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Distributed Systems

Chapter 4 Coordination and Agreement

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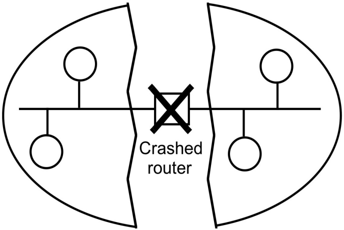
4.1: Introduction

- Coordination in the absence of master-slave relationship
- Failures and how to deal with it
- Distributed mutual exclusion
- Agreement is a complex problem
- Multicast communication
- Byzantine agreement

Assumptions

- Channels are reliable
- The network remains connected
- Process failures are not a threat to the communication
- Processes only fail by crashing



Failure Detectors

- Failure detector is a service answer queries about the failures of *other* processes
- Most failure detectors are *unreliable failure detectors*
  - Returning either *suspected* or *unsuspected*
  - *suspected*: some indication of process failure
  - *unsuspected*: no evidence for process failure
- *Reliable failure detector*
  - Returning either *failed* or *unsuspected*
  - *failed*: detector has determined that the process has failed
  - *unsuspected*: no evidence for failure

Example of an unreliable failure detector

- Each process p sends a 'p is here' message to every other process every T seconds
- If the message does not arrive within $T + D$ seconds then the process is reported as *Suspected*

4.2: Distributed Mutual Exclusion

- Problem known from operating systems (there: *critical sections*)
- How to achieve mutual exclusion only with messages

Application-Level Protocol

enter()	enter critical section – block if necessary
resourceAccesses()	access shared resources in critical section
exit()	leave critical section – other processes may enter

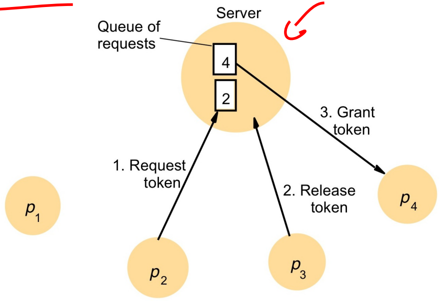
Essential Requirements

ME1: Safety	At most one process may execute the critical section at a time
ME2: Liveness	Requests to enter and exist the critical section eventually succeed
ME3: $\rightarrow$ ordering	requests enter the critical section according to the *happened-before* relationship

Performance of algorithms for mutual exclusion

- *Bandwidth* consumed: proportional to the number of messages sent in each *entry* and *exit* operation
- *Client delay* at each *entry* and *exit* operation
- *Throughput* rate of several processes entering the critical section
- Throughput is measured by the *synchronization delay* between one process exiting the critical section and the next process entering it
- short *synchronization delay* correspond to high *throughput*

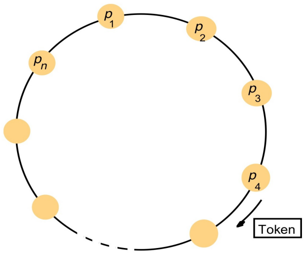
Central Server Algorithm



from *Distributed Systems – Concepts and Design*, Coulouris, Dollimore, Kindberg

- Simplest solution
- Request are handled by queues
- Performance
  - Entering the critical section: two messages (*request*, *grant*)
  - Leaving the critical section: one message (*release*)
- Server is performance bottleneck

Ring Based Algorithm



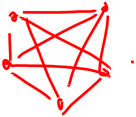
from *Distributed Systems – Concepts and Design*,

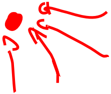
Coulouris, Dollimore, Kindberg

- Simplest distributed solution
- Arrange processes as ring (not related to physical network)
- A token (permission to enter critical section) is passed around
- Conditions ME1 (safety) and ME2 (liveness) are met
- ME3: $\rightarrow$ ordering is not fulfilled
- Continuous consumption of bandwidth
- Synchronisation delay is between 1 and n messages.

