Algorithms for Radio Networks

Wireless Sensor Networks: Sensor Coverage & Lifetime
Literature

- Handbook on Theoretical and Algorithmic Aspects of Sensor, Ad Hoc Wireless and Peer-to-Peer-Networks (Editor: Jie Wu)
  - Chapter 27: Models and Algorithms for Coverage Problems in Wireless Sensor Networks
Sensor Coverage

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Sensor Coverage

- **Problem**
  - Given an area
  - Cover the area with the smallest possible number of sensor nodes

- **Variants**
  - Circle Covering
    - 2-dimensional surface, sensor coverage is given by circles
  - Art Gallery Problem
    - Angled rooms: Sensor coverage and line of sight angle
      - e.g. camera surveillance
  - Arbitrarily complexer variants conceivable
Random Circle Covering

- **Naive approach**
  - Given a square of area $A$
  - How many randomly positioned the sensors with unit disk cover the square?

- **Naive calculation**
  - Area of the unit circle: $r^2\pi$
  - Number of sensors required: $n = A / (r^2\pi)$

- **Intuition**
  - $O(A/r^2)$ should be sufficient

- **But: intuition is wrong!**
Random Circle Covering

- **Naive approach**
  - Given a square of area $A$
  - How many randomly positioned sensors with unit disk cover the square?

- **Theorem**
  - Let $n = A / (r^2 \pi)$
    - where $A$ denotes the area of the square
    - and $r$ denotes the sensor radius
  - To cover such a square of $\Theta(n \log n)$ randomly placed sensors are necessary and sufficient
Random Circle Covering

- **Theorem**
  - Let \( n = \frac{A}{r^2 \pi} \)
    - where \( A \) denotes the area of the square
    - and \( r \) denotes the sensor radius
  - To cover such a square of \( \Theta(n \log n) \) randomly placed sensors are necessary and sufficient

- **Proof sketch (lower bound):**
  - The probability that a given point is not covered by a sensor is at least
    \( 1-r^2 \pi/A = 1-1/n \)
  - Consider \( n \) such points with distance \( r \)
  - The probability that at least \( 1/n \log n \) sensors do not cover one of these points is therefore
    \[
    \left(1 - \frac{1}{n}\right)^{\frac{1}{2} n \log n} \geq \left(\frac{1}{4}\right)^{\frac{1}{2} \log n} = \frac{1}{n}
    \]
  - Hence, the expected number of uncovered points is 1
Random Circle Covering

- **Theorem**
  - Let $n = \frac{A}{r^2 \pi}$
    - where $A$ denotes the area of the square
    - and $r$ denotes the sensor radius
  - To cover such a square of $\Theta(n \log n)$ randomly placed sensors are necessary and sufficient

- **Proof sketch (upper bound):**
  - By $cn \log n$ random sensors every square of size $r/3 \times r/3$ is covered with probability $1-n^{-k}$
    - where $k$ grows linear with $c$
  - Then the whole square is covered with probability $1-n^{-k-1}$
Optimal Deterministic Bound

- Nurmela, Östergard
  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteorian laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

- How many circles can cover a square?
  - A closed form solution is unknown
  - However for a small number of circles the problem can be solved by exhaustive search
Disk Coverage of a Square

\[ n = 1 \]

\[ \text{Figure 2: Coverings for } n \text{ to } n. \]

- Nurmela, Östergard
  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteorian laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)

Algorithms for Radio Networks
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Computer Networks and Telematics
University of Freiburg
Disk Coverage of a Square

\[ n = 2 \]

Nurmela, Östergard
- Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteorian laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)
Disk Coverage of a Square

Figure 2: Coverings for $n = 3$.

