Section 12.4 Invariants

Basics

- A Petri-net invariant is a property of a Petri-net, which holds for any marking, respectively transition word, of the net.
- We study place- and transition-invariants, which are based on a matrix representation of a net, respectively vector representation of markings and transitions.

Incidence Matrix

- Let $N = (P, T, F, V, m_0)$ a eS-Net, $T = \{t_1, \ldots, t_n\}$, $P = \{p_1, \ldots, p_m\}$, $n, m \geq 1$.
- A vector of dimension n (m) is called T- (P-)vector.
- For any $t \in T$, Δt can be represented as a column P-vector.
- The *incidence matrix* of N is given as a $m \times n$-matrix $C = (\Delta t_1, \ldots, \Delta t_n)$, respectively $C = (c_{i,j})_{1 \leq i \leq m, 1 \leq j \leq n}$, where $c_{ij} := \Delta t_j(s_i)$.

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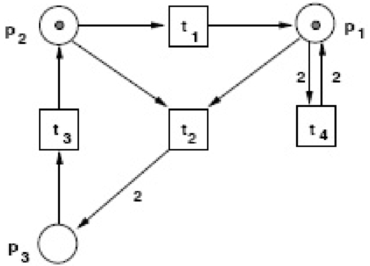
Example



$$C = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{pmatrix}$$

- Incidence matrices are independent of concrete markings,
- In case of loops, information concerning multiplicities is lost.

Parikh-Vektor

The transpose of a vector x, resp. matrix C is denoted by $x^\top$, bzw. $C^\top$.

The *Parikh-Vektor* $\bar{q}$ of some $q \in W(T)$ is a column T-vector, $n = |T|$, defined as follows:

$$\bar{q} : T \rightarrow NAT, \text{ where } \bar{q}(t) \text{ is the number of occurences of } t \text{ in } q.$$

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State Equation

Let $q \in W(T)$ and m, m' markings.

If $m[\, q \succ m'$, then $\sum_{t \in T} (\bar{q}(t) \cdot \Delta t) = C \cdot \bar{q} = \Delta q$.

Moreover, as $m[\, q \succ m'$, we have

- $m' = m + \Delta q^\top$.

  The equation:

  $$m' = m + (C \cdot \bar{q})^\top$$

  is called *state equation*.
- The system of linear equations given by

  $$C \cdot x = (m' - m)^\top$$

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however the following does not hold in general:

If $C \cdot x = (m' - m)^\top$ has an integer nonnegative solution then

$$\exists q \in W(T) : m[\, q \succ m',$$

I.e., the reachability problem cannot be solved, in general.

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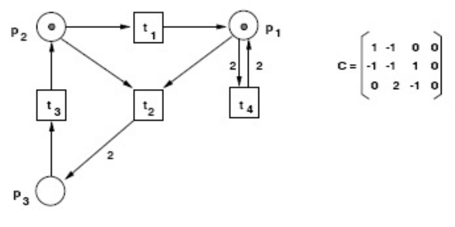

Let $m = (1, 0, 0)$, $m' = (0, 0, 1)$.
$x = (0, 1, 1, 0)^\top$ is a solution for $C \cdot x = (m' - m)^\top$, however we cannot find a word which can be fired at m.

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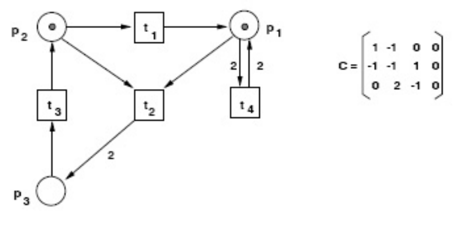

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$x = (0, 1, 1, 0)^\top$ is a solution for $C \cdot x = (m' - m)^\top$, however we cannot find a word which can be fired at m.

Theorem

Let N be a eS-Net and Δ a P-vector. There exists a marking m^* and a word $q \in W(T)$, such that $m^*[\,q \succ (m^* + \Delta)$, iff $C \cdot x = \Delta^\top$ has an integer nonnegative solution.

