Section 12.3 Analysis

Boundedness

Let $N = (P, T, F, V, m_0)$ be a eS-Net, m a marking, $p \in P$.

- Let $k \in NAT^+$. p is called *k-bounded*, if for each marking m' there holds:

$$m' \in R_N(m_0) \Rightarrow m'(p) \leq k.$$

- p is called *bounded*, if p k-bounded for some $k \in NAT^+$.
- N is called *bounded (k-bounded)*, if each place is bounded (k-bounded).
- A eS-net is called *safe*, if it is 1-bounded. Places of a bounded net may be interpreted as boolean conditions.

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Theorem

Let $N = (P, T, F, V, m_0)$ be a eS-Net. N is *unbounded*, i.e. not bounded, iff there exist $w \in W(T)$, $m, m' \in R_N(m_0)$, such that $m[\, w \succ m'$ and $m' > m$.

Proof $\Leftarrow$

Let $w \in W(T)$, $m, m' \in R_N(m_0)$, such that $m[\, w \succ m'$ and $m' > m$. It holds

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To proof $\Rightarrow$ we first proof:

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For each infinite sequence of markings (m_i) of markings there exists an infinite subsequencev (m'_j), which is weakly monotonic, i.e. $l < k$ implies $m'_l \leq m'_k$.

To prove the Lemma, first extract an infinite subsequence for which weak monotonicity holds for the first components of its markings. Then extract from that subsequence an infinite subsequence for which weak monotonicity holds for the second components of its markings, etc.

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Proof $\Rightarrow$

- Consider the reachability graph $EG(N)$, which has an infinite number of nodes. Starting from m_0 there exist a directed path to each node of the graph. Because of the finite number of transitions, each node has only a finite number of direct successors.
- Thus, at m_0 there start an infinite number of paths without cycles, however only a finite number of edges. Therefore, one of these edges must be part of infinitly many paths. Let $m_0 \rightarrow m_1$ be one such edge.
- The same argument can be applied w.r.t. m_1 such that we get $m_0 \rightarrow m_1 \rightarrow m_2$, where $m_1 \rightarrow m_2$ is part of an infinite number of paths.
- The above construction can be repeated infinitly many times. Therefore there exists an infinite sequence of markings (m_i) of pairwise distinct markings, such that m_k, m_l, $0 \leq k \leq l$ implies:

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because of the Lemma there exists an infinite weakly monotonic subsequence (m'_j) von (m_i). Let m'_1, m'_2 two successive elements. From construction we have $m_0[*\succ m'_1[*\succ m'_2$, $m'_1 \leq m'_2$ and even $m'_1 < m'_2$.

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Let $N = (P, T, F, V, m_0)$ be a eS-Net, $m \in NAT^{|P|}$ a marking. The decision problem:

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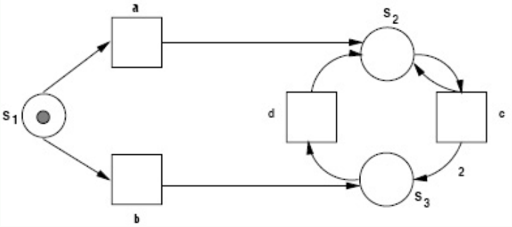
Consequence: Whenever a marking is not coverable w.r.t. some eS-Net N, it is not reachable in N.

Give examples.



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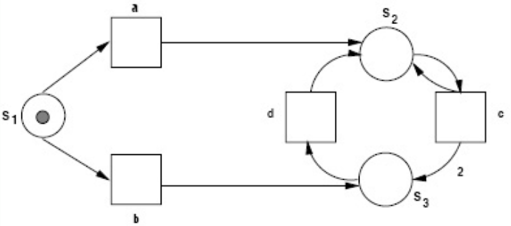

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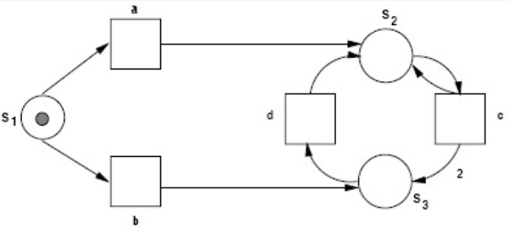

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Let $N = (P, T, F, V, m_0)$ a eS-Net. The *Coverability Graph* of N is given by $CG(N) := (R, B)$ as follows:

- *inductive definition of an auxiliary tree $T(N)$:*

  The values of the nodes in $T(N)$ are ω-markings of N. The value of the root node r is m_0. Let m be the value of some node n of $T(N)$, $t \in T$, and $m[t \succ m'$.