- **Nurmela, Östergard**
  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteorian laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)
Disk Coverage of a Square

Nurmela, Östergard
- Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteorian tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)
Disk Coverage of a Square

$\ n = 17 \$

- Nurmela, Östergard
  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteorian laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)
## Disk Coverage of a Square

<table>
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<th>$G$</th>
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</tr>
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</table>

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  - Covering a square with up to 30 equal circles (Teknillisen korkeakoulun tietojenkäsittelyteorian laboratorion tutkimusraportti 62, HUT-TCS-A62, Helsinki University of Technology, 2000)
Art Gallery Problem

- **Given**
  - a room (described by polygon)

- **Compute**
  - Minimum number of cameras and their placement
    - such that the entire space is covered

- **Results**
  - Every room with n edges can be monitored by at most n / 3
  - The exact solution is NP-hard
    - even in the two-dimensional case
  - Polynomial time approximation with a factor O (log n)
Energy Saving Methods

- **Schedule for sleep cycles**
  - MAC, routing protocol, sensing

- **Optimize transmission routes**
  - many hops of few hops

- **Selection of nodes depending on the charge battery status**
  - data acquisition
  - change of cluster heads
  - route choice may consider battery status

- **Reduction of the amount of data**
  - data aggregation
  - compression
  - filtering
Lifetime of a Sensor Network

- **Wireless Sensor Networks (WSN)**
  - cheap and energy optimized sensors
  - send data to sinks

- **Lifetime of the network**
  - is hard to analyze

- **Depends from**
  - network architecture, protocols
  - event or input behavior
  - definition of lifetime
  - hardware, channel characteristics
Lifetime

- On the Lifetime of Wireless Sensor Networks
  - Yunxia Chen, Qing Zhao, Communication Letters, Vol. 9, No. 11, Nov. 2005

- Theorem
  - For a WSN where
    - $E_0$: non-rechargable initial energy $E_0$
    - $P_c$: constant continuous power consumption in the complete network
    - $E[E_w]$: expected waste of energy
    - $\lambda$: average number of reported events
    - $E[E_r]$: expected energy necessary to report an event

$$E[\mathcal{L}] = \frac{E_0 - E[E_w]}{P_c + \lambda E[E_r]}$$
Greedy Lifetime Maximization

- **Question**
  - Which sensors should collect the data

- **Greedy Algorithmus**
  - Choose the sensor with the maximum energy efficiency index $\gamma_i$:
    - $\gamma_i = e_i - E_r(c_i)$
    - $E_r(c_i)$: Energy for the transport of a message for node $i$
    - $e_i$: Available energy at the node $i$
Equation (9) shows that the network lifetime decreases. Clearly, the better the channel gain, the smaller the reporting energy. It, however, provides insights on protocol design and makes the required transmission energy equal to the transmission time of one packet. The required energy for channel acquisition in the pure conservative and greedy protocols is the residual energy of sensor \( i \) as a function of its fading gain \( \gamma_i \). A sensor is considered optimal, greedy, or randomly chooses a sensor for transmission. The pure conservative protocol, which allows each sensor to determine whether it does or does not need channel information, maximizes the minimum residual energy across the network.

By utilizing CSI. To maximize the network lifetime, we propose a MAC protocol which selects the sensor with the maximum energy-efficiency defined as

\[
    E_{\text{eff}} = \frac{E_{\text{data}}}{E_{\text{total}}}
\]

where we have assumed, without loss of generality, that \( E_{\text{data}} \) and \( E_{\text{total}} \) are modelled as

\[
    E_{\text{data}} = E_{\text{trans}} + E_{\text{receive}}
\]

\[
    E_{\text{total}} = E_{\text{trans}} + E_{\text{receive}} + E_{\text{off}} + E_{\text{on}}
\]

where \( E_{\text{trans}} \) is the required transmission energy to achieve an accuracy of data collection (the first failure in data collection), whichever occurs first.

The network span until any sensor in the network dies (the first death) is the network lifetime and define the network lifetime as the time span until any sensor in the network dies (the first death) which sensor should be enabled in each data collection in order to maximize the network lifetime.

The “random” protocol which utilizes neither CSI nor REI can be modelled as

\[
    E_{\text{random}} = E_{\text{trans}} + E_{\text{receive}} + E_{\text{off}} + E_{\text{on}}
\]

We realize that this lifetime definition may not apply to many WSN applications. It, however, provides insights on protocol design and makes the required transmission energy equal to the transmission time of one packet. The required energy for channel acquisition in the pure conservative and greedy protocols is the residual energy of sensor \( i \) at the beginning of a data collection (the first failure in data collection), whichever occurs first.