The Algorithm of Ricart and Agrawala

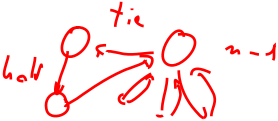
- Mutual exclusion between n peer processes $p_1, p_2, \ldots, p_n$ which
  - have unique numeric identifiers
  - possess communication channels to one another
  - keep Lamport clocks attached to the messages





- Process states
  - `released`: outside the critical section
  - `wanted`: wanting to enter critical section
  - `held`: being in the critical section
- Each process `released` immediately answers a request to enter the critical section
- The process with `held` does not reply to requests until it is finished
- If more than one process requests the entry, the first one collecting the $n-1$ replies is allowed to enter the critical section.
- If the Lamport clocks of the latest messages do not differ, the numeric ID is used to break the tie.

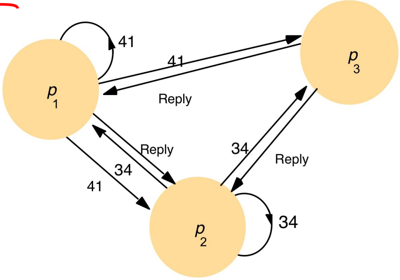
The Algorithm of Ricart and Agrawala



On initialization
  state := RELEASED;
To enter the section
  state := WANTED;
  Multicast *request* to all processes;
  T := request's timestamp;
  Wait until (number of replies received = $(N - 1)$);
  state := HELD;

request processing deferred here

On receipt of a request $<T_i, p_i>$ *at* p_j $(i \neq j)$
  if (*state* = HELD or (*state* = WANTED *and* $(T, p_j) < (T_i, p_i)$))
  then
    queue *request* from p_i without replying;
  else
    reply immediately to p_i;
  end if
To exit the critical section
  state := RELEASED;
  reply to any queued requests;

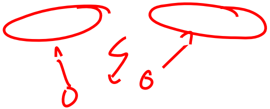


from *Distributed Systems – Concepts and Design*, Coulouris, Dollimore, Kindberg

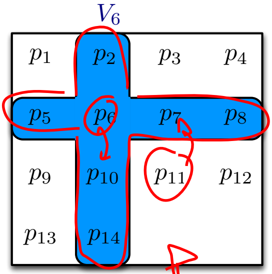
The Algorithm of Ricart and Agrawala

- Mutual exclusion properties
  - ME1 (safety): processes in state `held` prevent other ones from entering the CS
  - ME2 (liveness): follows from the ordering
  - ME3 (ordering): follows from the use of Lamport clocks
- Cost of gaining access: $2(n-1)$ messages
  - $n-1$ for multicast of request
  - $n-1$ for replies
- Client delay for requesting entry: a round-trip message
- Synchronization delay is one message transmission time

Maekawa's Voting algorithm



- Reduce the number of messages by asking a subset
- For each process p_i choose a *voting set* V_i such that
  1. $p_i \in V_i$
  2. $V_i \cap V_j \neq \emptyset$ for all i, j
  3. $|V_i| = k$ for all i (fairness)
  4. Each process occurs in at most m voting sets
- Minimal choice of $\max\{m, k\}$ is $k, m \in \Theta(\sqrt{n})$.
- The optimal solution can be approximated by placing all nodes in a square matrix and choosing the row and column as voting set.



from

optimal

Distributed Systems – Concepts and Design, Coulouris, Dollimore, Kindberg

Maekawa's Voting algorithm

On initialization
  state := RELEASED;
  voted := FALSE;

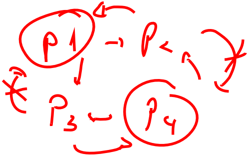
For p_i to enter the critical section
  state := WANTED;
  Multicast *request* to all processes in V_i;
  Wait until (number of replies received = K);
  state := HELD;

On receipt of a request from p_i at p_j
  if (*state* = HELD *or* *voted* = TRUE)
  then
    queue *request* from p_i without replying;
  else
    send *reply* to p_i;
    voted := TRUE;
  end if

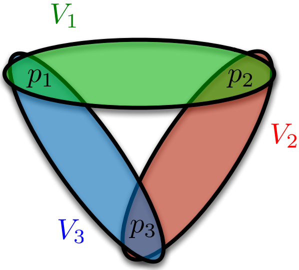
For p_i to exit the critical section
  state := RELEASED;
  Multicast *release* to all processes in V_i;