Proof:
"$\Rightarrow$": trivial.

"$\Leftarrow$": Let $m^* := \sum_{t \in T} x(t) \cdot t^-$.

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Corollary

Let $N = (P, T, F, V, m_0)$ be a eS-Net. There exists a marking m^* such that $N = (P, T, F, V, m^*)$ unbounded, iff $C \cdot x > 0$ has an integer nonnegative solution.

Useful application of the corollary:

If there does not exist an integer nonnegative solution for $C \cdot x > 0$, then for any initial marking, N is bounded.

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Transition-Invariants (T-Invariants)

Let $N = (P, T, F, V, m_0)$ be a eS-Net.

- Any nontrivial integer solution x of the homogenous linear equation system $C \cdot x = 0$ is called *transition-invariant (T-invariant)* of N.
- A T-invariant x is called *proper*, if $x \geq 0$.
- A T-invariant x is called *realizable* in N, if there exists a word $q \in W(T)$ with $\bar{q} = x$ and a reachable marking m such that $m[\, q \succ m$.
- N is called *covered with T-invariants*, if there exists a T-invariant x of N with all components positive, i.e. greater than 0.

Proper T-invariants denote *possible* cycles of the reachability graph - realizable T-invariants denote cycles which indeed may occur.

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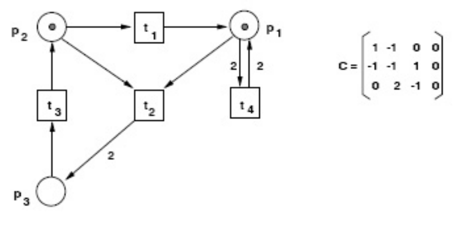

are as follows:

$$x = \lambda_1 \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

where λ_1, λ_2 integers.

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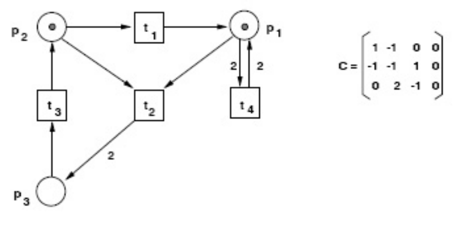

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Theorem

Let $N = (S, T, F, V, m_0)$ be a eS-Net. If there exists a marking m, such that N live and bounded at m, then N covered by T-invariants.

Proof: Let N live and bounded at some m.

As N is live at m, there exists a word $q_1 \in L_N(m)$, which contains all transitions in T and the marking $m + \Delta q_1$ is reachable from m.

Moreover, N is live at $m + \Delta q_1$ as well. Therefore, there exits a word $q_2 \in L_N(m)$, which contains all transitions in T and N is live at the marking $m + \Delta q_1 q_2$.

There exists an infinite sequence of markings (m_i), where $m_i := m + \Delta q_1 \dots q_i$, such that:

$$m[\, q_1 \succ m_1 [\, q_2 \succ m_2 \dots m_i [\, q_{i+1} \succ m_{i+1} \dots$$

As N is bounded at m, there is only a finite number of markings which are reachable. Therefore, there exist $i, j \in NAT : i < j$ such that $m_i = m_j$. Thus

$$m_i [\, q_{i+1} \dots q_j \succ m_j = m_i$$

As all these q_i mention all transitions, we finally conclude

$$x = \bar{q}_{i+1} + \dots + \bar{q}_j$$

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Useful application of the theorem:

Whenever N is not covered by T-invariants, then for every marking it holds N not live or not bounded.

Place-Invariants (P-Invariants)

Let $N = (P, T, F, V, m_0)$ be a eS-Net.

- Any nontrivial integer solution y of the homogeneous linear equation system $y \cdot C = 0$ is called *place-invariant (P-invariant)* of N.
- A P-invariant y is called *proper P-invariant*, if $y \geq 0$.
- N is called *covered with P-invariants*, if there exists a P-invariant y with all components positive, i.e. greater than 0.

