Chapter 12: Modeling and Analysis of Distributed Applications

Petri-Nets

- Petri-nets are abstract formal models capturing the flow of information and objects in a way which makes it possible to describe distributed systems and processes at different levels of abstraction in a unified language.

- Petri-nets have the name from their inventor Carl Adam Petri, who introduced this formalism in his PhD-thesis 1962.
Processing of complaints: informal description.

- Customer inquiring
- Archive
- Complaint registration
- Complaint processing
Complaints processing: formal Petri-net orchestration.\textsuperscript{1}

\begin{center}
\includegraphics[width=\textwidth]{complaints_petri_net.png}
\end{center}

Complaints processing: more than one complaint
Complaints processing: how to distinguish complaints
Complaints processing: keeping things together

Petri-Nets

Distributed Systems Part 2
Transactional Distributed Systems
Advanced Information Systems, SS 2011
Petri-nets model system dynamics.

- Activities trigger state transitions,
- Activities impose control structures,
- Applicable for modelling discrete systems.

Benefits

- Uniform language,
- Can be used to model sequential, causal independent (concurrent, parallel, nondeterministic) and monitored exclusive activities.
- Open for formal analysis, verification and simulation,
- Graphical intuitive representation.

The name *Petri-net* denotes a variety of different versions of nets - we will discuss the special case of *System Nets* following the naming introduced by W. Reisig.
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## Section 12.1 Elementary System Nets

### Basic elements of an elementary System Net (eS-Net)

- System states are represented by *places*, graphically circles or ovals.
- A place may be marked by an arbitrary number of *tokens* graphically represented by black dots.
- System dynamics is represented by *transitions*, graphically rectangles.
- *Transitions* represent activities (events) and the causalities between such activities (events) are represented by edges.
- *Multiplicities* represent the consumption, respectively creation of resources which are caused by the *occurrence* of activities.
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3-Philosopher-Problem

\( b_j \): philosopher starts eating; \( e_j \): philosopher stops eating;
\( i_j \): philosopher is eating; \( g_j \): fork on the desk;
\( 1 \leq j \leq 3 \).
A transition may occur when certain conditions with respect to the markings of its directly connected places are fulfilled; the occurrence of a transition - also called its firing - effects the markings of its directly connected edges, i.e. has local effects.

The surrounding of a transition $t$ is given by $t$ and all its directly connected places:

$s_1, \ldots, s_k$ are called preconditions (pre-places), $s_{k+1}, \ldots, s_n$ postconditions (post-places).

A place which is pre- and post-place at the same time is called a loop.
A transition *may* occur when certain conditions with respect to the markings of its directly connected places are fulfilled; the *occurrence* of a transition - also called its *firing* - effects the markings of its directly connected edges, i.e. has local effects.

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A *net* is given as a triple $N = (P, T, F)$, where

- $P$, the set of *places*, and $T$, the set of *transitionen*, are non-empty disjoint sets,
- $F \subseteq (P \times T) \cup (T \times P)$, is the set of directed edges, called *flow relation*, which is a binary relation such that $\text{dom}(F) \cup \text{cod}(F) = P \cup T$.

Let $N = (P, T, F)$ be a net and $x \in P \cup T$.

- $xF := \{y \mid (x, y) \in F\}$
- $Fx := \{y \mid (y, x) \in F\}$

For $p \in P$, $pF$ is the set of *post-transitions* of $p$; $Fp$ is the set of *pre-transitions* of $p$. For $t \in T$, $tF$ is the set of *post-places* of $t$; $Ft$ is the set of *pre-places* of $t$. 
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For \( t \in T \), \( tF \) is the set of post-places of \( t \); \( Ft \) is the set of pre-places of \( t \).
Let $N = (P, T, F)$ be a net. Any mapping $m$ from $P$ into the set of natural numbers $\text{NAT}$ is called a marking of $P$.

A mapping $P \rightarrow \text{NAT} \cup \{\omega\}$ is called $\omega$-marking. $\omega$ represents an infinitely large number of tokens.

Arithmetic of $\omega$:

$$\omega - n = \omega, \omega + n = \omega, n \cdot \omega = \omega, 0 \cdot \omega = 0, \omega > n$$

where $n \in \text{NAT}, n > 0$.

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A eS-Net is given as \( N = (P, T, F, V, m_0) \), where

- \((P, T, F)\) a net,
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A transition may fire once it is enabled.

Let $N = (P, T, F, V, m_0)$ a eS-Net, $m$ a marking and $t \in T$ a transition.

- $t$ is enabled at $m$, if for all pre-places $p \in Ft$ there holds:
  \[ m(p) \geq V(p, t). \]

- Whenever $t$ is enabled at $m$, then $t$ may fire at $m$. Firing $t$ at $m$ transforms $m$ to $m'$, $m[t \succ m']$, in the following way:
  \[
  m'(p) := \begin{cases} 
  m(p) - V(p, t) + V(t, p) & \text{falls } p \in Ft, p \in tF, \\
  m(p) - V(p, t) & \text{falls } p \in Ft, p \not\in tF, \\
  m(p) + V(t, p) & \text{falls } p \not\in Ft, p \in tF, \\
  m(p) & \text{sonst.}
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Transitions and markings in terms of vectors

Let places in \( P \) be linearly ordered.