  - Whenever on the path from the root r to n there exists a node n'' with value m'' such that $m'' < m'$, then update m' by $m'(s) := \omega$ for all places p with $m''(p) < m'(p)$.
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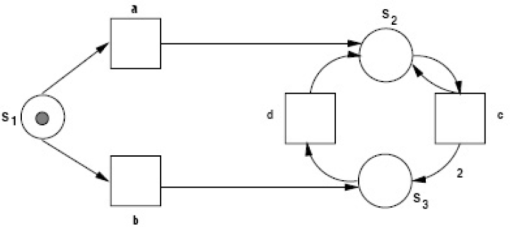
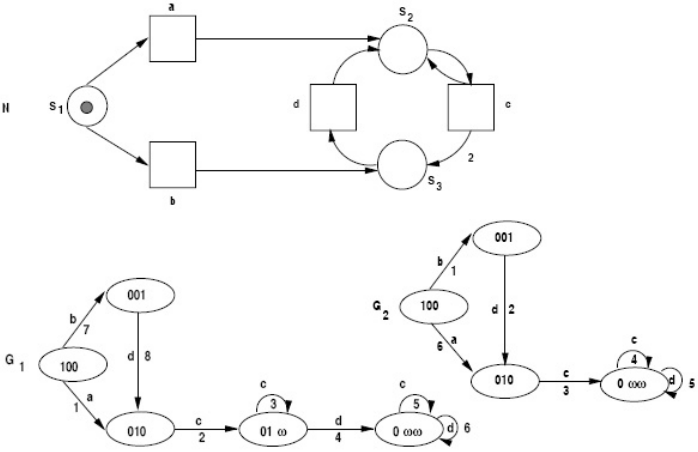
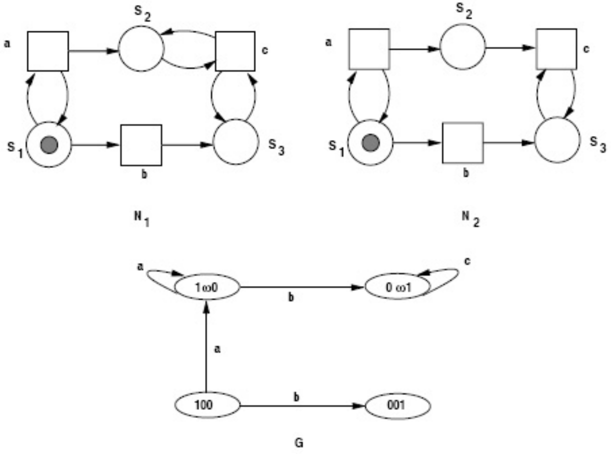
Give a coverability tree.



A eS-net with two different coverability graphs.



Two eS-Nets with identical coverability graphs.



Theorem

The coverability graph $CG(N) = (R, B)$ of a eS-net N is finite.

Proof:
Assume $CG(N)$ is not finite. Then it contains an infinite number of nodes. Thus there exists an infinite, weakly monotonic sequence of ω-markings, i.e. values of the nodes in the tree. Because of the construction of the auxiliary tree $T(N)$, such an infinite sequence cannot exist, as we can introduce ω only a finite number of times.