### Performance Greedy-Algorithm

![Graph showing network lifetime comparison]

**Fig. 1.** Comparison of the network lifetime.

**On the Lifetime of Wireless Sensor Networks**

Yunxia Chen, Qing Zhao,
Communication Letters, Vol. 9, No. 11, Nov. 2005
Lifetime Maximization by Scheduling

- Cardei, Du

- **Problem**
  - Measurement points are covered by more than one sensors
  - Multiple measurements waste energy

- **Solution**
  - Activate only the nodes with minimum set-cover
Multiple Coverage of Sensors

A measures B

A measures C

B measures C

1 + 1 + 1
Covering Set

A

B

C

sleep

active

measures

dsleeps

sleep

active

measures

dsleeps
Disjoint Set-Cover

1st round

2nd round

3rd round

measures

always sleeps

1st round

1st round

1st round

always sleeps

measures

measures

measures

A

B

C
Definition Disjoint Set-Cover (DSC)

- **Given**
  - $n$ sensors $S=\{S_1, S_2, \ldots, S_n\}$
  - $m$ measurement points $T=\{T_1, T_2, \ldots, T_m\}$
  - Sensor coverage $S_i \subseteq T$

- **Compute**
  - Maximal number of disjoint coverings, i.e.
    - disjoint sets $M_1, \ldots, M_k$ from $S$, such that each set covers the set $T$

- **Motivation**
  - The network lifetime increases by a factor of $k$
Complexity von Disjoint Set-Cover (DSC)

- **Theorem**
  - DSC is NP-hard for two sets
  - DSC is in general NP-hard
  - DSC can not be approximated by a factor of 2 without solving an NP-hard problem

- **Several heuristics are known**
Heuristics for DSC

- Slijepcevic Potkonjak 2001
  - *Power Efficient Organization of Wireless Sensor Networks*, IEEE International Conference on Communications
  - Greedy algorithm
    - Greedily selects a minimal covering set
    - Removed this one and repeated until no more covering set is found

- Cardei, Du 2006
  - Problem is represented as flow problem
  - This is solved as linear problem
  - The solution gives an approximation of the disjoint set-cover problem
Comparison

- **Slijepcevic Potkonjak 2001**
  - simple distributed greedy solution
- **Cardei, Du 2006**
  - MC-MIP complex central algorithm

\[ \text{Improving Wireless Sensor Network Lifetime through Power Aware Organization, Wireless Networks 11, 333–340, 2005} \]
Outlook

- **Disjoint sets of network nodes may not be useful**
  - might be too far away from each other
  - important relay nodes are not activated

- **Extension**
  - Disjoint Connected Set Problem::
  - Find vertex-connected subgraph
    - Also NP-hard

- **Similar heuristics exist**
Disjoint Connected Set Problem
Disjoint Connected Set Problem

1st round

A

2nd round

B

C

3rd round

NP-hard

Greedy

Heuristics
Literature Energy Harvesting

- Kansal, Hsu, Zahedi, Srivastava
Motivation

- **Energy harvesting**
  - can remove batteries from WSNs
  - potentially infinite lifetime
  - active time can be increased (or reduced)

- **Example**
  - solar energy only available at daylight

- **Energy concept**
  - necessary for the entire period
  - regulates interplay of sleep phase, data rate and short term energy source
Harvesting Paradigma

- Typical task in battery operated WSN
  - minimize energy consumption
  - maximize lifetime

- Task in harvesting-WSN
  - continuous operation
    - i.e. infinite lifetime
  - term: energy-neutral operation
Possible Sources

- **Piezoelectric effect**
  - mechanical pressures produces voltage

- **Thermoelectric effect**
  - temperature difference of conductors with different thermal coefficient

- **Kinetic energy**
  - e.g. self-rewinding watches

- **Micro wind turbines**

- **Antennas**

- **Chemical sources,...**
Differences Compared to Batteries

- **Time dependent**
  - form of operation has to be adapted over time
  - sometimes not predictable