- Markings of a net can be considered as vectors of nonnegative integers of dimension \(|P|\), called place-vectors.

- Transitions \( t \) can be characterized as vectors of nonnegative integers of dimension \(|P|\), called transition vectors \( \Delta t, t^+, t^- \):

Let \( N = (P, T, F, V, m_0) \) a eS-Net, \( p \in P \) and \( t \in T \).

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\begin{align*}
t^+(p) &:= \begin{cases} 
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\Delta t(p) &:= t^+(p) - t^-(p).
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Place and transition vectors at work:

- \( m \leq m' \), if \( m(p) \leq m'(p) \) for \( \forall p \in P \),
- \( m < m' \), if \( m \leq m' \), however \( m \neq m' \).
- \( t \) is enabled at \( m \) iff \( t^- \leq m \),
- \( m[t \triangleright m'] \) iff \( t^- \leq m \) and \( m' = m + \Delta t \).
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- $m[t \succ m'$ iff $t^- \leq m$ and $m' = m + \Delta t$. 
Reachability

Let \( N = (S, T, F, V, m_0) \) a eS-Net.

We denote \( W(T) \) the set of words with finite length over \( T \); \( \epsilon \in W(T) \) is called the empty word.

The length of a word \( w \in W(T) \) is given by \( l(w) \). We have \( l(\epsilon) = 0 \).

Let \( m, m' \) be markings of \( P \) and \( w \in W(T) \). We define a relation \( m[w > m'] \) inductively:

- \( m[\epsilon > m'] \) iff \( m = m' \),
- Let \( t \in T, w \in W(T) \). \( m[wt > m'] \) iff \( \exists m'' : m[w > m''], m''[t > m'] \).

The reachability relation \([* >] \) of \( N \) is defined by

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\( m' \) is reachable from \( m \) in \( N \).
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$m'$ is \textit{reachable} from $m$ in $N$. 
\( R_N(m) := \{ m' \mid m[* \succ m'] \} \), the set of markings reachable from \( m \) by \( N \),

\( L_N(m) := \{ w \mid \exists m' : m[w \succ m'] \} \), the set of all words representing firing sequences of transitions of \( N \) starting at \( m \),

\( \Delta w := \sum_{i=1}^{n} \Delta t_i \), wobei \( w = t_1 t_2 \ldots t_n \).

**Results**

- \([ * \succ \) is reflexiv and transitiv.
- \( m[w \succ m'] \Rightarrow (m + m^*)[w \succ (m' + m^*)], \forall m^* \in \text{NAT}^{|S|}. \) (Monotonie)
- \( m[w \succ m'] \Rightarrow m' = m + \Delta w. \)
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\item $R_N(m) := \{ m' \mid m[*\Rightarrow m'] \}$, the set of markings reachable from $m$ by $N$.
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\end{itemize}
- $R_N(m) := \{ m' \mid m[\ast \succ m'] \}$, the set of markings reachable from $m$ by $N$,
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12. Petri-Nets

12.1. Elementary System Nets

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- \( m [ w \succ m' \Rightarrow m' = m + \Delta w. \)
Reachability graph

Let $N = (P, T, F, V, m_0)$ a eS-Net. The *Reachability graph* of $N$ is a directed graph $EG(N) := (R_N(m_0), B_N)$; $R_N(m_0)$ is the set of nodes and $B_N$ is the set of annotated edges as follows:

$$B_N = \{ (m, t, m') \mid m, m' \in R_N(m_0), t \in T, m[t \triangleright m'] \}. $$
Exercise: Give the reachability graph of the following eS-Net:

\[
\begin{align*}
R_N(m_0) &= \{(1, 0, 0, 0), (1, 1, 0, 0), (1, 2, 0, 0), (1, 3, 0, 0), \ldots, \\
&\quad (0, 0, 1, 0), (0, 1, 1, 0), (0, 2, 1, 0), (0, 3, 1, 0), \ldots, \\
&\quad (0, 0, 1, 1), (0, 1, 1, 1), (0, 0, 1, 2), (0, 2, 1, 1), (0, 1, 1, 2), (0, 0, 1, 3), \ldots\} \\
L_N(m_0) &= \{\epsilon, t_1, t_1 t_1, t_1 t_1 t_1, \ldots, \\
&\quad t_2, t_1 t_2, t_1 t_1 t_2, t_1 t_1 t_1 t_2, \ldots, \\
&\quad t_1 t_2 t_3, t_1 t_1 t_2 t_3, t_1 t_1 t_2 t_3 t_3, t_1 t_1 t_1 t_2 t_2 t_3, t_1 t_1 t_1 t_1 t_2 t_3 t_3 t_3, \ldots\} 
\end{align*}
\]
Section 12.2 Control Patterns

- eS-nets can be used to model *causal dependencies*; for modelling temporal aspects extensions of the formalism are required.
- Whenever between some transitions there are no causal dependencies, the transitions are called *concurrent*; concurrency is a prerequisite for parallelism.
Some typical causalities

**Sequence**

![Sequence Diagram](image1.png)

**Iteration**

![Iteration Diagram](image2.png)
AND-join, OR-join, AND-split, OR-split

AND-join

OR-join

AND-split

OR-split
OR-Split with regulation
OR-Join with regulation
A eS-Net with concurrency