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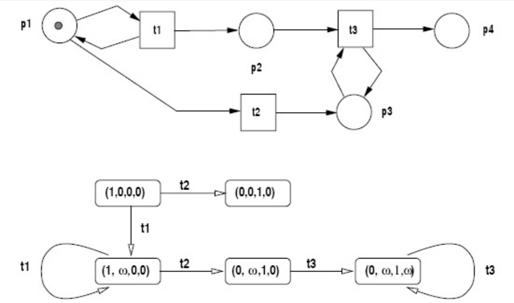

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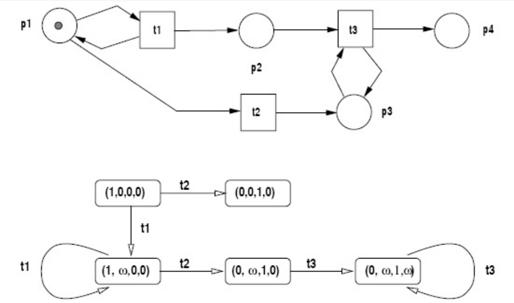

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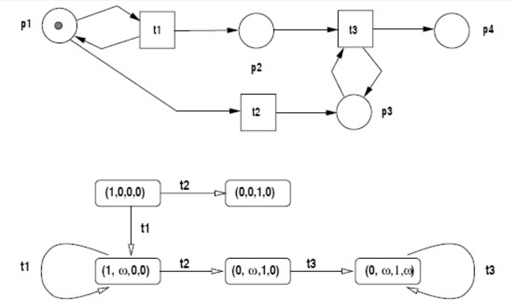

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Live, dead and deadlockfree

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- A marking m is called *dead* in N, if there is no $t \in T$ which is enabled at m.
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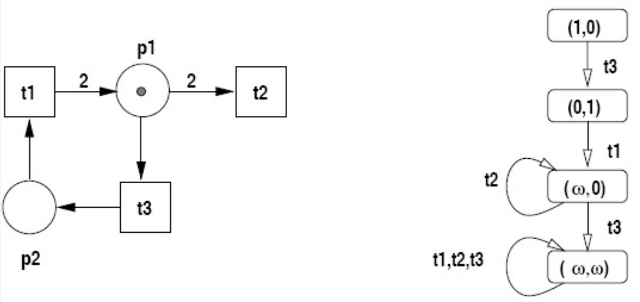

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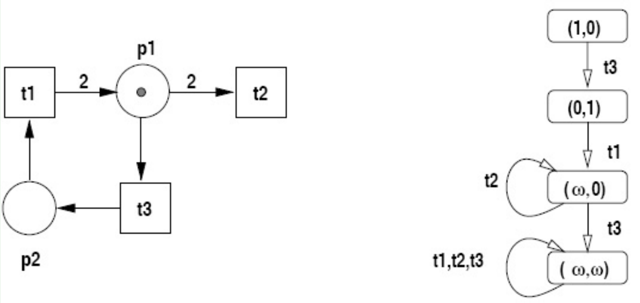

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Basics

- A Petri-net invariant is a property of a Petri-net, which holds for any marking, respectively transition word, of the net.
- We study place- and transition-invariants, which are based on a matrix representation of a net, respectively vector representation of markings and transitions.

Incidence Matrix

- Let $N = (P, T, F, V, m_0)$ a eS-Net, $T = \{t_1, \ldots, t_n\}$, $P = \{p_1, \ldots, p_m\}$, $n, m \geq 1$.
- A vector of dimension n (m) is called T- (P-)vector.
- For any $t \in T$, Δt can be represented as a column P-vector.
- The *incidence matrix* of N is given as a $m \times n$-matrix $C = (\Delta t_1, \ldots, \Delta t_n)$, respectively $C = (c_{i,j})_{1 \leq i \leq m, 1 \leq j \leq n}$, where $c_{ij} := \Delta t_j(s_i)$.

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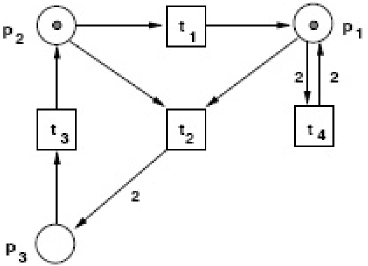
- Let $N = (P, T, F, V, m_0)$ a eS-Net, $T = \{t_1, \ldots, t_n\}$, $P = \{p_1, \ldots, p_m\}$, $n, m \geq 1$.
- A vector of dimension n (m) is called T- (P-)vector.
- For any $t \in T$, Δt can be represented as a column P-vector.
- The *incidence matrix* of N is given as a $m \times n$-matrix $C = (\Delta t_1, \ldots, \Delta t_n)$, respectively $C = (c_{i,j})_{1 \leq i \leq m, 1 \leq j \leq n}$, where $c_{ij} := \Delta t_j(s_i)$.