If y is a P-invariant, then for any marking m the sum of the number of tokens on the places p is invariant with respect to the firing of the transitions weighted by $y(p)$.

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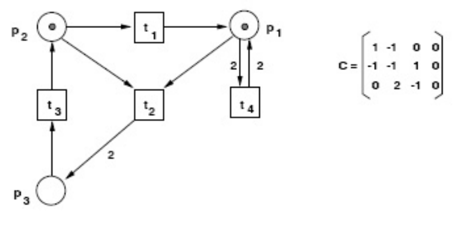

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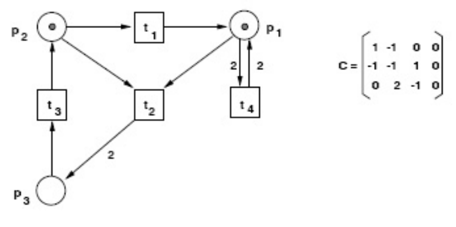

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Theorem

Let $N = (P, T, F, V, m_0)$ a eS-Net and let y a P-invariant of N. Then:

$$m \in R_N(m_0) \Rightarrow y \cdot m^\top = y \cdot m_0^\top .$$

Proof:
Assume $m_0[\, q \succ m$. Then $m = m_0 + (C \cdot \bar{q})^\top$ and also:

$$y \cdot m^\top = y \cdot m_0^\top + y \cdot (C \cdot \bar{q}) =$$

$$= y \cdot m_0^\top + (y \cdot C) \cdot \bar{q} = y \cdot m_0^\top + 0 \cdot \bar{q} = y \cdot m_0^\top .$$

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Corollary:

- Let y P-invariante of N, m marking.

  $y \cdot m^\top \neq y \cdot m_0^\top \Rightarrow m \notin R_N(m_0)$.

- Let y proper P-invariant of N. Let $p \in P$ such that $y(p) > 0$.

  Then, for any initial marking, s is bounded.

  Proof: $y \cdot m_0^\top = y \cdot m^\top \geq y(p) \cdot m(p) \geq m(p)$.

- Let N be covered by P-invariants. N is bounded for any initial marking.

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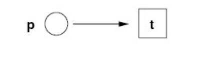
Note, the following net is bounded for any initial marking, however does not have a P-invariant:



P-invariants allow sufficient tests for non-reachability and boundedeness.

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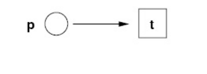
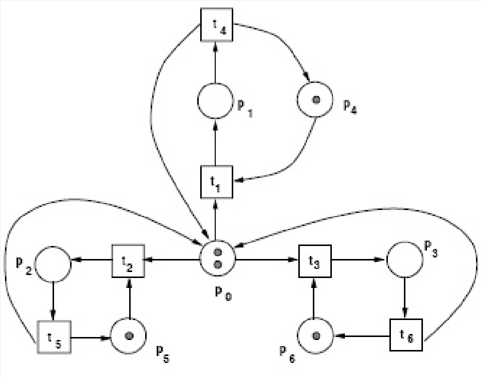


P-invariants allow sufficient tests for non-reachability and boundedeness.

Example: Prove freedom from deadlocks.



$$C = \begin{bmatrix} -1 & -1 & -1 & 1 & 1 & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

P-invariants:

$$Y_1 = (0,1,0,0,1,0,0)$$
$$Y_2 = (0,0,1,0,0,1,0)$$
$$Y_3 = (0,0,0,1,0,0,1)$$
$$Y_4 = (1,1,1,1,0,0,0)$$

Initial marking is given by $m_0 = (2,0,0,0,1,1,1)$. Assume there exist a dead marking m, $m_0[\,q \succ m$. Then it must hold $m(p_1) = m(p_2) = m(p_3) = 0$. Because of Y_4 it follows $m(p_0) = 2$. As m dead it follows $m(p_4) = m(p_5) = m(p_6) = 0$. However this contradicts $Y_1 m_0 = Y_1 m$.

