Section 12.7 Workflow-Nets

Literature:

van der Aalst, Hofstede: http://is.tm.tue.nl/staff/wvdaalst/publications/p174.pdf

Workflow (WF)-Net

A eS-Net $N = (P, T, F)$ is a WF-Net, if

- There exists an *input-place* $i \in P$ where $Fi = \emptyset$.
- There exists an *output-place* $o \in P$ where $oF = \emptyset$.
- In N, every $x \in P \cup T$ is contained in a path from i to o.

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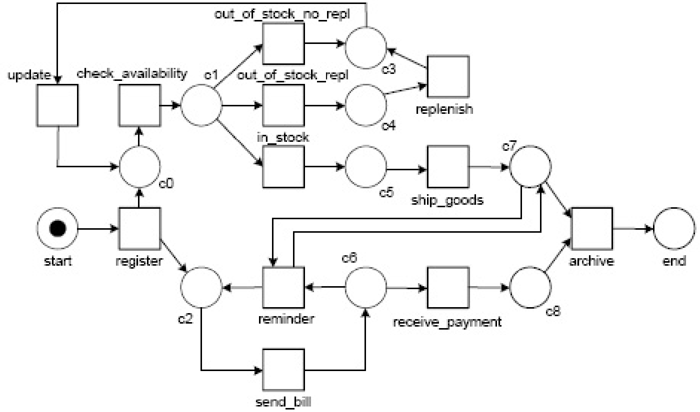
Example: WF-net order handling



Properties of a WF-Net

Let $N = (P, T, F)$ a WF-Net with input-place $i$and output-place o.

- For $p \in P$ there holds $Fp \neq \emptyset$ or $p = i$.
- For $p \in P$ there holds $pF \neq \emptyset$ or $p = o$.
- Let $\overline{N} = (\overline{P}, \overline{T}, \overline{F})$, where $\overline{P} = P$, $\overline{T} = T \cup \{t^*\}$ and $\overline{F} = F \cup \{(o, t^*), (t^*, i)\}$.

  $\overline{N}$ is called the *shortcut* net of N.

  $\overline{N}$ is strongly connected.

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Sound WF-Nets

A WF-Net is called *sound*, if the following holds.

Let m_i be a initial marking, such that only the input place i is marked.
Let m_o be a output marking, such that only the out-put place o is marked.

- From every marking m, which is reachable from m_i, marking m_o is reachable.
- m_o is the only marking reachable from m_i for which o is marked.
- The WF-Net does not contain dead transitions.

Theorem

A WF-Net N is sound iff $(\overline{N}, m_i)$ is life and bounded.

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Lemma

A WF-Net N is sound, if $(\overline{N}, m_i)$ live and bounded.

Proof

As $(\overline{N}, m_i)$ live there exists for any reachable marking m (including m_i) a firing word leading to a marking m' such that t^* is enabled. Therefore o is marked in m'.

Consider an arbitrary such marking m' which is reachable from m_i, i.e. $m' = m'' + m_o$. t^* is enabled in m'. Thus marking $m'' + m_i$ is reachable from m_i. As $(\overline{N}, m_i)$ is bounded we have $m'' = 0$.

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Lemma

Whenever a WF-Net N is sound, then $(\overline{N}, m_i)$ is bounded.

Proof

We show (N, m_i) bounded.

Assume (N, m_i) is not bounded. Then there exist markings m_1, m_2, such that $m_i[*\succ m_1$, $m_1[*\succ m_2$ and $m_2 > m_1$.

As N sound we have $m_1[q\succ m_o$. Moreover, because of $m_2 > m_1$, there exists a marking m with $m_2[q\succ m$ and $m > m_o$. This is a contradiction to N sound.

N sound and (N, m_i) bounded implies $(\overline{N}, m_i)$ bounded.

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Lemma

If a WF-Netz N is sound, then $(\overline{N}, m_i)$ is life.

Proof

As N sound, from any marking m' which is reachable from m_i, we can reach m_o.

Therefore, from any m', which is reachable in $(\overline{N}, m_i)$, we can reach m_i. As N does not have any dead transitions w.r.t. m_i, it follows $(\overline{N}, m_i)$ is live.

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Excursus: Net-Classes

Let $N = (P, T, F, V, m_0)$.