Example



$$C = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{pmatrix}$$

- Incidence matrices are independent of concrete markings,
- In case of loops, information concerning multiplicities is lost.

Parikh-Vektor

The transpose of a vector x, resp. matrix C is denoted by $x^\top$, bzw. $C^\top$.

The *Parikh-Vektor* $\bar{q}$ of some $q \in W(T)$ is a column T-vector, $n = | T |$, defined as follows:

$$\bar{q} : T \rightarrow NAT, \text{ where } \bar{q}(t) \text{ is the number of occurences of } t \text{ in } q.$$

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State Equation

Let $q \in W(T)$ and m, m' markings.

If $m[\, q \succ m'$, then $\sum_{t \in T} (\bar{q}(t) \cdot \Delta t) = C \cdot \bar{q} = \Delta q$.

Moreover, as $m[\, q \succ m'$, we have

- $m' = m + \Delta q^{\top}$.

  The equation:

  $$m' = m + (C \cdot \bar{q})^{\top}$$

  is called *state equation*.
- The system of linear equations given by

  $$C \cdot x = (m' - m)^{\top}$$

  has an integer nonnegative solution x.

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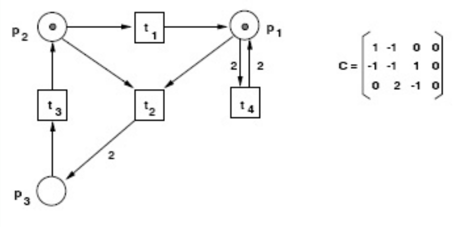

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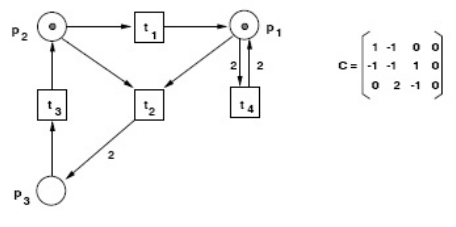

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Theorem

Let N be a eS-Net and Δ a P-vector. There exists a marking m^* and a word $q \in W(T)$, such that $m^*[\, q \succ (m^* + \Delta)$, iff $C \cdot x = \Delta^\top$ has an integer nonnegative solution.

Proof:
"$\Rightarrow$": trivial.

"$\Leftarrow$": Let $m^* := \sum_{t \in T} x(t) \cdot t^-$.

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Corollary

Let $N = (P, T, F, V, m_0)$ be a eS-Net. There exists a marking m^* such that $N = (P, T, F, V, m^*)$ unbounded, iff $C \cdot x > 0$ has an integer nonnegative solution.

Useful application of the corollary:

If there does not exist an integer nonnegative solution for $C \cdot x > 0$, then for any initial marking, N is bounded.

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Transition-Invariants (T-Invariants)

Let $N = (P, T, F, V, m_0)$ be a eS-Net.

- Any nontrivial integer solution x of the homogenous linear equation system $C \cdot x = 0$ is called *transition-invariant (T-invariant)* of N.
- A T-invariant x is called *proper*, if $x \geq 0$.
- A T-invariant x is called *realizable* in N, if there exists a word $q \in W(T)$ with $\bar{q} = x$ and a reachable marking m such that $m[\, q \succ m$.
- N is called *covered with T-invariants*, if there exists a T-invariant x of N with all components positive, i.e. greater than 0.

Proper T-invariants denote *possible* cycles of the reachability graph - realizable T-invariants denote cycles which indeed may occur.