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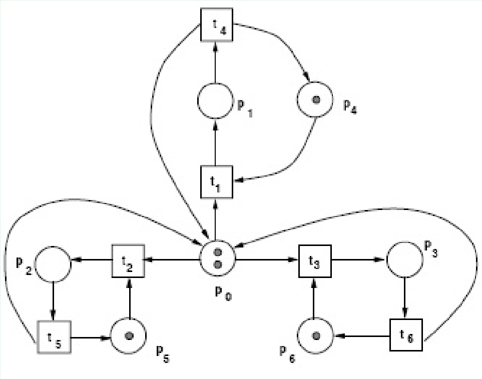

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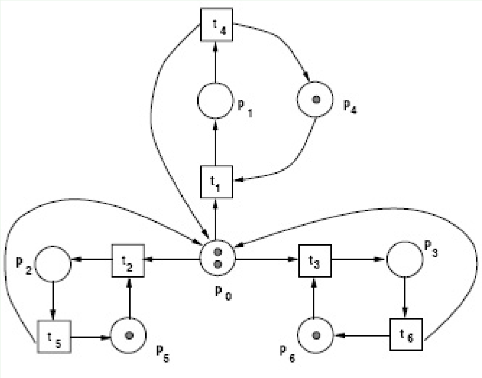

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Section 12.5 Place Capacities

Sometimes when modelling we would like to fix an upper bound for the number of tokens in a place.

- Let $N = (P, T, F, V, m_0)$ be a eS-Net, c a ω-marking of P and let $m_0 \leq c$. (N, c) is called *eS-Net with capacities*. $c(p), p \in P$ is called *capacity* of p.
- For eS-nets with capacities the notion of being enabled is adapted:

  a transition $t \in T$ *is* enabled at marking m, *if* $t^- \leq m$ *and* $m + \Delta t \leq c$.

- Capacities graphically are labels of places - no label means capacity ω.

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Any eS-net with capacities can be simulated by a eS-Net without capacities.

Construction

- Let p a palce with capacity $k = c(p), k \geq 1$. Let p^{co} be the complementary place of p which is assigned the initial marking $k - m_0(p)$.
- Whenever for a transition t we have $\Delta t(p) > 0$, we introduce an arc from p^{co} to t with multiplicity $\Delta t(p)$;
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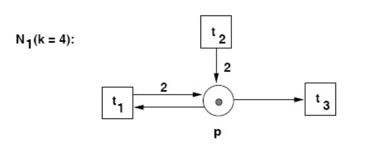
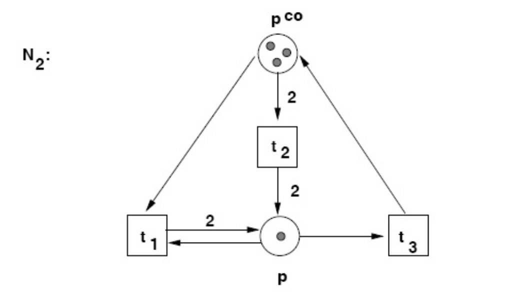
A eS-Net with capacities and its simulation by a bounded eS-Net.

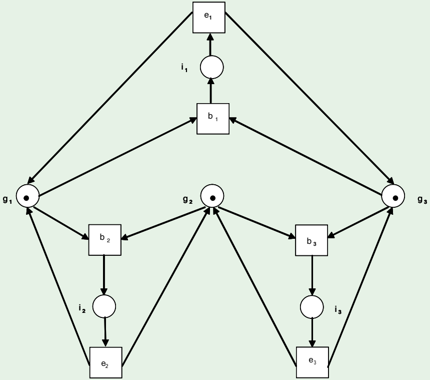




Section 12.6 S-Nets with Colors

- eS-Nets in practice may become huge and difficult to understand.
- Sometimes such nets exhibit certain regularities which give rise to questions how to reduce the size of the net without losing modeling properties.