On receipt of a release from p_i at p_j
  if (queue of requests is non-empty)
  then
    remove head of queue – from p_k, say;
    send *reply* to p_k;
    voted := TRUE;
  else
    voted := FALSE;
  end if

Maekawa's Voting algorithm



- Mutual exclusion properties
  - ME1 (safety): follows from the intersections of V_i and V_j
  - ME2 (liveness): not guaranteed.
- Sanders improved this algorithm to achieve ME2 and ME3 (not presented here)
- Cost
  - $2k$ per entry to the critical section
  - k for exit
  - $O(\sqrt{n})$ messages
- Client delay for requesting entry: a round-trip message
- Synchronization delay is a round-trip message



Mutual Exclusion

Fault Tolerance

- What happens when messages are lost
- What happens when process crashes

- All of the above algorithms presented fail
- We will revisit this problem

4.3: Elections

Election Algorithm

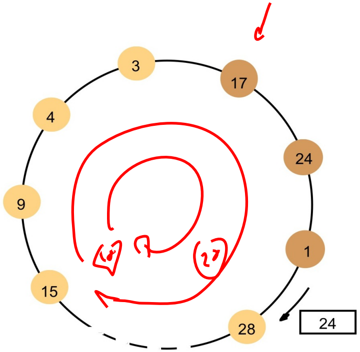
- An algorithm for choosing a unique process from a set of processes $p_1, \ldots, p_n$.
- A process *calls the election* if it initiates a run of an election algorithm
- Several elections could run in parallel where subset of processes are *participants* or *non-participants*.
- We assume processes have numeric IDs and that wlog. the process with the highest will be chosen.

Requirements

E1: Safety	During the run each participant has either $\text{elected}_i = \perp$ or $\text{elected}_i = P$, where P is the non-crashed process with the largest ID
E2: Liveness	All participating processes p_i eventually set $\text{elected}_i \neq \perp$ or crash.

Ring-Based Election: Algorithm of Chang and Roberts

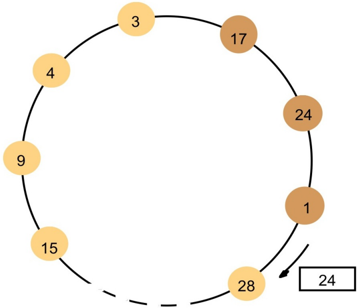
- Each process p_i has a communication channel to the next process in the ring $p_{(i+1) \bmod n}$
- Messages are sent clockwise
- Assumption: no failures occur
- Non-participants are marked
- When a process receives an election message, it compares the identifier
  - If the arrived ID is greater, it forwards it
  - if the arrived ID is smaller and the process participates, it replaces it with its ID
  - if the arrived ID equals the process ID, the process is elected and sends an elected message around (with its ID).



Note: The election was started by process 17.
The highest process identifier encountered so far is 24.
Participant processes are shown darkened

Ring-Based Election: Algorithm of Chang and Roberts

- E1 (Safety): follows directly
- E2 (Liveness): follows in the absence of crashes and communication errors
- Worst-case performance if a single node participates in the process
- Time: $\overset{2}{\cancel{3}}n - 1$ messages for the election
- Not very practical algorithm fault-prone and high communication overhead
- assumes a-priori knowledge (ring topology)



Note: The election was started by process 17.
The highest process identifier encountered so far is 24.
Participant processes are shown darkened

The Bully Algorithm of Garcia & Molina

- The distributed system is assumed to be synchronous
  - i.e. after a timeout period T a missing answer is interpreted as crash
  - reliable failure detector
  - fail-stop model
- Message types
  - *election*: Announces an election
  - *answer*: Answers *election* message (contains ID)
  - *coordinator*: Announces the identity of the elected process
- Any process may trigger an *election*
- Every process receiving an *election* messages sends an *answer* and starts a new one (if it has not started one before).
- If a process knows it has the highest ID (based on the answers) it sends the *coordinator* message to all processes
- If answers of lower IDs fail to arrive within time T the sender considers itself a coordinator and sends the *coordinator* message