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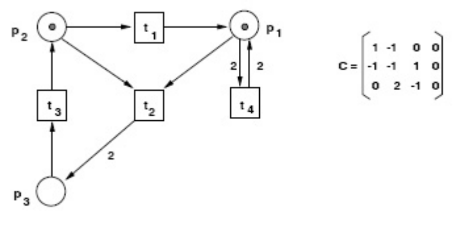

are as follows:

$$x = \lambda_1 \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

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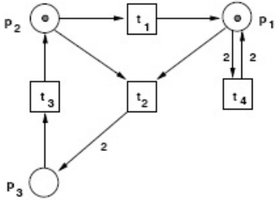

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Theorem

Let $N = (S, T, F, V, m_0)$ be a eS-Net. If there exists a marking m, such that N live and bounded at m, then N covered by T-invariants.

Proof: Let N live and bounded at some m.

As N is live at m, there exists a word $q_1 \in L_N(m)$, which contains all transitions in T and the marking $m + \Delta q_1$ is reachable from m.

Moreover, N is live at $m + \Delta q_1$ as well. Therefore, there exits a word $q_2 \in L_N(m)$, which contains all transitions in T and N is live at the marking $m + \Delta q_1 q_2$.

There exists an infinite sequence of markings (m_i), where $m_i := m + \Delta q_1 \dots q_i$, such that:

$$m[\, q_1 \succ m_1 [\, q_2 \succ m_2 \dots m_i [\, q_{i+1} \succ m_{i+1} \dots$$

As N is bounded at m, there is only a finite number of markings which are reachable. Therefore, there exist $i, j \in NAT : i < j$ such that $m_i = m_j$. Thus

$$m_i [\, q_{i+1} \dots q_j \succ m_j = m_i$$

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Useful application of the theorem:

Whenever N is not covered by T-invariants, then for every marking it holds N not live or not bounded.

Place-Invariants (P-Invariants)

Let $N = (P, T, F, V, m_0)$ be a eS-Net.

- Any nontrivial integer solution y of the homogeneous linear equation system $y \cdot C = 0$ is called *place-invariant (P-invariant)* of N.
- A P-invariant y is called *proper P-invariant*, if $y \geq 0$.
- N is called *covered with P-invariants*, if there exists a P-invariant y with all components positive, i.e. greater than 0.

If y is a P-invariant, then for any marking m the sum of the number of tokens on the places p is invariant with respect to the firing of the transitions weighted by $y(p)$.

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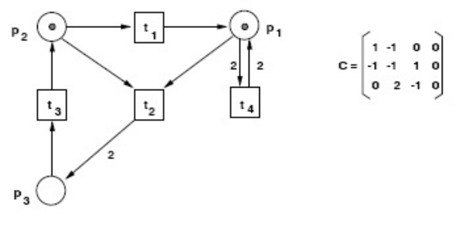

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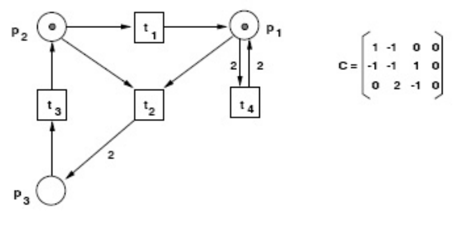

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Let $N = (P, T, F, V, m_0)$ a eS-Net and let y a P-invariant of N. Then:

$$m \in R_N(m_0) \Rightarrow y \cdot m^\top = y \cdot m_0^\top.$$

Proof:
Assume $m_0[\, q \succ m$. Then $m = m_0 + (C \cdot \bar{q})^\top$ and also:

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Corollary:

- Let y P-invariante of N, m marking.

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- Let y proper P-invariant of N. Let $p \in P$ such that $y(p) > 0$.

  Then, for any initial marking, s is bounded.

  Proof: $y \cdot m_0^\top = y \cdot m^\top \geq y(p) \cdot m(p) \geq m(p)$.

- Let N be covered by P-invariants. N is bounded for any initial marking.