- **Location dependent**
  - different nodes have different energy
    - load balancing necessary

- **Never ending supply**

- **New efficiency paradigm**
  - utilization of energy for maximum performance
  - energy saving may result in unnecessary opportunity costs
Solutions without Power Management

- **Without energy buffer**
  - harvesting hardware has to supply maximal necessary energy level at minimum energy input
  - only in special situation possible
    - e.g. light switch

- **With energy buffer**
  - power management system necessary
Power Management System

- **Target**
  - Providing the necessary energy from external energy source and energy buffer
Energy Sources

- **Uncontrolled but predictable**
  - e.g. daylight

- **Uncontrolled and unpredictable**
  - e.g. wind

- **Controllable**
  - energy is produced if necessary
  - e.g. light switch, dynamo on bike

- **Partially controllable**
  - energy is not always available
  - e.g. radio source in the room with changing reception
Harvesting Theory

- **$P_s(t)$**: Power output from energy source at time $t$
- **$P_c(t)$**: Energy demand at time $t$
- **Without energy buffer**
  - $P_s(t) \geq P_c(t)$: node is active
- **Ideal energy buffer**
  - Continuous operation if
    \[
    \int_0^T P_c(t) dt \leq \int_0^T P_s(t) dt + B_0 \quad \forall T \in [0, \infty)
    \]
  - where $B_0$ is the initial energy
  - energy buffer is lossless, store any amount of energy
Harvesting Theory

- $P_s(t)$: Power output from energy source at time $t$
- $P_c(t)$: Energy consumed at time $t$
- Let $[x]^+ = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$

Non-ideal energy buffer
- Continuous operation if

$$B_0 + \eta \int_0^T [P_s(t) - P_c(t)]^+ dt - \int_0^T [P_c(t) - P_s(t)]^+ dt - \int_0^T \bar{P}_{\text{leak}}(t) dt \geq 0$$

- $B_0$ is the initial energy
- $\eta$: efficiency of energy buffer
- $\bar{P}_{\text{leak}}(t)$: energy loss of the memory
Harvesting Theory

- **$P_s(t)$**: Power output from energy source a time $t$
- **$P_c(t)$**: Energy consumed at time $t$
- Let
  $$[x]^+ = \begin{cases} x & x \ge 0 \\ 0 & x < 0 \end{cases}$$

- **Non-ideal energy buffer with limited reception $B$**
  - Continuous operation if
    $$B_0 + \eta \int_0^T [P_s(t) - P_c(t)]^+ dt - \int_0^T [P_c(t) - P_s(t)]^+ dt - \int_0^T P_{\text{leak}}(t) dt \ge 0$$
  - $B_0$ is the initial energy of the buffer
  - $\eta$: efficiency of energy buffer
  - $P_{\text{leak}}(t)$: leakage power of the energy buffer

  $$B_0 + \eta \int_0^T [P_s(t) - P_c(t)]^+ dt - \int_0^T [P_c(t) - P_s(t)]^+ dt - \int_0^T P_{\text{leak}}(t) dt \le B$$
Model of Benign Energy Behavior

- If the power source $P_s(t)$ occurs regularly, then it satisfies the following equations

$$\int_\tau^{\tau+T} P_s(t) dt \leq \rho_1 T + \sigma_1$$
$$\int_\tau^{\tau+T} P_s(t) dt \geq \rho_1 T - \sigma_2$$

![Graph showing harvested power over time](image_url)

Fig. 2. Solar energy based charging power recorded for 9 days
Model of Benign Energy Behavior

- Benign energy consumption:
  - $P_c(t)$ satisfies the following
    \[
    \int_{\tau}^{\tau+T} P_c(t) \, dt \leq \frac{\rho T}{2} + \sigma_3
    \]
    \[
    \int_{\tau}^{\tau+T} P_c(t) \, dt \geq \frac{\rho T}{2} - \sigma_4
    \]
Energy Neutrality for Benign Sources