The Bully Algorithm of Garcia & Molina

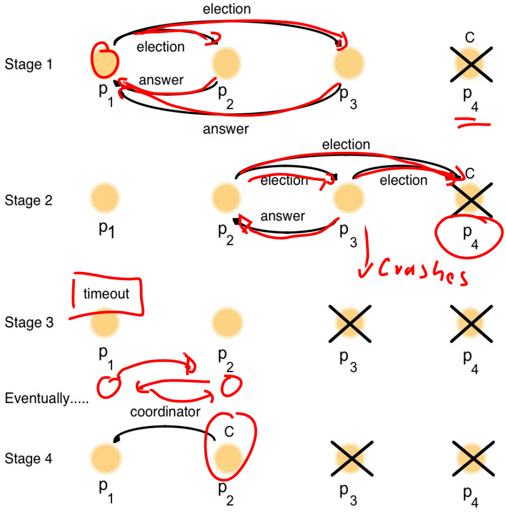
n processes

n^2 # message

$n^2 + (n-1)^2 + (n-2)^2 + \ldots + 1$

$= O(n^3)$

- If a process receives an *election* message it sends back an *answer* messages and begins another election — if it has not begun an election
- If a process knows it has the highest ID it sends the *coordinator* message
- New arriving processes with higher ID „bully" existing cordinators



The Bully Algorithm of Garcia & Molina

- E2: liveness condition is guaranteed if messages are transmitted reliably
- E1: safety condition: Not guaranteed if processes are replaced by processes with the same identifier
- different conclusions on which is the coordinator process
- E1 not guaranteed if the timeout value is too small
- In the worst case the algorithm needs $O(n^2)$ messages for n processes

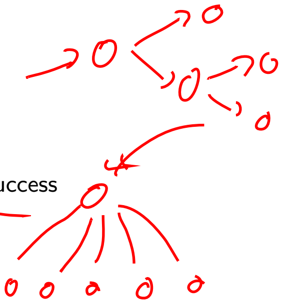
if they stay alive

4.4: Multicast communication

- broadcast — one to all
- multicast — one to a group
- unicast — one to one

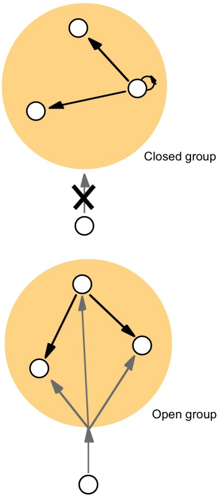
- With a single call of *multicast*(g, m) a process sends a message to all members of the group g
- Using *deliver*(m), received messages are delivered on participating processes
- *Efficiency*
  - Number of messages, transmission time
- *Delivery guarantees*
  - ordering
  - receipt
  - e.g. IP Multicast does not guarantee ordering ~~of~~ or success

IP: Internet Protocol



4.4: Multicast communication

- *System Model*
  - *multicast*(g, m): sends the message m to all members of group g
  - *deliver*(m): delivers a message to the process (message has been received by lower level)
  - *sender*(m): sender of a message m (within the message header)
  - *group*(m): group of a message m (within the message header)
- Allowed senders
  - closed group: senders must be members of a group
  - open group: any process can send a message to the group



Basic Multicast

- *B-multicast*(g, m): for each process $p \in g$, *send*(p, m)
- *B-deliver*(m): if message m is received at p return the message m

Ack Implosion

- if too many processes participate
- if*send* uses acknowledgments, some of them could be dropped
- then the messages could be retransmitted
- further *acks* are lost due to full buffers etc.

Reliable Multicast

- *Safety: Integrity*
  - Every message is delivered at most once
  - Receiver of m is a member of $group(m)$
  - Sender has initiated a $multicast(g, m)$
- *Liveness: Validity*
  - If a correct process multicasts a messages then it eventually delivers m (to itself)
- *Agreement*
  - If a correct process delivers m then all other processes eventually deliver m

finally

Implementing Reliable Multicast over Basic Multicast

On initialization
    $Received := \{\}$;

For process p to R-multicast message m to group g
    B-multicast(g, m);   // $p \in g$ is included as a destination

On B-deliver(m) at process q with g = group(m)
    if ($m \notin Received$)
    then
        $Received := Received \cup \{m\}$;
        if ($q \neq p$) *then B-multicast(g, m); end if*
        R-deliver m;
    end if