What about a n-philosopher problem with $n >> 3$?



Why not introduce tokens with individual information?

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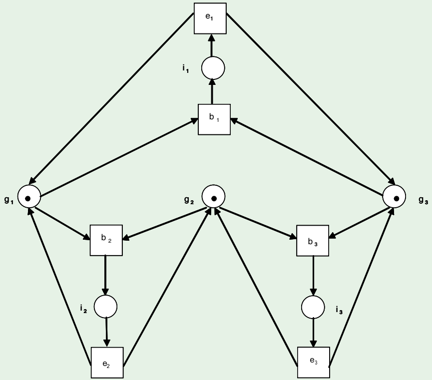
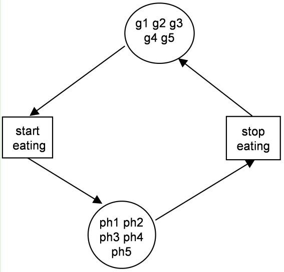


Why not introduce tokens with individual information?

Abstraction 5-philosopher problem

Note: the intention of the marking shown only is to demonstrate „individual“ tokens.



What about being enabled and firing?

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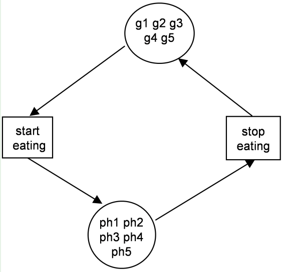

What about being enabled and firing?

Colored System-Nets

A colored System-Net distinguishes different kinds of sorts for markings - the so called *colors* - and functions over these sorts which are used to label the edges of the net.

Generalizing eS-Nets, in a colored net a transition will be called enabled, if certain conditions are true, which are based on the functions which are assigned to the edges of the transitions surrounding.

Thus, we have colors, to characterize markings (*place colors*), and colors, to characterize the firing of transitions (*transition colors*).

As a marking of a place now can be built out of different kind of tokens, we introduce multisets.

- Let A be a set. A *multiset m* over A is given by a maping $m : A \to NAT$.
- Let $a \in A$. If $m[a] = k$ then there exist k occurences of a in m.
- A multiset oftenly is written as a (formal) sum, e.g. $[Apple, Apple, Pear]$ is written as $2 \cdot Apple + 1 \cdot Pear$.

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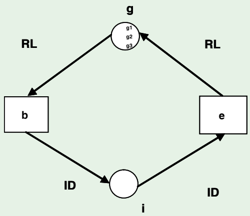
A colored version of the 3-Philosopher-Problem



Colors

$$C(g) = \{g_1, g_2, g_3\},\ C(i) = \{ph_1, ph_2, ph_3\} \quad \text{place colors}$$
$$C(b) = \{ph_1, ph_2, ph_3\},\ C(e) = \{ph_1, ph_2, ph_3\} \quad \text{transition colors}$$

Functions

$$ID(ph_j) := 1 \cdot ph_j, 1 \le j \le 3$$
$$RL(ph_j) := \begin{cases} 1 \cdot g_1 + 1 \cdot g_3 & \text{if } j = 1, \\ 1 \cdot g_{j-1} + 1 \cdot g_j & \text{if } j \in \{2, 3\}. \end{cases}$$

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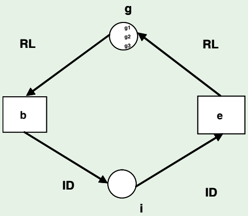

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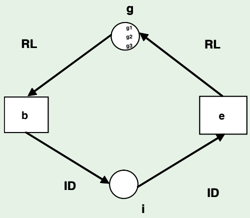

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Multiplicities

A *multiplicity* assigned to an edge between a place p and a transition t is a mapping from the set of transition colors of t into the set of multisets over the colors of p.