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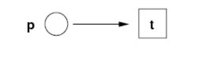

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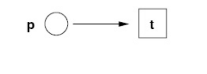

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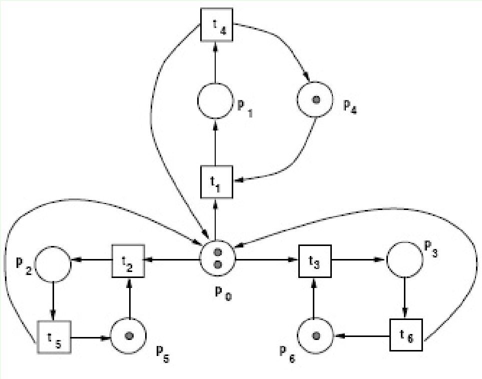
Example: Prove freedom from deadlocks.



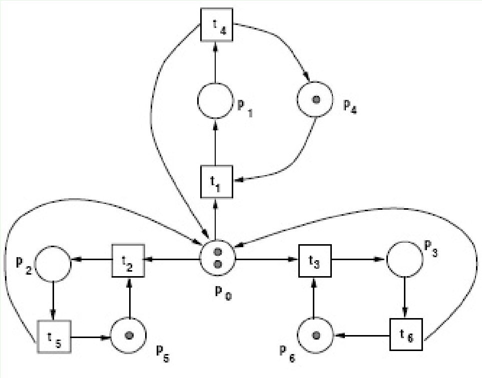
$$C = \begin{bmatrix} -1 & -1 & -1 & 1 & 1 & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

P-invariants:

$$\begin{aligned} Y_1 &= (0,1,0,0,1,0,0) \\ Y_2 &= (0,0,1,0,0,1,0) \\ Y_3 &= (0,0,0,1,0,0,1) \\ Y_4 &= (1,1,1,1,0,0,0) \end{aligned}$$

Initial marking is given by $m_0 = (2,0,0,0,1,1,1)$. Assume there exist a dead marking m, $m_0[\,q \succ m$. Then it must hold $m(p_1) = m(p_2) = m(p_3) = 0$. Because of Y_4 it follows $m(p_0) = 2$. As m dead it follows $m(p_4) = m(p_5) = m(p_6) = 0$. However this contradicts $Y_1 m_0 = Y_1 m$.

Example: Prove freedom from deadlocks.



$$C = \begin{bmatrix} -1 & -1 & -1 & 1 & 1 & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

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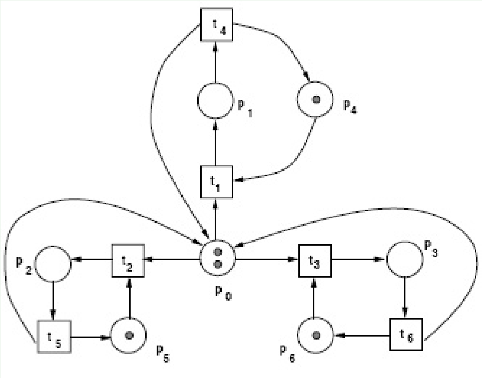

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Section 12.5 Place Capacities

Sometimes when modelling we would like to fix an upper bound for the number of tokens in a place.

- Let $N = (P, T, F, V, m_0)$ be a eS-Net, c a ω-marking of P and let $m_0 \leq c$. (N, c) is called *eS-Net with capacities*. $c(p), p \in P$ is called *capacity* of p.
- For eS-nets with capacities the notion of being enabled is adapted:

  a transition $t \in T$ *is* enabled at marking m, *if* $t^- \leq m$ *and* $m + \Delta t \leq c$.

- Capacities graphically are labels of places - no label means capacity ω.

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Any eS-net with capacities can be simulated by a eS-Net without capacities.

Construction

- Let p a palce with capacity $k = c(p), k \geq 1$. Let p^{co} be the complementary place of p which is assigned the initial marking $k - m_0(p)$.
- Whenever for a transition t we have $\Delta t(p) > 0$, we introduce an arc from p^{co} to t with multiplicity $\Delta t(p)$;
  whenever $\Delta t(p) < 0$, we introduce an arc from t to p^{co} with multiplicity $-\Delta t(p)$.

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A eS-Net with capacities and its simulation by a bounded eS-Net.