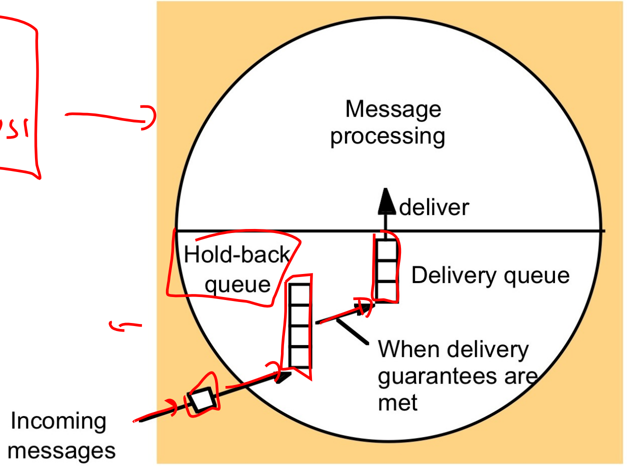
Each message needs to be sent $|g|$ times!

Implementing Reliable Multicast over IP Multicast

- *R-multicast*(g, m) for sending process p
  - Sender increments a (sending) sequence number S_g^p for group g after each messages
  - Sequence number sent with message
  - Acknowledgements of all received messages with $\langle q, R_g^q \rangle$ are piggy backed with message
  - Negative Acknowledgments: by received sequence number R_g^q causes retransmission of message
- *R-deliver*(g) for receiving process q
  - R_g^q is the sequence number of the latest message it has delivered.
  - it is send with each acknowledgment and allows the sender (and all receivers) to learn about missing messages
  - Process q *delivers* a message m (with piggy backed S) only if $S = R_g^q + 1$.
  - messages with $S > R_g^q + 1$ are kept in a hold-back queue
  - messages with $S < R_g^g + 1$ are erased
  - After delivery $R_g^q := R_g^q + 1$

Hold-Back Queue for Arriving Multicast Messages

one process

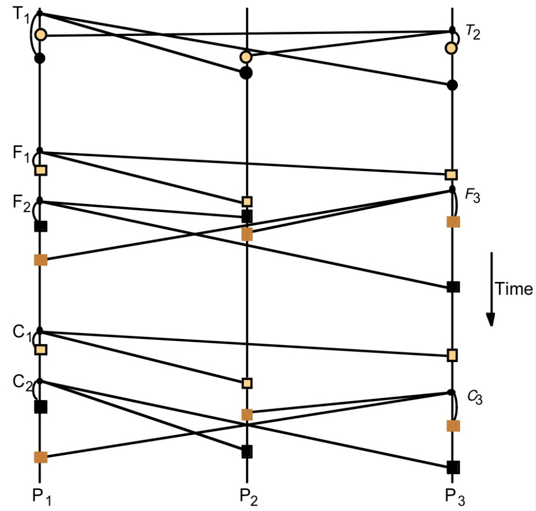


Ordered Multicast

- *FIFO Ordering*
  - If a process casts `multicast`(g, m) before `multicast`(g, m')
  - then m is delivered before m'
  - in each process of group g
- *Causal Ordering:*
  - If `multicast`$(g, m) \rightarrow$ `multicast`(g, m')
  - then m is delivered before m'
  - $\rightarrow$ is based only on messages within the group g
- *Total Ordering:*
  - If a process delivers m before m'
  - then m is delivered before m' on any other process of g

Total, FIFO and Causal Ordering

- *Total Ordering*
- *FIFO Ordering*
- *Causal Ordering*



Bulletin Board

Bulletin board: *os.interesting*		
Item	From	Subject
23	A.Hanlon	Mach
24	G.Joseph	Microkernels
25	A.Hanlon	Re: Microkernels
26	T.L'Heureux	RPC performance
27	M.Walker	Re: Mach
end		

- *FIFO Ordering*
- *Causal Ordering*
- *Total Ordering*

Implementing FIFO Ordering Multicast

- Use sequence numbers for each message
  - S_g^p for each sender process p and group g
  - R_g^p for the last message delivered to process p of group g
- Multiast over IP Multicast satsifies FIFO ordering
- Essential components for FIFO ordering:
  - Sender piggy backs S_g^p on the message
  - Receiver checks wether received message satisfies $S = R_g^q + 1$
  - and delivers m and sets $R_g^q := R_g^q + 1$.
  - if $S > R_g^q + 1$ it puts m into the hold-back queue
- In combination of a reliable multicast we obtain a reliable FIFO ordering multicast algorithm