- N is called *Synchronization-Graph*, if for each place p it holds $\mid Fp \mid = \mid pF \mid = 1$.
- N is called *Statemachine*, if for each transition t it holds $\mid Ft \mid = \mid tF \mid = 1$.
- N is called *Free-Choice-Net (FC-Net)*, if $t, t' \in pF \Rightarrow Ft = \{s\} = Ft'$.
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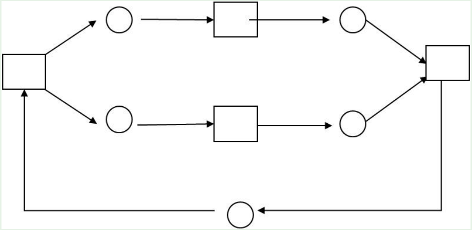
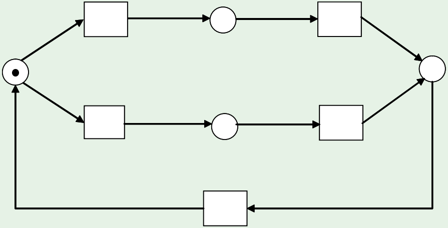
- A synchronization-graph is also a FC-Net.
- A statemachine is also a FC-Net.
- A FC-Net is also a EFC-Net.

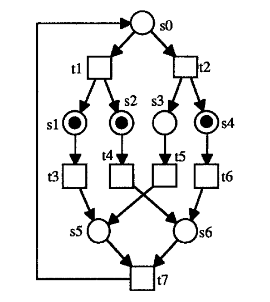
Synchronization-Graph



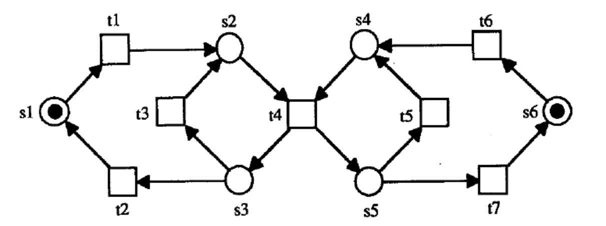
Statemachine



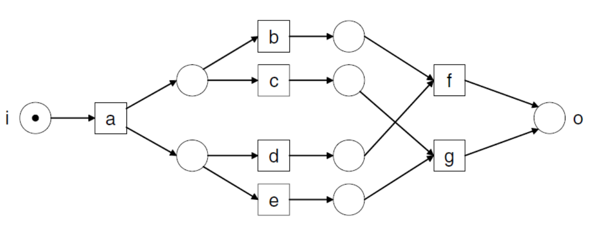
FC-Net


s0
t1
t2
s1
s2
s3
s4
t3
t4
t5
t6
s5
s6
t7

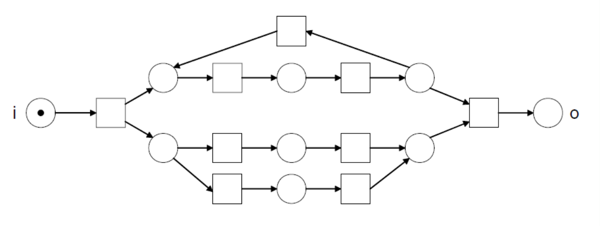
FC-Net


t1
s2
s4
t6
s1
t3
t4
t5
s6
t2
s3
s5
t7

A not sound WF-Net; the WF-Net is free-choice



A WF-Net which is sound, however not free-choice



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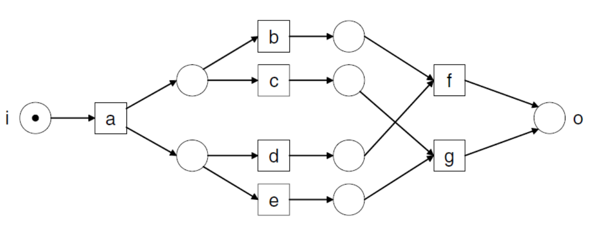

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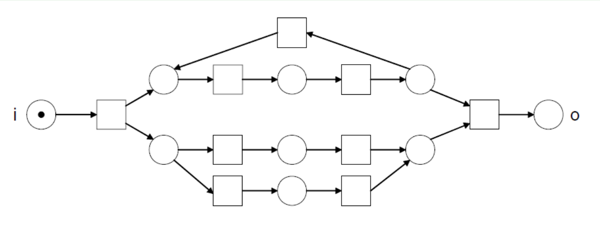

Soundness of a WF-Net

A WF-Net, which is a FC-Net, can be checked for soundness in polynomial time.

... from practical experiences:

For modeling in practical applications FC-Nets are sufficient.

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Example: WF-Net order handling - make it free-choice!