- Substitution into the non-ideal energy source inequality:

\[ B_0 + \eta \cdot \min\{ \int_T P_s(t) dt \} - \max\{ \int_T P_c(t) dt \} - \int_T P_{\text{leak}}(t) dt \geq 0 \]

\[ \Rightarrow B_0 + \eta(\rho_1 T - \sigma_2) - (\rho_2 T + \sigma_3) - \rho_{\text{leak}} T \geq 0 \]

- This inequality must hold for \( T=0 \)

\[ B_0 \geq \eta \sigma_2 + \sigma_3 \]

- This condition must hold for all \( T \)

\[ \eta \rho_1 - \rho_{\text{leak}} \geq \rho_2 \]

- If these inequalities hold then continuous operation can be guaranteed
Necessary Energy Buffer for Benign Energy Sources

- Substituting in the second equation

\[
B_0 + \eta \cdot \max\{\int_T P_s(t)dt\} - \min\{\int_T P_c(t)dt\} - \int_T P_{\text{leak}}(t)dt \leq B
\]

\[\Rightarrow B_0 + \eta(\rho_1 T + \sigma_1) - (\rho_2 T - \sigma_4) - \rho_{\text{leak}} T \leq B\]

- For \(T=0\) we need

\[B_0 + \eta(\sigma_1 - \sigma_4) \leq B\]

- Substitution of \(B_0 \geq \eta \sigma_2 + \sigma_3\) yields

\[B \geq \eta(\sigma_1 + \sigma_2) + \sigma_3 - \sigma_4\]

- For \(T \to \infty\) we have

\[\eta \rho_1 - \rho_{\text{leak}} \leq \rho_2\]

- This condition may be violated without problems
Energy Neutral Operation

- **Theorem**

  For benign energy sources the energy neutrality can be satisfied if the following conditions apply:

  - $\rho_2 \leq \eta \rho_1 - \rho_{\text{leak}}$
  - $B \geq \eta \sigma_1 + \eta \sigma_2 + \sigma_3$
  - $B_0 \geq \eta \sigma_2 + \sigma_3$
3.1 Buffer Size and Related Considerations

The first direct implication is on the design of the energy buffer required in the harvesting system. As an example consider a harvesting system that harvests solar energy. The power output from a solar cell [Kansal et al. 2004] is plotted in Figure 2 for nine days. Assuming this data is representative of the solar energy received on typical days of operation, this energy generation profile may be characterized by the $(\rho_1, \sigma_1, \sigma_2)$ model in Table I.

Table I. Solar cell parameters in experimental environment

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<th>Parameter</th>
<th>Value</th>
<th>Units</th>
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<td>23.6</td>
<td>mW</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>$1.4639 \times 10^3$</td>
<td>J</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>$1.8566 \times 10^3$</td>
<td>J</td>
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</table>

Let us assume that the load can be designed to operate at $\eta \rho_1 - \rho_{\text{leak}}$, where $\rho_{\text{leak}}$ will depend on energy storage technology used. Then, the battery size required according to equation (19) is $\eta (\sigma_1 + \sigma_2)$. Several technologies are available to implement this energy buffer, such as NiMH batteries, Li-ion batteries, ultracapacitors or NiCd batteries. For instance, for NiMH batteries, $\eta = 0.7$ and the required size is $3.32 \times 10^3$ Joules. This can be easily provided by an AA sized NiMH battery which has a capacity of 1800mAh, i.e., $7.7 \times 10^3$ Joules.

Note that using a larger battery than the above size does not help improve the supported energy neutral performance level. A larger battery than that calculated above may however be used to provide for practical considerations.
Further Considerations

- The behavior of energy sources can be learned
  - As a result, the available energy can be calculated
  - The task can be adapted to the energy supply
- Thereby
  - Nodes with better energy situation can take over routing
  - Measurements can occur seldomer, but will never stop
Algorithms for Radio Networks

Wireless Sensor Networks: Sensor Coverage & Lifetime

University of Freiburg
Technical Faculty
Computer Networks and Telematics
Christian Schindelhauer
\[ B_0 - \int_{t=0}^{T} P_c(t) \, dt \]
Problem