In the example:

$$V(b,i) = V(i,e) = ID, \; V(g,b) = V(e,g) = RL,$$

where:

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ID denotes the identity mapping.

Marking

Markings are multisets over the respective place colors.

In the example:

$$m_0(p) := \begin{cases} 1 \cdot g_1 + 1 \cdot g_2 + 1 \cdot g_3 & \text{if } p = g, \\ 0 & \text{otherwise.} \end{cases}$$

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A *colored Net* $CN = (P, T, F, C, V, m_0)$ is given by:

- A net (P, T, F).
- A mapping C which assignes to each $x \in P \cup T$ a finite nonempty set $C(x)$ of *colors*.
- Mapping V assignes to each edge $f \in F$ a mapping $V(f)$.

  Let f be an edge connecting palce p and transition t.
  $V(f)$ is a mapping from $C(t)$ into the set of multisets over $C(p)$.
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$$V(p,t)(d) \leq m(p).$$

- Assume t is enabled in color d at marking m. Firing of t in color d transforms m to a marking m':

$$m'(p) := \begin{cases} m(p) - V(p,t)(d) + V(t,p)(d) & \text{if } p \in Ft, \\ & p \in tF, \\ m(p) - V(p,t)(d) & \text{if } p \in Ft,, \\ & p \notin tF, \\ m(p) + V(t,p)(d) & \text{if } p \notin Ft,, \\ & p \in tF, \\ m(p) & \text{otherwise.} \end{cases}$$

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$$m'(p) := \begin{cases} m(p) - V(p,t)(d) + V(t,p)(d) & \text{if } p \in Ft, \\ & p \in tF, \\ m(p) - V(p,t)(d) & \text{if } p \in Ft,, \\ & p \notin tF, \\ m(p) + V(t,p)(d) & \text{if } p \notin Ft,, \\ & p \in tF, \\ m(p) & \text{otherwise.} \end{cases}$$

Let $CN = (P, T, F, C, V, m_0)$ be a colored System-Net.

- A marking m of P is mapping which assignes to each place p a multiset $m(p)$ over $C(p)$.
- A transition t *is enabled in color* $d \in C(t)$ at m, if for all pre-places $p \in Ft$ there holds:

$$V(p,t)(d) \leq m(p).$$

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Fold and Unfold of a Colored System-Net

Folding

By folding of a eS-Net we can reduce the number of places and transitions; places and transitions are represented by appropriate place and transition colors, on which certain functions defining the multiplicities are defined.

Let $N = (P, T, F, V, m_0)$ a eS-Net. A folding is defined by π and τ:

- $\pi = \{q_1, \ldots, q_k\}$ a (disjoint) partition of P,
- $\tau = \{u_1, \ldots, u_n\}$ a (disjoint) partition of T.

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Two special cases

Call $GN(\pi, \tau) := (P', T', F', C', V', m_0')$ the result of folding.

- All elements of π, τ are one-elementary:

$$\Rightarrow N \text{ and } GN(\pi, \tau) \text{ are isomorph,}$$

- π, τ contain only one element:

$$\Rightarrow |P'| = |T'| = 1, \text{"the model is represented by the labellings".}$$

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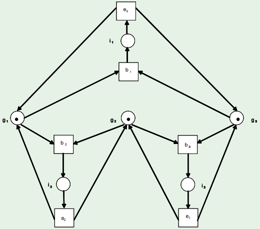
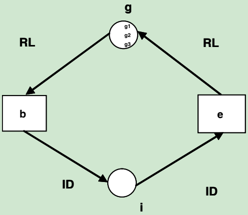
3-Philosopher-Problem



Folding $\pi = \{\{g_1, g_2, g_3\}, \{i_1, i_2, i_3\}\}$, $\tau = \{\{b_1, b_2, b_3\}, \{e_1, e_2, e_3\}\}$.