Implementing Total Ordering Multicast with a Sequencer

- A sequencer is an extra process taking care about ordering
- A sender process sends message with unique ID *i* to sequencer
- Sequencer marks message with ordering and multicasts the message

1. Algorithm for group member *p*

On initialization: $r_g := 0$;

To TO-multicast message m to group g
    B-multicast$(g \cup \{sequencer(g)\}, <m, i>)$;

On B-deliver$(<m, i>)$ *with* $g = group(m)$
    Place $<m, i>$ in hold-back queue;

On B-deliver$(m_{order} = <\text{"order"}, i, S>)$ *with* $g = group(m_{order})$
    wait until $<m, i>$ in hold-back queue and $S = r_g$;
    TO-deliver m;   // (after deleting it from the hold-back queue)
    $r_g = S + 1$;

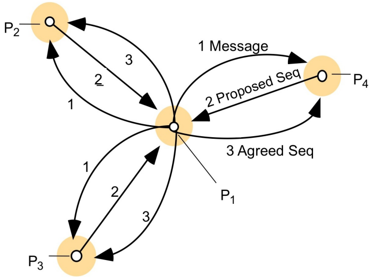
2. Algorithm for sequencer of *g*

On initialization: $s_g := 0$;

On B-deliver$(<m, i>)$ *with* $g = group(m)$
    B-multicast$(g, <\text{"order"}, i, s_g>)$;
    $s_g := s_g + 1$;

Implementing Total Ordering Multicast using ISIS

- Used in the ISIS toolkit of Birman & Joseph
- Each participating process proposes a sequence number for a messages
  - All proposed message numbers are unique
  - The sender chooses the maximum of all proposals and sends this information (piggy backed with the next messages)
  - This agreed sequence number defines the ordering of the hold-back-queue
  - The smallest elements of the hold-back queue can be delivered as the first element
- Does not imply causal nor FIFO ordering



Implementing Causal Ordering

- Uses vector clocks to keep causal ordering (piggy backed to messages)
- Vector clock $V_i^g[i]$ counts all multicast messages of process i in group g
- hold-back queue reflects vector clocks

Algorithm for group member p_i $(i = 1, 2 \ldots, N)$

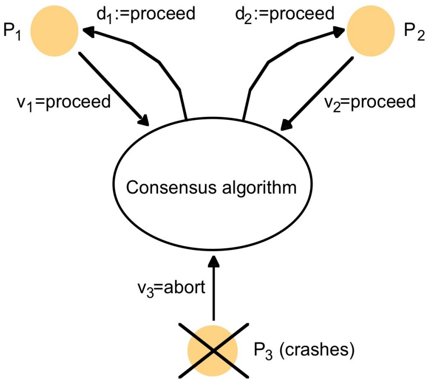
On initialization
$V_i^g[j] := 0 \ (j = 1, 2 \ldots, N);$

To CO-multicast message m to group g
$V_i^g[i] := V_i^g[i] + 1;$
B-multicast$(g, <V_i^g, m>);$

On B-deliver$(<V_j^g, m>)$ *from* p_j, *with* $g = group(m)$
place $<V_j^g, m>$ in hold-back queue;
wait until $V_j^g[j] = V_i^g[j] + 1$ and $V_j^g[k] \leq V_i^g[k] \ (k \neq j);$
CO-deliver m; // after removing it from the hold-back queue
$V_i^g[j] := V_i^g[j] + 1;$

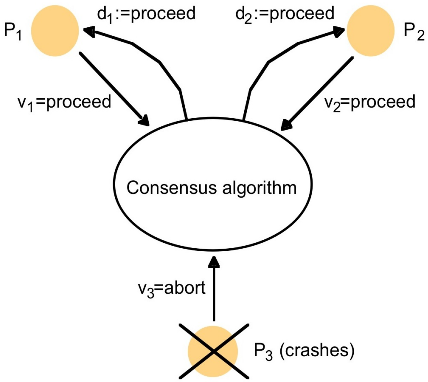
4.5: Consensus

- n processes $p_1, \ldots, p_n$
- at most f processes have arbitrary (Byzantine) failures
- Every process starts in the *undecided*state and *proposes* a value v_i
- Eventually all correct processes p_i
  - choose the *decided* state
  - and choose the same value $d_i \in \{v_1, \ldots, v_n\}$
  - and stay in this state



