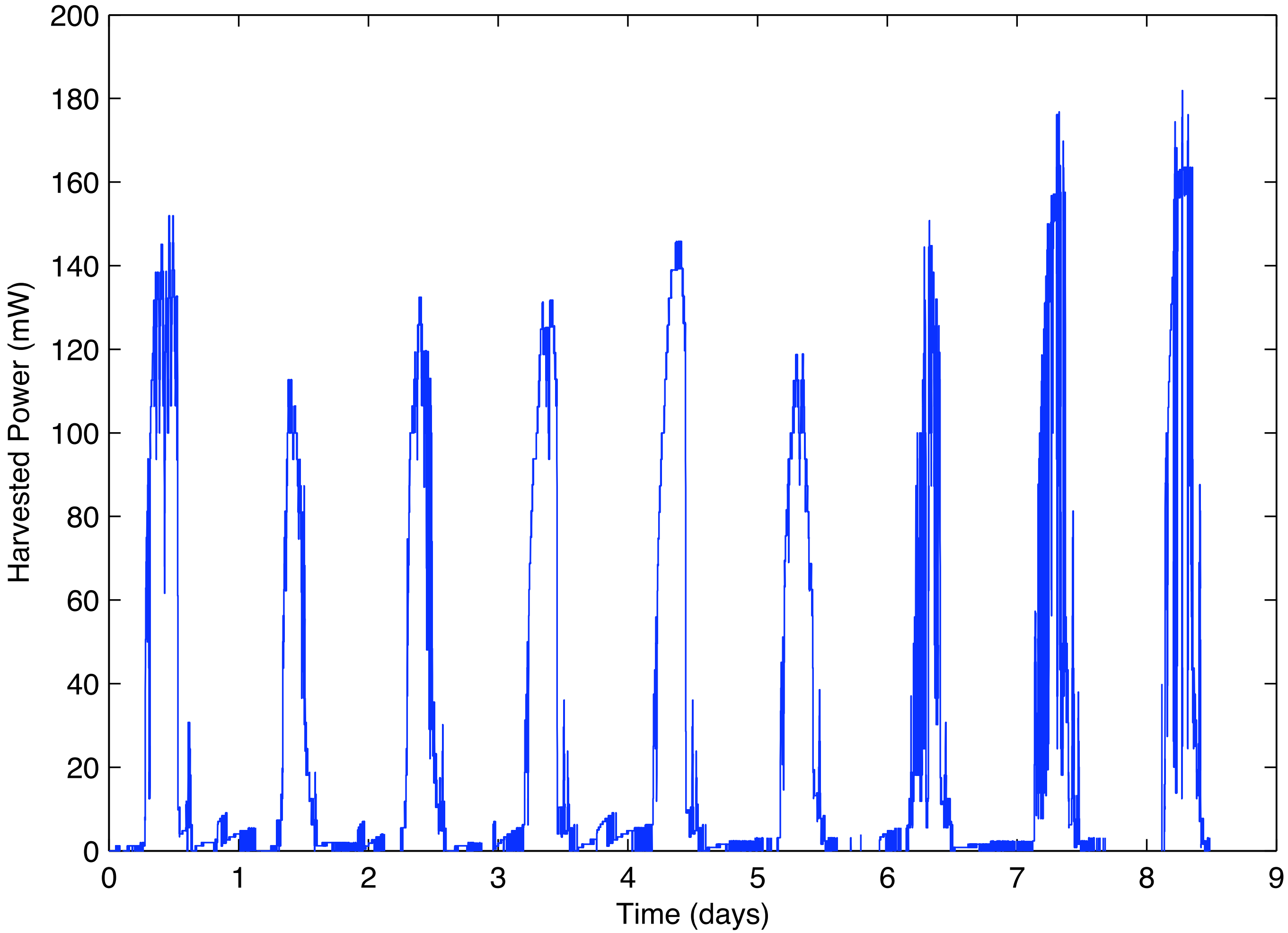


Fig. 2. Solar energy based charging power recorded for 9 days

Parameter	Value	Units
ρ_1	23.6	mW
σ_1	1.4639×10^3	J
σ_2	1.8566×10^3	J

Further Considerations

- **The behavior of energy sources can be learned**
  - As a result, the available energy can be calculated
  - The task can be adapted to the energy supply
- **Thereby**
  - Nodes with better energy situation can take over routing
  - Measurements can occur seldomer, but will never stop

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Algorithms for Radio Networks

Wireless Sensor Networks: Sensor Coverage & Lifetime

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