Colors from folding:
$C(g) = \{g_1, g_2, g_3\}, C(i) = \{i_1, i_2, i_3\}, C(b) = \{b_1, b_2, b_3\}, C(e) = \{e_1, e_2, e_3\}$

Multiplicities: ID, RL analogously to previous version.



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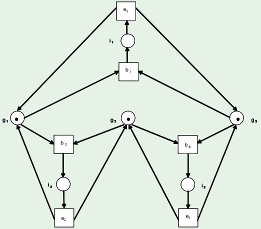

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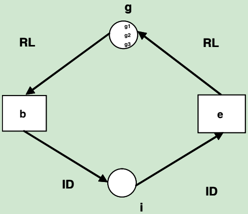

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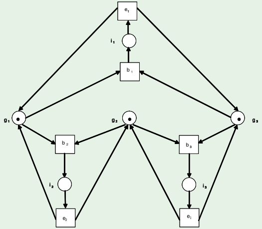

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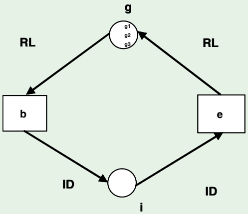
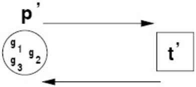


3-Philosopher-Problem?

$\pi = \{P\},\ \tau = \{T\}$:
$S' = \{s'\},\ T' = \{t'\}$,
$C(s') = \{g_1, g_2, g_3, i_1, i_2, i_3\}$,
$C(t') = \{b_1, b_2, b_3, e_1, e_2, e_3\}$,
$m'_0(s') = g_1 + g_2 + g_3$,



$$V'(s',t')(t) = \begin{cases} g_1 + g_3 & \text{falls } t = b_1, \\ g_1 + g_2 & \text{falls } t = b_2, \\ g_2 + g_3 & \text{falls } t = b_3, \\ i_1 & \text{falls } t = e_1, \\ i_2 & \text{falls } t = e_2, \\ i_3 & \text{falls } t = e_3, \end{cases} \qquad V'(t',s')(t) = \begin{cases} g_1 + g_3 & \text{falls } t = e_1, \\ g_1 + g_2 & \text{falls } t = e_2, \\ g_2 + g_3 & \text{falls } t = e_3, \\ i_1 & \text{falls } t = b_1, \\ i_2 & \text{falls } t = b_2, \\ i_3 & \text{falls } t = b_3. \end{cases}$$

Given $\pi = \{q_1, \ldots, q_k\}$, $\tau = \{u_1, \ldots, u_n\}$.

The *folding* $GN(\pi, \tau) := (P', T', F', C', V', m'_0)$ of N is defined as follows:

- $P' := \{p'_1, \ldots, p'_k\}$; $T' := \{t'_1, \ldots, t'_n\}$,
- $C'(p'_i) = q_i$ für $i = 1, \ldots, k$; $C'(t'_j) = u_j$ für $j = 1, \ldots, n$,
- $F' := \{(p', t') \mid C'(p') \times C'(t') \cap F \neq \emptyset\} \cup \{(t', p') \mid C'(t') \times C'(p') \cap F \neq \emptyset\}$,
- $f' = (p', t') \in F'$: $V'(f')$ is defined $(t \in C'(t'))$:

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Unfolding

Let $GN = (P, T, F, C, V, m_0)$ a CN-Net.

The *Unfolding* of GN is a eS-Net $GN^* := (P^*, T^*, F^*, V^*, m_0^*)$ given as follows:

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Definition

Let E be a certain property of a net, e.g. boundedness, liveness, or reachability.

A CS-Net GN has property E, whenever its unfolding GN^* has property E.

Analysis of colored System Nets

- Analyse unfolding:

  Advantage: Methods exist,
  Pitfall: Unfoldings may be huge eS-Nets.

- Analyse colored net:
  - Reachability graph and coverability graph can be defined in analogous way to eS-Nets.
  - There exists a theory for invariants, as well.
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