Consensus Problem

- *Termination:* Eventually each correct process p_i is *decided* by setting variable d_i
- *Agreement:* The decision value d_i of all correct processes is the same
- *Integrity:* If all correct process proposed the same value v, then $d_i = v$ for all correct p_i



- Possible decision functions: *majority, minimum, maximum, ...*
- Byzantine failures can cause irritating and adversarial messages
- System crashes may not be detected

Byzantine Generals Problem

- n generals have to agree on attack or retreat
- one of them is the commander and issues the order
- at most f generals are traitors (possibly also the commander) and have adversarial behavior
- all correct generals have eventually to agree on the commander decision if he acts correctly

Consensus Problem

- *Termination:* Eventually each correct process p_i is *decided* by setting variable d_i
- *Agreement:* The decision value d_i of all correct processes is the same
- *Integrity:* If the commander is correct then all correct processes choose the commander's proposal

Interactive Consistency

- n processes need to agree on a *vector* of values
- Each process proposes a value v_i
- A correct processes eventually decide on a vector $d_i = \{d_{i,1}, \ldots, d_{i,n}\}$ where

$$d_{i,j} = v_j \quad \text{if } p_j \text{ is correct}$$

Interactive Consistency

- *Termination:* Eventually each correct process p_i is *decided* by setting variable d_i
- *Agreement:* The decision value d_i of all correct processes is the same
- *Integrity:* If the p_j is correct then all correct processes p_i set $d_{i,j} = v_j$

The Relationship between Consensus Problems

Assume solutions to Consensus (C), Byzantine generals (BG), interactive consistency (IC)

$$
\begin{aligned}
C_i(v_1, \ldots, v_n) &= \text{consensus decision value of } p_i \text{ for proposals } v_i \\
BG_i(j, v) &= \text{BG decision value of } p_i \text{ for commander } p_j \text{ proposal } v_j \\
IC_i(v_1, \ldots, v_n)[j] &= j\text{-th position of interactive consistency} \\
&\quad\ \text{decision vector of } p_i \text{ for proposals } v_i
\end{aligned}
$$

Solving IC from BG

- In parallel n Byzantine generals problems are solved
- each process p_j acts as commander once

$$IC_i(v_1, \ldots, v_n)[j] = BG_i(j, v)$$

The Relationship between Consensus Problems

Solving C from IC

- *majority* returns the most often parameter or $\perp$ if no such value exists
- for all $i = 1, \ldots, n$

$$C_i(v_1, \ldots, v_n) = majority(IC_i(v_1, \ldots, v_n)[1], \ldots, IC_i(v_1, \ldots, v_n)[n])$$

Solving BG from C

- The commander p_j sends its proposed value to itself and each other process
- All processes run consenus with the values $v_1, \ldots, v_n$ received from the commander
- for all $i = 1, \ldots, n$

$$BG_i(j, v) = C_i(v_1, \ldots, v_n)$$

Consensus in a Synchronous System

- Assume that there are no arbitrary (Byzantine) errors
- Given a synchronous distributed systems (fail-stop model)
- Use basic multicast for $f + 1$ rounds
- Multicast all known values of all participants
- $Values_i^r$ denotes the set of proposed variables at the beginning of round r
- Reduce communication overhead by multicasting only freshly arrived variables $Values_i^r - Values_i^{r-1}$
- Choose the minimum of all known values as final value

Consensus in a Synchronous System

Algorithm for process $p_i \in g$; algorithm proceeds in $f + 1$ rounds

On initialization
$Values_i^1 := \{v_i\};\ Values_i^0 = \{\};$

In round r $(1 \le r \le f + 1)$
B-multicast$(g,\ Values_i^r - Values_i^{r-1})$; // Send only values that have not been sent
$Values_i^{r+1} := Values_i^r;$
while (in round r)
{
On B-deliver(V_j) *from some* p_j
$Values_i^{r+1} := Values_i^{r+1} \cup V_j;$
}

After $(f + 1)$ *rounds*
Assign $d_i = minimum(Values_i^{f+1});$

Consensus in a Synchronous System

- There are no arbitrary errors only processes that crash and are correctly detected
- Given a synchronous distributed systems (fail-stop model)
- Correctness
  - Assume that two processes p_i and p_j have different values at round r
  - Then, in round $r - 1$ at least one process p_k has sent different values to p_i and p_j
  - Then, p_k has crashed in this round
  - Since the number of crashes is limited to f there are not enough crashes to cover each of the $f + 1$ rounds

Byzantine Generals Problem in a Synchronous System

- Assume that there are **Byzantine** errors
- Given a synchronous distributed system
  - crashes are detected
  - other wrong behavior can not detected, e.g. strange messages
- messages are not (digitally) signed
- at most f faulty processes

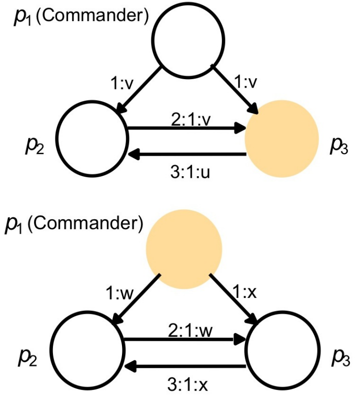
Impossibility of a solution of the Byzantine generals problem [Lamport, Shostak, Pease 1982]

- The byzantine generals problem cannot be solved for $n = 3$ and $f = 1$.
- The byzantine generals problem cannot be solved for $n \leq 3f$.

Byzantine Generals Problem in a Synchronous System

Impossibility of a solution of the Byzantine generals problem for $n = 3$

- The byzantine generals problem with arbitrary failures cannot be solved for $n = 3$ and $f = 1$ in a synchronous system.
  - a faulty commander sending different values to his generals
  - cannot be distinguished from a faulty general forwarding wrong values



Solution of the Byzantine Generals Problem

- Assume that there are **Byzantine** errors
- Given a synchronous distributed system
- messages are not (digitally) signed
- at most f faulty processes

Solution of the Byzantine generals problem [Pease, Shostak, Lamport 1980]

- The byzantine generals problem can be solved for $n = 4$ and $f = 1$.
- The byzantine generals problem can be solved for $n \geq 3f + 1$.

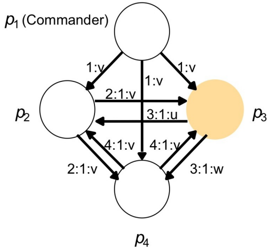
Solution for Four Generals and One Faulty Process

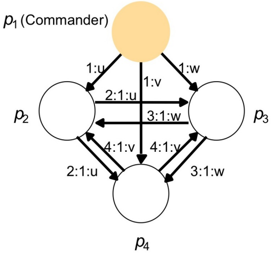
- The byzantine generals problem can be solved for $n \geq 4$ and $f = 1$.

Algorithm of Pease et al.

1. The commander sends a value to all other generals (lieutenants)
2. All lieutenants send the received value to all other lieutenants
3. The commander chooses its value; the lieutenants compute the majority of all received values

- Since $n \geq 4$ the majority function always can be computed if at most one process is faulty
- If the commander crashes very early then all lieutenants agree on $\perp$





More About the Byzantine Generals Problems

- For $f > 1$ the algorithm can be used recursively
  - Complexity: $f + 1$ rounds and $O(n^{f+1})$ messages
  - The time complexity of $f + 1$ rounds is optimal
- With the help of signed messages
  - any number of faulty generals $f < n$ can be dealt with
  - with signed messages the Byzantine Generals problem can be solved in $f + 1$ rounds with $O(n^2)$ messages [Dolev & Strong 1983]
- For asynchronous systems with crash failures
  - No algorithm can reach consensus even if only **one processor** is faulty [Fischer, Lynch, Paterson 1985]
  - Each algorithm that tries to reach consensus can be confronted with a faulty process which influences the result if it continues (instead of crashing)

End of Section 4